Physics of AGN
Schock Acceleration

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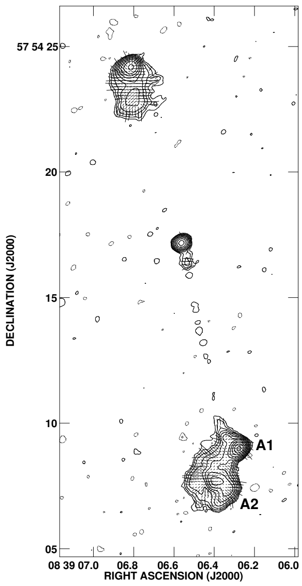

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- Shock
- Energy Gain
- Escape
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Literature:
"An introduction to the theory of diffusive shock acceleration of energetic particles in tenuous plasmas", L O'C Drury, Rep. Prog. Phys., Vol. 46, 973 (1983)
"Particle acceleration at astrophysical shocks: a theory of cosmic ray origin", R. Blandford & D. Eichler, Phys. Rep. 154, 1 (1987)
"High energy astrophysics", M.S. Longair, Cambridge University Press (1981)

Shock Acceleration

■ Outline

How do particles in AGN achieve such high energies and are redistributed into a power law.

The possible answer: first and second order Fermi acceleration or diffusive shock acceleration (Fermi invented this for the acceleration of cosmic rays in colliding ISM clouds).

Outline of Physical Mechanism:

- decelleration of a supersonic flow produces a thin shock
- in the shock frame this produces a converging flow
- particles scatter repeatedly across the shock
- each shock crossing cycle results in an energy gain
- the total energy gain is limited by the finite escape probability of particles from the shock region

Shocks

■ Basic Structure

When a flow moves supersonically the information of a decelleration in the flow velocity cannot travel **upstream**, since the signal speed is slower than the flow speed.

Consequently particles will “bump” into one another and the decelleration will happen almost instantly.

For a collisionless, magnetized plasma the size of the shock region is given by the gyro radius.

$$r_\mathrm{g} = \frac{\gamma_\perp \beta_\perp m c^2}{eB} \quad \text{(cgs)}$$

$$r_\mathrm{g} = 3 \times 10^6\,\mathrm{cm} \cdot \gamma_\perp \beta_\perp \left(\frac{B}{\mathrm{Gauss}}\right) \quad (m = m_\mathrm{p})$$

A shock front is a surface of discontinuity across which there is a steady flow of mass, momentum, and energy

Important paramters:

- velocities $u_1,\ u_2$
- mass densities $\rho_1,\ \rho_2$
- particle numbers $N_1,\ N_2$
- energy density $\varepsilon_1,\ \varepsilon_2$
- pressure p_1, p_2
- enthalpy density $h_{1,2} = p_{1,2} + \varepsilon_{1,2}$

(Subscripts refer to up- and downstream parts of the flow.)

Simplification of a strong shock: thermodynamics of upstream region is dominated by the shock (e.g. ram pressure $\gg$ internal pressure)

$$\Rightarrow p_1 = 0,\ \varepsilon = 0$$

$\boxed{\text{Shocks}}$

■ Conservation laws

Particle conservation (consider mass flow through cylinder):

$$\dot{N} = \pi r^2 u_{1,2} \rho_{1,2} / m_\mathrm{p} = \mathrm{const}$$

Define mass flux density as: $\dot{\Sigma}_\mathrm{m} = m_\mathrm{p} \dot{N} / (\pi r^2) = u_{1,2} \rho_{1,2}$

$$\Rightarrow \rho_1 u_1 = \rho_2 u_2 = \dot{\Sigma}_\mathrm{m}$$

Momentum conservation:

$$F = \dot{P} = (\pi r^2)(\dot{\Sigma}_\mathrm{m} u_{1,2} + p_{1,2}) = \mathrm{const}$$

$$\Rightarrow \dot{\Sigma}_\mathrm{m} u_1 = \dot{\Sigma}_\mathrm{m} u_2 + p_2 \Leftrightarrow \rho_1 u_1^2 = \rho_2 u_2^2 + p_2$$

$$\Rightarrow \dot{\Sigma}_\mathrm{m}(u_1 - u_2) = p_2$$

(ρu^2 corresponds to a ram pressure)

Energy conservation:

$$\frac{1}{2}\dot{\Sigma}_\mathrm{m} u_1^2 = \frac{1}{2}\dot{\Sigma}_\mathrm{m} u_2^2 + u_2 h$$

(p/ρ corresponds to PV work)

$$\Rightarrow \frac{1}{2}\dot{\Sigma}_\mathrm{m}(u_1^2 - u_2^2) = u_2 h$$

$$\Rightarrow \frac{1}{2}\dot{\Sigma}_\mathrm{m}(u_1 - u_2)(u_1 + u_2) = u_2 h$$

from momentum conservation follows

$$\Rightarrow \frac{1}{2} p_2 (u_1 + u_2) = u_2 h$$

and algebraic transformation yields

$$\Rightarrow (u_1 + u_2) p_2 = 2 u_2 (\varepsilon_2 + p_2)$$

■ Rankine-Hugoniot Relations

The jump conditions then yield

$$\Rightarrow \frac{u_1}{u_2} + \frac{u_2}{u_2} = 2\frac{\varepsilon_2}{p_2} + 2\frac{p_2}{p_2}$$

$$\Rightarrow R = 2\frac{\varepsilon_2}{p_2} + 1$$

Here we define the compression ratio

$$R = \frac{u_1}{u_2}$$

which only depends on the hydrodynamic quantities of the flow. The equation essentially details what fraction of the bulk kinetic energy of the flow is converted into internal energy.

