Compact Radio Cores in AGN

■ Basics

How to calculate the spectrum of an astrophysical object from zero knowledge?

Define the basic energetics:
We assume that the internal energy density $U_{\rm j}$ of the jet is dominated by the magnetic energy density $U_{\rm B}$ and the energy density in relativistic particles $U_{\rm e+p}$.
A general starting point in astrophysics is the assumption of equipartition (sometimes just for the sake of parameterization), and one can set

$$U_{\rm e+p} = kU_{\rm B} = kB^2/8\pi, \quad k \lesssim 1$$
$$U_{\rm j} = (1+k)U_{\rm B}$$

Define the particle spectrum:
As usually observed in quasars, the relativistic particles are distributed as a power law in energy,

$$\frac{dN(\gamma)}{d\gamma} = K\gamma^{-p}$$

where we ignore the protons for simplicity. This is usually justified for calculating synchrotron radiation, but may be wrong when calculating high-energy emission (e.g., hadronic interactions can be quite important).

The total energy density in relativistic particles is then obtained by integrating over the energy distribution.

$$U_{\mathrm{e+p}} = K \int_{\gamma_1}^{\gamma_2} \gamma^{-p} \cdot \gamma m c^2 d\gamma \Rightarrow K = kB^2/(8\pi\Lambda m_{\mathrm{e}} c^2)$$

$$\lambda = \left[\begin{array}{c} (\gamma_1^{2-p} - \gamma_2^{2-p})/(p-2) \ \text{für } p \neq 2 \\ \ln(\gamma_2/\gamma_1) \ \text{für } p = 2 \end{array} \right.$$

$$(\gamma_1^{2-p} - \gamma_2^{2-p})/(p-2) \propto \left[\begin{array}{ll} \gamma_1^{2-p} & \text{für } p > 2 \\ \gamma_2^{2-p} & \text{für } p < 2 \end{array} \right.$$

The equipartition assumption provides us with the proper normalization constant when integrating over the entire energy range.

$$n_{\mathrm{e}} = K \int_{\gamma_1}^{\gamma_2} \gamma^{-p} d\gamma \simeq -K\gamma_1^{1-p}/(1-p) \ \ (\gamma_2 \gg \gamma_1)$$

$$\Rightarrow n_{\mathrm{e}} = \frac{kB^2}{8\pi\gamma_1^{p-1}\Lambda m_{\mathrm{e}} c^2 (p-1)}$$

Not all the particles are necessarily relativistic and contribute to visible (synchrotron) radiation. We suppose that the *total* particle density n is given by:

$$n = n_{\mathrm{e}}/x_{\mathrm{e}} \text{ and } x_{\mathrm{e}} \lesssim 1$$

However, we can arrive at the total particle density also on a different route, simply by specifying a certain mass loss rate. In this case the particle density is determined by the bulk velocity v, the radius of the outflow cylinder/cone r, and the mass outflow rate $\dot{M}$ of the jet. From mass conservation we get

$$\dot{M}_{\mathrm{j}} = m_{\mathrm{p}} n \pi r^2 v.$$

Combining the two equations for the density, yields a relation between the outflow rate and the magnetic field as a function of the scaling parameters

$$\dot{M}_\mathrm{j} = m_\mathrm{p}\pi r^2 v \frac{kB^2}{8\pi x_\mathrm{e}\gamma_1^{p-1}\Lambda m_\mathrm{e} c^2 (p-1)}.$$

This allows one to express the magnetic field as

$$B = \sqrt{\frac{8\pi x_\mathrm{e}\gamma_1^{p-1}\Lambda m_\mathrm{e} c^2 (p-1)\dot{M}_\mathrm{j}}{k m_\mathrm{p}\pi v}} \cdot r^{-1}$$

and we get for a power law index of $p \neq 2$

$$B = 0.3\,\mathrm{G}\sqrt{\frac{x_\mathrm{e}}{\beta k}\left(\frac{p-1}{p-2}\right)\left(\frac{\gamma}{100}\right)\left(\frac{\dot{M}_\mathrm{j}}{M_\odot/\mathrm{yr}}\right)}\left(\frac{r}{\mathrm{pc}}\right)^{-1}.$$

This is scaled to the situation of a jet in a luminous quasar. Sensible physical parameters are obtained by assuming that the dimensionless parameters are of order unity and the outflow rate is a small fraction of the inflow/accretion rate

$$\dot{M}_\mathrm{j} = q_\mathrm{m}\dot{M}_\mathrm{acc} \ \text{ and } \ q_\mathrm{m} \ll 1.$$

In many jet sources the fraction of outflowing gas is of order of a few percent.

