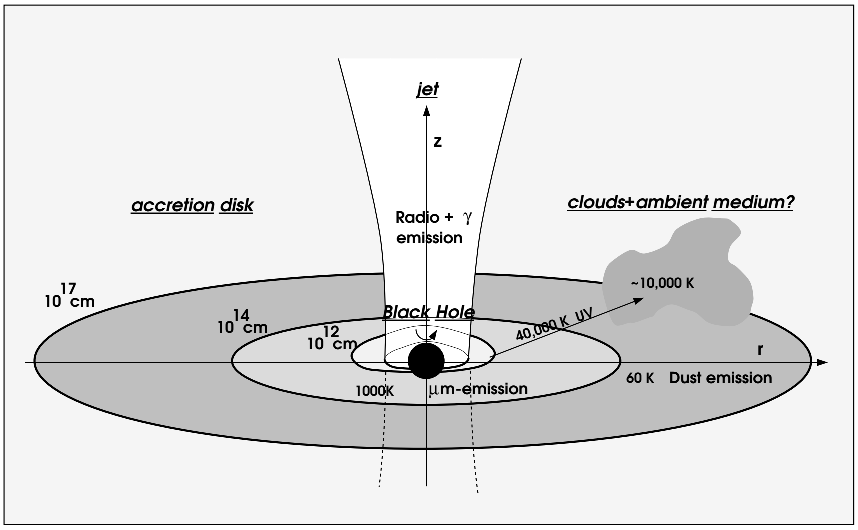
Physics of AGN
Accretion Disks

Heino Falcke
MPIfR Bonn



Contents:

- thin disk scenario
- α-viscosity
- emitted spectra
- effective temperature

Literature:
"An Introduction to Active Galactic Nuclei", Bradley M. Peterson, Cambridge University Press, Cambridge (Chap. 3)
"Active Galactic Nuclei", Ian Robson, John Wiley & Sons, Chichester (Chap. 7)
J. Frank, A.R. King, & D.J. Raine "Accretion Power in Astrophysics", 2nd edition, Cambridge University Press, Cambridge

Standard Accretion Disk

■ Basic Assumptions

Scenario: We assume that the luminosity (and the UV-bump) of AGN is produced by accretion of matter within an disk onto a black hole, where all dissipated energy is radiated locally.

Basic assumptions for the disk are:

- stationary ($\frac{\partial}{\partial t} = 0$)
- axial symmetry ($\frac{\partial}{\partial \phi} = 0$)
- geometrically thin (thickness $<$ 10% of radius)
- optically thick (radiation transport in z)
- gravitational potential dominated by central point mass

Basic Parameters:

- point mass $M_\bullet$
- mass accretion rate $\dot{m}$
- cylindrical coordinates: radius R, height Z

Fundamental papers:

– Weizsäcker, C. F. (1948): "Die Rotation kosmischer Gasmassen" *Z.Naturf.*,**3 a**: 524-539 (initial formulation)

– Lynden-Bell, D., Pringle, J. E. (1974): "The Evolution of viscous Discs and the Origin of the Nebular Variables" *Mon.Not.R.astr.Soc.*, **168**(3): 603-637 (re-discovery)

– Shakura, N. I., Sunyaev, R. A. (1973): "Black Holes in Binary Systems. Observational Appearance" *Astron.Astrophys.*,**24**: 337-355 (α-viscosity)

Standard Accretion Disk

■ Angular Momentum Transport

The biggest problem of an accretion disk is angular momentum transport!

Consider isolated rings of width ΔR in a disk around a point mass. The Kepler velocity is

$$v_\phi = \sqrt{\frac{GM_\bullet}{R}}.$$

The specific angular momentum (a.m. per unit mass) is

$$l = Rv_\phi = \sqrt{GM_\bullet R}.$$

$\Rightarrow$ angular velocity (v_ϕ/R) increases inwards (differential rotation)

$\Rightarrow$ specific angular momentum increases outwards

$\Rightarrow$ to move a ring further in its angular momentum has to be decreased

To transport mass in a disk towards the center angular momentum has to be transported outwards!

change of angular momentum = torque

Possible mechanisms to produce a torque:

- magnetic fields (Balbus & Hawley)
- MHD winds
- spiral waves (self-gravitation)
- tidal forces
- viscous friction

Standard Accretion Disk

■ Turbulent Viscosity

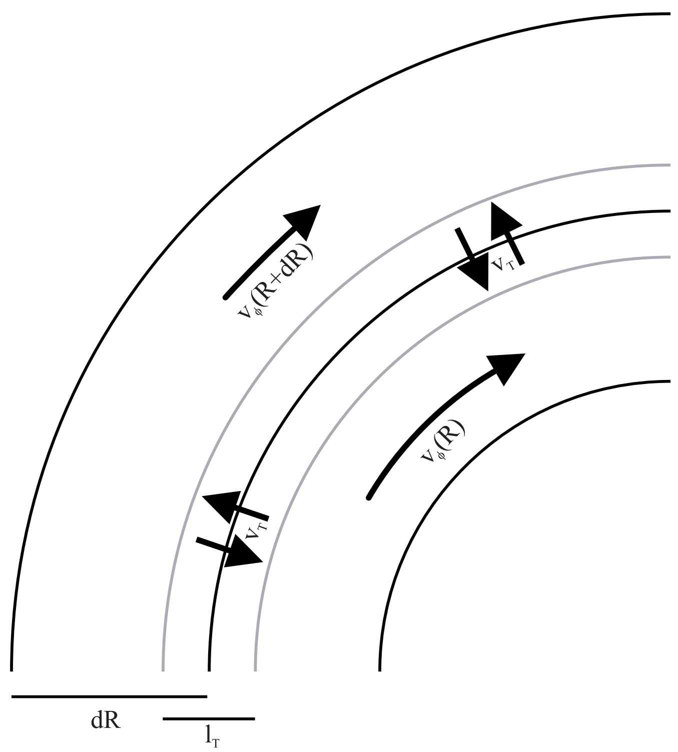
Angular momentum transport: if two neighbouring Keplerian rings are coupled by viscous forces, the outer, slower ring will brake the inner faster ring and thereby gain angular momentum while the inner disk looses it.

Energy dissipation: the friction between the rings will inevitably lead to energy dissipation.

angular momentum transport ⇔ energy dissipation

Molecular viscosity is too low by many orders of magnitude!

Postulate a **turbulent mass transport** between two rings with eddies of characteristic length l_{T} and velocity v_{T}—energy is transported in a cascade from large to small eddies where it is dissipated.



The net mass flow between both rings will be zero, however, when slower material is shuffled to a lower orbit it will interact and slow down the inner ring, while faster material put onto a larger orbit will speed up the outer ring.

Standard Accretion Disk

■ Turbulent Viscosity

The torque between two rings is given by $G = \frac{dL}{dt}$. Let

$$\Sigma(R) = \frac{dm}{2\pi R dR}$$

be the surface mass density of a ring and ω is the angular velocity v_ϕ/R.

Calculate now the angular momentum transport effect due to an even exchange of mass between the two rings. Ignore mass accretion for now, i.e. $\dot{m} = 0$, just concentrate on the micro-physics.
Angular momentum of the interaction region is

$$L = m v_\phi R = m\omega R^2 = 2\pi R l_{\mathrm{T}} \Sigma\left(R\right)\omega R^2$$

The change of angular momentum of this, i.e. the torque, is due to the change in ω only

$$G = \dot{L} = \frac{d\left(2\pi R l_{\mathrm{T}} \Sigma\left(R\right)\omega R^2\right)}{dt}$$

$$G = 2\pi R^3 l_{\mathrm{T}} \Sigma\left(R\right)\frac{d\omega(R)}{dR}\frac{dR}{dt}$$