For the Galactic Center, the inflow rate is determined to be $\sim 10^{-6} M_\odot/\mathrm{yr}$ and for a 1% efficiency for expelling mass, we are left with only $\dot{M}_\mathrm{j} = 10^{-8} M_\odot/\mathrm{yr}$ and at 10 $R_\mathrm{g} \sim 3.8 \cdot 10^{12}$ cm we obtain (1 pc $= 3 \cdot 10^{18}$ cm)

$$B = 30\,\mathrm{G}\sqrt{\frac{x_\mathrm{e}}{\beta k}\left(\frac{p-1}{p-2}\right)\left(\frac{\gamma}{100}\right)\left(\frac{\dot{M}_\mathrm{j}}{10^{-8} M_\odot/\mathrm{yr}}\right)}\left(\frac{r}{10 R_\mathrm{g}}\right)^{-1}$$

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■ Hydrodynamics

Pressure, density, sound speed, and energy density are related through the adiabatic index.

$$P = (\Gamma - 1)U$$

$$\rho c_{\mathrm{s}}^2 = \Gamma P = \Gamma(\Gamma - 1)U$$

We can treat the mix of relativistic particles and magnetic fields as a "photon gas" with adiabatic index $\Gamma = 4/3$.

$$c_{\mathrm{s}} = \sqrt{\rho^{-1}\Gamma(\Gamma - 1)U}$$

The energy density is set by the magnetic field and the mass density by the number of relativistic particles (divided by x_{e})

$$c_{\mathrm{s}} = \frac{2}{3}\sqrt{(m_{\mathrm{p}} n/x_{\mathrm{e}})^{-1}\,(1+k)\frac{B^2}{8\pi}}$$

The relativistic particle density is again related to the magnetic field through equipartition.

$$c_{\mathrm{s}} = \frac{2}{3}\sqrt{x_{\mathrm{e}}(1+k)\left(\frac{m_{\mathrm{p}} k B^2}{8\pi\gamma_1^{p-1}\Lambda m_{\mathrm{e}} c^2 (p-1)}\right)^{-1}\frac{B^2}{8\pi}}$$

$$\Rightarrow c_{\mathrm{s}} = \frac{2}{3}\sqrt{\frac{\gamma_1 m_{\mathrm{e}} c^2}{m_{\mathrm{p}}}\left(\frac{p-1}{p-2}\right)\left(\frac{1+k}{k}\right)x_{\mathrm{e}}}$$

A photon gas has a maximal sound speed of $c_{\mathrm{s}} = c\sqrt{\Gamma - 1} = c/\sqrt{3}$ (this is reached when the internal energy of electrons is larger than the rest mass of protons and start to dominate the total mass and energy). The limit of

$$c_{\rm s} \leq c/\sqrt{3} \sim 0.6c$$

gives and upper limit for the minimum electron energy, or the number of electrons

$$\Rightarrow x_{\rm e}\gamma_1 \lesssim \left(\frac{1}{3}\right)\left(\frac{9}{4}\right)\left(\frac{m_{\rm p}}{m_{\rm e}}\right)\left(\frac{p-2}{p-1}\right)\left(\frac{k}{(1+k)}\right)$$

This is simply a matter of energy conservation: one cannot increase the internal energy density indefinitely without significantly increasing the total energy (which is limited). At the maximum sound speed internal and bulk kinetic energy rival each other.

This limit translates into an energy limit for relativistic electrons

$$x_{\rm e}\gamma_1 \lesssim 344 \;\text{ für } p=3 \text{ and } k \lesssim 1.$$

Turned around, this means that a *maximal jet* with $c_{\rm s} = c/\sqrt{3}$, requires either $\gamma_1 \sim 300$ or $x_{\rm e} \sim 300$. The latter is only possible if one has more electrons than protons (i.e. in a pair-plasma dominated jet), or one has an electron distribution with a low-energy cut-off in the electron distribution around several hundreds ($\gamma_{\rm e} \sim 100$, $T_{\rm e} \gtrsim 10^{11}$ K).

■ Maximal Sound Speed

Meaning of maximal sound speed:
The relativistic enthalpy density of the jet is given by

$$U_{\mathrm{j}} = \gamma(\Gamma U) + \gamma\rho c^2$$

and because of $\rho c_{\mathrm{s}}^2 = \Gamma P = \Gamma(\Gamma - 1)U$

$$\Rightarrow U_{\mathrm{j}} = \gamma\frac{\Gamma\rho\beta_{\mathrm{s}}^2 c^2}{\Gamma(\Gamma - 1)} + \gamma\rho c^2$$

$$U_{\mathrm{j}} = \gamma\rho c^2\left(\frac{\beta_{\mathrm{s}}^2}{\Gamma - 1} + 1\right).$$

This means that for the maximal sound speed ($c_{\mathrm{s}} = \sqrt{(\Gamma - 1)}$ we have

$$U_{\mathrm{j}} = 2\gamma\rho c^2$$

and indeed the *internal* energy density rivals the rest mass energy density.