Here we have $\frac{dR}{dt} = v_{\mathrm{T}}$

$$G = 2\pi R^3 l_{\mathrm{T}} v_{\mathrm{T}} \Sigma\left(R\right)\omega'(R)$$

The term $\nu = v_{\mathrm{T}} l_{\mathrm{T}}$ is called the **viscosity**.

$$G = 2\pi R^3 \nu \Sigma\left(R\right)\omega'(R)$$

For differential rotation in a Keplerian disk we have

$$\omega' = -\frac{3}{2}\sqrt{GM_\bullet}R^{-5/2}$$

and hence

$$G = -3\pi\sqrt{GM_\bullet R}\,\nu\Sigma$$

Standard Accretion Disk

■ Mass & Angular Momentum Conservation

Mass conservation: A stationary solution requires that

$$\dot{m} = -\frac{2\pi R\, dR\, \Sigma(R)}{dt} = -2\pi R v_r \Sigma(R) = \text{const}$$

$$v_{\mathrm{r}} = -\frac{\dot{m}}{2\pi R\Sigma}$$

Angular momentum conservation:

$$\dot{L} = R v_\phi \dot{m} = R v_\phi \cdot 2\pi R \Sigma v_{\mathrm{r}} = G - C$$

where G is the torque between the rings and C is the torque of the innermost ring (inner boundary condition).

If we assume that at the innermost orbit all inflowing mass and angular momentum are swallowed by the black hole we can set

$$C = \sqrt{GM_\bullet R_{\mathrm{in}}}\dot{m}$$

(this is most likely wrong, e.g. it ignores jet formation).

$$-2\pi\sqrt{GM_\bullet}R^{3/2}\Sigma v_{\mathrm{r}} = \sqrt{GMR_{\mathrm{in}}}\dot{m} + 3\pi\sqrt{GM_\bullet R}\nu\Sigma$$

Combining this with the mass conservation equation we get

$$2\pi\sqrt{GM_\bullet}R^{3/2}\Sigma \cdot \left(\frac{\dot{m}}{2\pi R\Sigma}\right) = \sqrt{GMR_{\mathrm{in}}}\dot{m} + 3\pi\sqrt{GM_\bullet R}\nu\Sigma$$

$$R^{1/2}\dot{m} = \sqrt{R_{\mathrm{in}}}\dot{m} + 3\pi\sqrt{R}\nu\Sigma$$

$$\dot{m} = \sqrt{\frac{R_{\mathrm{in}}}{R}}\dot{m} + 3\pi\nu\Sigma$$

$$\frac{\dot{m}}{3\pi}\left(1 - \sqrt{\frac{R_{\mathrm{in}}}{R}}\right) = \nu\Sigma$$

The product of viscosity and surface mass density is proportional to the mass accretion rate. For a constant accretion rate, an increasing viscosity will lead to a decreasing surface mass density of the disk and vice versa.

Standard Accretion Disk

■ Dissipation Rate

The viscosity between the rings will lead to energy dissipation at a rate

$$D = \frac{\Delta E}{\Delta t \Delta A}$$

This energy relation is directly related to the (differential) torque ($G = F \cdot R$) between the rings, since this implies that a force is acting between the (differential) rings, and

$$\Delta E = F \cdot s = F \cdot 2\pi R = 2\pi G$$

$$\Delta A = 2 \cdot 2\pi R \Delta R$$

$$\Delta t = \frac{2\pi}{\Delta \omega}$$

$$D = \frac{2\pi G}{8\pi^2 R}\frac{\Delta\omega}{\Delta R} = \frac{G\omega'}{4\pi R}$$