⇒ Efficient jets are always "light" (matter is photon-like)!

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■ Spectrum

Through the help of the previous equations we can express the particle density, magnetic field, and electron Lorentz factor as a function of dimensionless parameters and the total energy of the system. We can use this to calculate the jet spectrum.

We describe the jet as a freely expanding flow, beyond its collimation point (nozzle). The geometry of the jet is then given by the forward velocity $\gamma_{\rm j}\beta_{\rm j}c$ and the side-ways velocity, i.e. the sound-speed $\gamma_{\rm s}\beta_{\rm s}c$. An isothermal, non-accelerating jet will then simply take the shape of a cone. The half-opening angle will be given by the Mach cone

$$\sin\phi \sim \phi = \frac{1}{\mathcal{M}} = \frac{\gamma_{\rm j}\beta_{\rm j}}{\gamma_{\rm s}\beta_{\rm s}}.$$

The diameter of the jet as a function of the longitudinal coordinate is

$$\Rightarrow r = \sin\phi z \sim \frac{z}{\mathcal{M}}$$

and for a relativistic, maximal jet we have

$$\frac{1}{\mathcal{M}} = \frac{\gamma_{\rm j}}{\beta\sqrt{3}} \sim \frac{1}{10}\left(\frac{\gamma_{\rm j}}{6}\right) \simeq 6^\circ$$

For the particle density and the magnetic field, particle and energy conservation determine the scaling (which we also see from the previous equations)

$$n \propto z^{-2} \;\; \& \;\; B \propto z^{-1}$$

This determines the basic parameters and the geometry of the jet at each point beyond the nozzle and we can calculate an emission spectrum. The dominating process in the radio is synchrotron. The absorption coefficient is given for $p = 2$ as

$$\alpha = 4.5 \cdot 10^{-12}\mathrm{cm}^{-1} \frac{k}{\ln(\gamma_2/\gamma_1)} \left(\frac{B}{\mathrm{G}}\right)^4 \left(\frac{\nu}{\mathrm{GHz}}\right)^{-3}$$

(assuming a mean pitch angle of $\alpha \sim 54°$).
Writing this in a more compact form (with $B \propto z^{-1}$)

$$\alpha = k_1 b_1^4 z^{-4} \nu^{-3}$$

$$k_1 = 4.5 \cdot 10^{-12}\mathrm{cm}^{-1} \frac{k}{\ln(\gamma_2/\gamma_1)} GHz^{-3}$$

$$k_1 = 13.9 \cdot 10^{6}\mathrm{pc}^{-1} \frac{k}{\ln(\gamma_2/\gamma_1)} GHz^{-3}$$

$$b_1 = 0.3\,\mathrm{G} \sqrt{\frac{x_\mathrm{e} \ln(\gamma_2/\gamma_1)}{\beta k} (p-1) \left(\frac{\gamma}{100}\right) \left(\frac{\dot{M}_\mathrm{j}}{M_\odot/\mathrm{yr}}\right)}.$$

In an 'atmosphere' most of the radiation is usually emitted at the frequency and the location where the optical depth becomes unity (e.g., at the peak of the black body).
The optical depth is given by the absorption coefficient to be

$$\tau = 2r\alpha/\sin i$$

The $\tau = 1$ surface (i.e., the visual surface) can thus be determined:

$$1 = 2\frac{z}{\mathcal{M}} k_1 b_1^4 z^{-4} \nu^{-3} / \sin i$$

$$\Rightarrow z_{\tau=1} = \left(2 \frac{1}{\sin i \mathcal{M}} k_1\right)^{1/3} b_1^{4/3} \nu^{-1}$$

$$\Rightarrow z_{\tau=1} = 50\ pc \left(\frac{1}{\beta}\frac{k_1}{13.9 \cdot 10^6}\right)^{1/3} \frac{b_1}{0.3}^{4/3} \nu^{-1}$$

This means frequency and size are inversely related to each other. One never looks at the same position at different frequencies!

Note: $z_{\tau=1} \propto \nu^{-1}\dot{M}^{2/3}$ and for $\dot{M} \sim 0.03$ and $\nu = 10$ GHz $\Rightarrow z \sim 0.5$ pc. At z=0.5, d=1.5 Gpc, i.e., 0.5pc∼0.5mas, which is just the limit for the resolution of VLBI experiments.

The synchrotron emissivity is given as

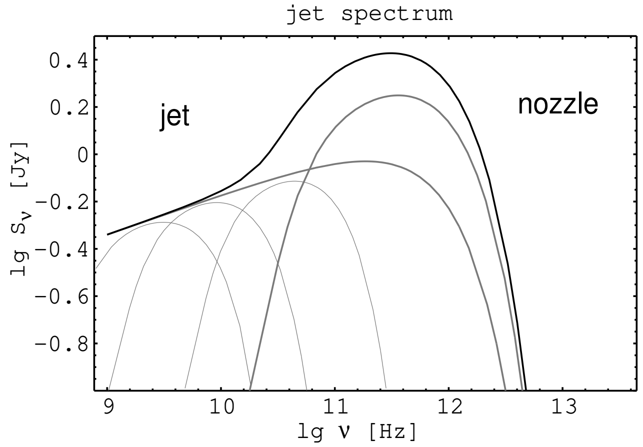
$$\epsilon = 5.5 \cdot 10^{-19} \frac{\mathrm{erg}}{\mathrm{s\,cm^3\,Hz}} \left(\frac{k}{\ln(\gamma_2/\gamma_1)}\right) \left(\frac{B}{\mathrm{G}}\right)^{3.5} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-0.5}$$

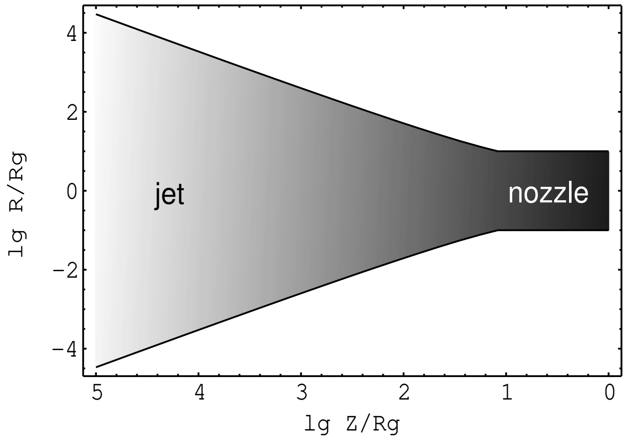
or in a compact form as

$$\epsilon = k_2 b_1^{3.5} r^{-3.5} \nu^{-0.5}$$

$$k_2 = 5.5 \cdot 10^{-19} \frac{\mathrm{erg}}{\mathrm{s\,cm^3\,Hz}} \left(\frac{k}{\ln(\gamma_2/\gamma_1)}\right) GHz^{0.5}$$

$$k_2 = 1.6 \cdot 10^{37} \frac{\mathrm{erg}}{\mathrm{s\,pc^3\,Hz}} \left(\frac{k}{\ln(\gamma_2/\gamma_1)}\right) GHz^{0.5}$$





To get the total emission spectrum we integrate the emissivity at a given frequency ν in slices along the entire jet, starting at $z_{\tau=1}$:

$$L_\nu = 4\pi \int_{z_{\tau=1}}^{\infty} \epsilon(z)\pi r^2(z)/\sin(i)dz$$

$$\Rightarrow L_\nu = 4\pi^2 \mathcal{M}^{-2} \sin^{-1}(i) k_2 b_1^{3.5} \mathcal{M}^{3.5} \nu^{-0.5} \int_{z_{\tau=1}}^{\infty} z^{-3.5} z^2 dz$$

$$\Rightarrow L_\nu = 4\pi^2 \sin^{-1}(i) k_2 b_1^{3.5} \mathcal{M}^{1.5} \nu^{-0.5} z_{\tau=1}^{-0.5}$$

since $z_{\tau=1} \propto \nu^{-1} \Rightarrow$ ν cancels and we obtain a 'flat' spectrum $S_\nu \propto \nu^0$.

A flat radio spectrum is a natural signature of a freely expanding outflow.

To obtain the absolute flux level one needs to divide the luminosity by $4\pi D^2$ and then specify the total power in the system:

$$b_1 \propto \sqrt{\dot{M}} \quad z_{\tau=1} \propto \dot{M}^{2/3}$$

$$\Rightarrow S_\nu \propto \dot{M}^{3.5/2} \dot{M}^{-1/2 \cdot 2/3} \propto \dot{M}^{1.75-0.33} \propto \dot{M}^{1.42}$$

This non-linear scaling of the radio flux with accretion rate/power remains true until the radiation becomes a significant source of cooling (which we have ignored so far). From this point on, an increase in power can at best lead to a linear increase in jet luminosity.