For Kepler rotation we have

$$\omega' = -\frac{3}{2}\sqrt{GM_\bullet}R^{-5/2}$$

and

$$G = -3\pi\sqrt{GM_\bullet R}\nu\Sigma$$

yielding

$$D = \frac{3\pi\sqrt{GM_\bullet R}\nu\Sigma \cdot \frac{3}{2}\sqrt{GM_\bullet}R^{-5/2}}{4\pi R}$$

$$D = \frac{9}{8}GM_\bullet \nu\Sigma R^{-3}$$

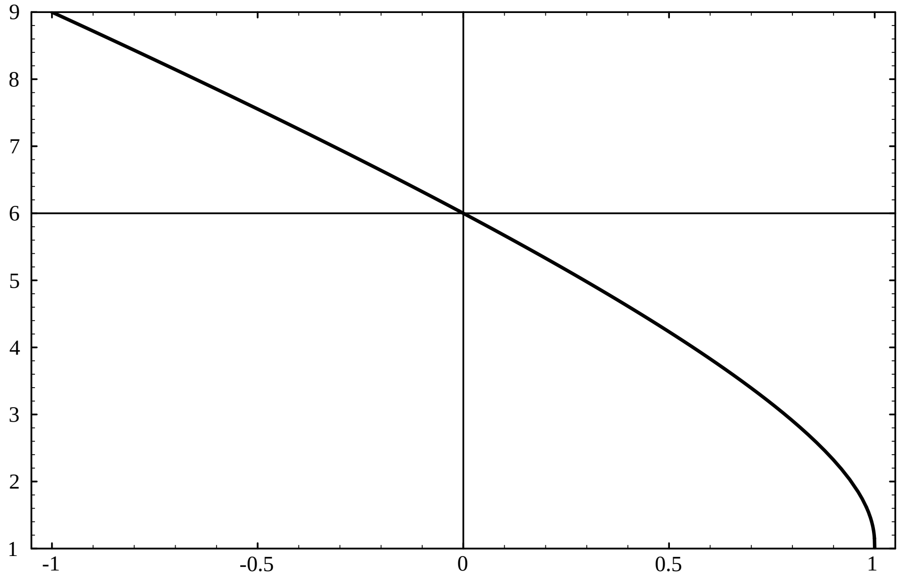
and with

$$\nu\Sigma = \frac{\dot{m}}{3\pi}\left(1 - \sqrt{\frac{R_{\rm in}}{R}}\right)$$

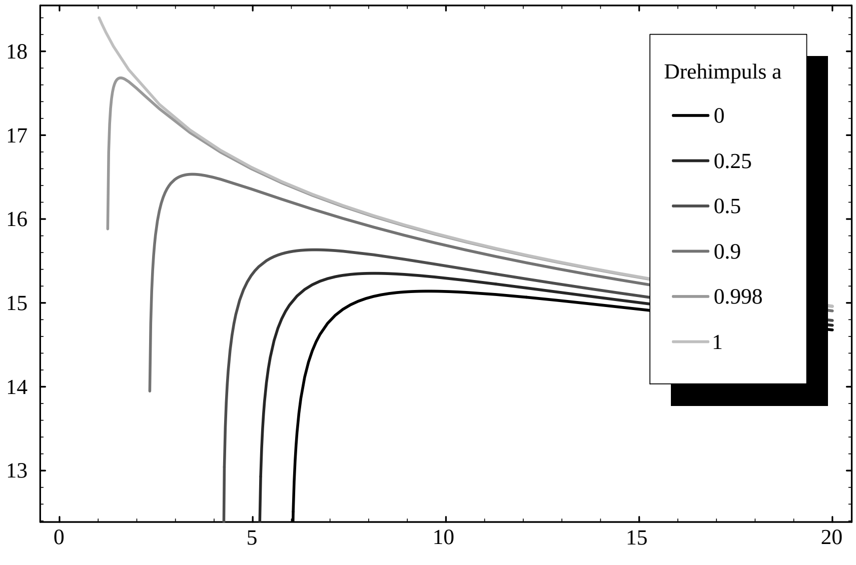
$$\Rightarrow D = \frac{3GM_\bullet \dot{m}}{8\pi R^3}\left(1 - \sqrt{\frac{R_{\rm in}}{R}}\right)$$

Standard Accretion Disk

■ Dissipation Rate



The inner-most, marginal stable orbit around a black hole as a function of its spin (x-axis: spin parameter a, y-axis: $R_{\mathrm{ms}}/R_{\mathrm{g}}$).



Dissipationrate of a stationary accretion disk around a Kerr hole for different spin parameters a (x-axis: R/R_{g}, y-axis: dissipation rate).

Standard Accretion Disk

■ Accretion disk luminosity

From the assumption of local dissipation and radiation of the dissipated energy and from the assumption of an optically thick disk we can calculate the luminosity and spectrum of the disk.

The total luminosity of the accretion disk outside an inner radius R_1 is given by integrating the dissipation rate over the two disk surfaces:

$$L = 2\int_{R_1}^{\infty} D\, 2\pi R\, dR$$

$$L = \frac{3}{2} G M_\bullet \dot{m} \int_{R_1}^{\infty} R^{-2}\left(1 - \sqrt{\frac{R_{\rm in}}{R}}\right) dR$$

$$D = \frac{3}{2} G M_\bullet \dot{m} R_1^{-1}\left(1 - \frac{2}{3}\sqrt{\frac{R_{\rm in}}{R_1}}\right)$$

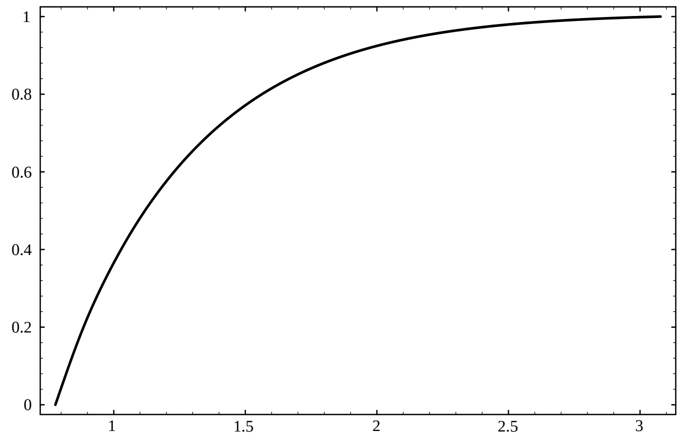
For $R_1 = R_{\rm in}$ we have

$$D = \frac{G M_\bullet \dot{m}}{2 R_{\rm in}}$$

and for $R_{\rm in} = R_{\rm g} = G M_\bullet / c^2$

$$D = \frac{1}{2}\dot{m} c^2$$

(but relativistic corrections will reduce this value slightly)



Integrated luminosity of an accretion disk around a Schwarzschild black hole for the range $R_{in} = 6R_{\rm g}$ to R as a function of R, normalized by the total luminosity, i.e. $r \to \infty$ (x-axis: $R/R_{\rm g}$, y-axis: $L_{\rm int}(R) = \int_{R_{\rm in}}^{R} 4\pi D(R)\, dr$).

Standard Accretion Disk

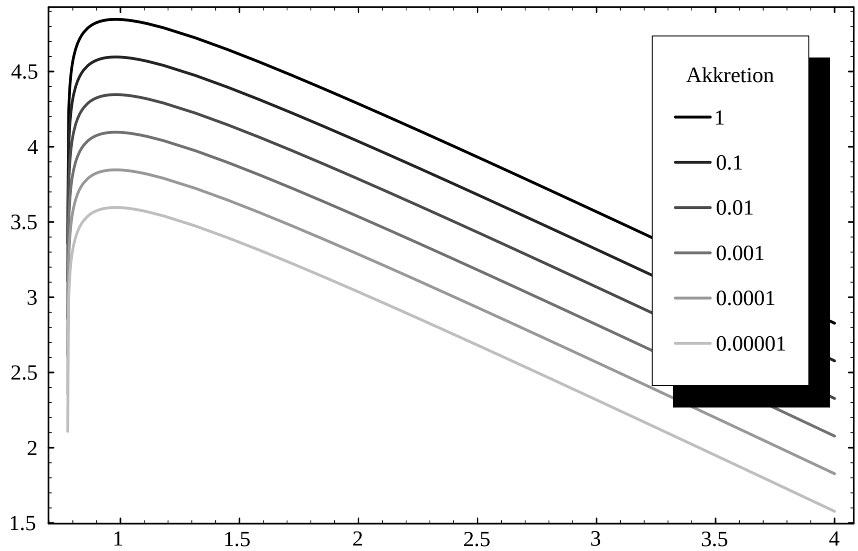
■ Effective Temperature

To first order we can assume that the disk radiates locally like a black body and we can use the Stefan-Boltzman law:

$$\sigma T_{\text{eff}}^4 = D$$

giving us the run of effective temperature along the disk

$$T_{\text{eff}} = \left(\frac{3GM_\bullet \dot{m}}{8\pi\sigma} \left(1 - \sqrt{\frac{R_{\text{in}}}{R}} \right) \right)^{1/4} R^{-3/4}$$



Effektive temperature of a stationary accretion disk around a Schwarzschild black hole for different (Eddington) accretion rates $\dot{m}_0/m_9$ (x-axis: R/R_{g}, y-axis: T_{eff} [K]).

■ Hertzsprung-Russell-Escher diagram

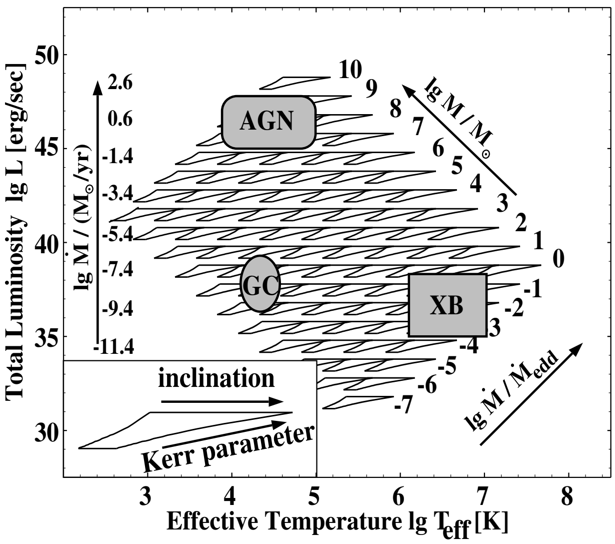
From the run of effective temperatures we can see that the highest temperature is reached near the innermost radius, e.g. for $R_{\rm in} = R_{\rm g}$ we have (ignoring the correction factor):

$$T_{\rm eff} = \left(\frac{3c^6\dot{m}}{G^2 M_\bullet^2 8\pi\sigma}\right)^{1/4}$$

$$T_{\rm eff} = 8.6 \cdot 10^5\,{\rm K}\ \left(\frac{M_\bullet}{10^8 M_\odot}\right)^{-1/2} \left(\frac{\dot{m}}{M_\odot/{\rm yr}}\right)^{1/4}$$

Hence, the temperature will decrease with increasing mass and increase with increasing accretion rate!

Unlike stars, accreting black holes have two important parameters determining $T_{\rm eff}$ and L: the central mass and the accretion rate. Therefore, the Hertzsprung-Russell diagram will turn from a one-dimensional main sequence into a plane. 'Minor' additional parameters: inclination angle, spin of BH



Standard Accretion Disk

■ Accretion disk spectrum

In order to obtain the accretion disk spectrum we simply integrate for each frequency over the whole disk, approximating the locally emitted spectrum once more by a black body.

$$I_\nu = \frac{2h\nu^3}{c^2\left(e^{h\nu/k_{\rm B}T_{\rm eff}(R)} - 1\right)}$$

$$T_{\rm eff} = \left(\frac{3GM_\bullet \dot{m}}{8\pi\sigma}\left(1 - \sqrt{\frac{R_{\rm in}}{R}}\right)\right)^{1/4} R^{-3/4}$$

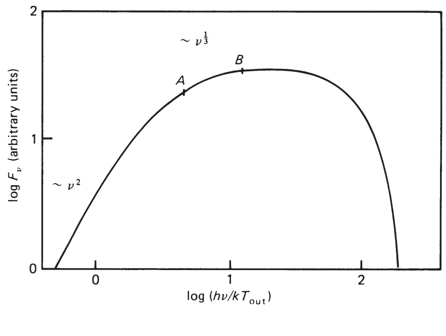
$$F_\nu = \frac{2\pi \cos i}{D}\int_{R_{\rm in}}^{R_{\rm out}} I_\nu R\, dR$$

$$F_\nu = \frac{4\pi \cos i \nu^3}{c^2 D^2}\int_{R_{\rm in}}^{R_{\rm out}} \frac{R\, dR}{e^{h\nu/k_{\rm B}T_{\rm eff}(R)} - 1}$$

This spectrum is independent of the viscosity, since here we only consider the locally dissipated potential energy. However, the exact disk structure (atmosphere or corona) might well modify the pure black body spectrum. The properties of the disk structure are a function of the viscosity.
The overall spectrum will consist of three parts:

- **low frequencies:** steep ν^2 rise due the black-body at $R_{\rm out}$
- **intermediate frequencies:** flat $\sim \nu^{1/3}$ spectrum due to superposition of black-bodies
- **high frequencies:** exponential cut-off due the black-body at $R_{\rm in}$

Figure 20. The continuum spectrum F_ν of a steady optically thick accretion disc radiating locally as a black body. Here the outer radius $R_{\rm out} = 250R_*$, and $T_{\rm out} = T(R_{\rm out})$. Points A and B have 'spectral index' $d(\ln F_\nu)/d(\ln \nu)$ equal to 0.38 and 0.28 respectively, showing the relative shortness of the '$\nu^{1/3}$' section of the continuum.



Standard Accretion Disk

■ Disk structure

The height of an accretion disk is simply determined by a hydrostatic pressure equilibrium.

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = g_z^{grav}$$

The z-component of the gravitational attraction (r-component is balanced by centrifugal force) is given by

$$g_z^{grav} = \frac{z}{\left(r^2 + z^2\right)^{\frac{3}{2}}} \approx \frac{z}{r^3}$$

If we express the pressure in terms of the local sound speed and mass density

$$P = \rho {c_s}^2$$

and replace the derivative by division through a scale height z_0, we get

$$z_0 = c_s r^{3/2} = \frac{c_s}{\omega}$$

i.e. the height of the disk is directly proportional to the local sound speed of the disk.

Unfortunately, the local sound-speed cannot be calculated easily since we need to know the surface density to calculate the vertical radiation transport through the disk. We know T_{eff}, i.e. the surface temperature, but the temperature in the optically thick part has to be much higher. The equations so far only give us $\nu\Sigma$ and we need a powerful prescription of the viscosity to solve for Σ and in turn for the other important properties of the disk.

Standard Accretion Disk

■ α-Parameter

As shown, the viscosity parameter is a product of a characteristic velocity and a characteristic lengths scale.
These scales are attributed to the scale of the largest eddies.
Shakura & Sunyaev (1973) therefore proposed the following parametrization:

$$\nu = \frac{2}{3} c_s z_0 \left(\frac{3 l_T v_T}{2 c_s z_0} \right) = \frac{2}{3} \alpha c_s z_0,$$

arguing that the lengths scale for eddies has to be smaller than the disk height and the velocity of the turbulence has to be smaller than the local sound speed.

Hence

$$\alpha = \frac{3 l_T v_T}{2 c_s z_0} \lesssim 1.$$

Even though it is by no means clear that α has to be a constant, it is assumed to be so for a first order approximation. This can be used to fully solve for the full accretion disk structure in the vertical direction.