From the mass conservation equation we know that

$$\rho_1 u_1 = \rho_2 u_2 \Rightarrow \frac{\rho_2}{\rho_1} = R$$

For a gas with adiabatic index Γ we have $p = (\Gamma - 1)\varepsilon$ and hence we get

$$R = \frac{2}{(\Gamma - 1)} + 1.$$

For a relativistic (photon) gas we have $\Gamma = 4/3$ and for a monatomic, non-relativistic gas we have $\Gamma = 5/3$, i.e. compression ratios of $R = 7$ and $R = 4$ respectively.

Relativistic Particles

■ Diffusive Scattering

Consider a situation where particles are gyrating in a magnetized plasma with a largely homogenous magnetic field.
The particles will gyrate around the fieldlines with a pitch angle θ and $\mu = \cos\theta$ – they follow a helical path.

$$\mu = \frac{\vec{p} \cdot \vec{B}}{|\vec{p}| \left|\vec{B}\right|}$$

The momenta are given by

$$p_{||} = \mu p \quad \text{and} \quad p_{\perp} = \sqrt{1 - \mu^2} p$$

and the corresponding gyroradius is

$$r_{\mathrm{g}} = \frac{\gamma_{\perp} \beta_{\perp} m c^2}{eB}.$$

For an ideal conductor as an astrophysical plasma, the electric field is identical zero and particles cannot gain energy, i.e. their energy is (largely) conserved.
Waves and irregularities can, however, scatter the particle and change the pitch angle. This does **not** change the particle energy or momentum.
After sufficient scatterings the distribution of pitch angles will be completely randomized, yielding an isotropic particle distribution.
Scattering is most effective when the MHD waves (or irregularities) are those of the length scale of the gyroradius of a particle.
The particles will diffuse in the plasma with a diffusion coefficient

$$\kappa \simeq \frac{1}{3} r_{\mathrm{g}} v$$

which is the 'Bohm' diffusion coefficient, which is the smallest diffusion coefficient in a completely randomized B-field.
The diffusion equation for the isotropic particle distribution is simply (here only one-dimensional)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial}{\partial x} f \right).$$

Relativistic Particles

■ Energy Gain

Assume basic geometry of the shock: B-field is parallel to the shock front, i.e. vertical to the shock normal.
Particles will diffuse across the shock. Upstreams properties: momentum p, pitch μ, velocity v.
Transform to shock frame:

$$p_{\text{shock}} = mv + \mu m \Delta u = mv\left(1 + \mu \frac{u_1 - u_2}{v}\right)$$

(ignore pitch angle transformation and anisotropy caused by shock)
The average change in momentum is given by integrating this factor over all angles:

$$\langle \Delta p \rangle = p \int_0^1 \mu \frac{u_1 - u_2}{v} 2\mu d\mu$$

(the probability of crossing the shock is proportional to $\cos\theta$ and for an isotropic distribution $1 \geq \mu \geq -1$ we get a weighting factor of 2μ.)

$$\Rightarrow \langle \Delta p \rangle = \frac{2}{3} p \frac{u_1 - u_2}{v}$$

We obtain the same answer for the other direction since u_1 and u_2 are interchanged and μ runs from -1 to 0.
Hence, the mean energy/momentum gain per **cycle** is

$$\frac{\langle \Delta p \rangle}{p} = \frac{4}{3} \frac{u_1 - u_2}{v}$$

(for most astrophysical situations $v \sim c$)

Relativistic Particles

■ Stochastic Accelerations and Leaky Box

While the particles have a certain probability to cross the shock repeatedly and gain energy in every cycle, they also have a certain probability of leaving the shock region ("leaky box").
The velocity with which particles are leaving the shock region is given by the post-shock velocity u_2. They are advected with the flow. The particle loss flux density is then given by

$$nu_2.$$

Of course, particles will move back and forth into the downstream region from upstream with their velocity v, on average:

$$\langle v\rangle = \frac{1}{2}\int_0^1 n\mu v d\mu = \frac{nv}{4}$$

(one half of particles are moving to the right with average velocity $v/2$).
The probability of not returning is then:

$$\mathcal{P} = \frac{nu_2}{nv/4} = \frac{4u_2}{v} \ll 1 \quad (\text{für } u_2 \ll v)$$

(i.e., many particles cross the shock many times)

A particle of initial momentum p_0 that crosses the shock region $n_{\rm sc}$ times gains each time a factor $\frac{4}{3}\frac{u_1-u_2}{v}$, i.e. the momentum p_n after $n_{\rm sc}$ cycles is

$$p_n \sim \Pi_{i=1}^{n_{\rm sc}}\left(1+\frac{4}{3}\frac{u_1-u_2}{v_i}\right)p_0$$

(since this is a stochastic process, this is never exactly true)
yielding

$$\ln\left(\frac{p_n}{p_0}\right) \sim \Sigma_{i=1}^{n_{\rm sc}}\ln\left(1+\frac{4}{3}\frac{u_1-u_2}{v_i}\right)$$

since $\ln(1+\epsilon)\sim\epsilon$ for $\epsilon \ll 1$ (First order Taylor expansion around $x=1$: $\ln x \to x-1$) we have

$$\ln\left(\frac{p_n}{p_0}\right) \sim \Sigma_{i=1}^{n_{\rm sc}} \frac{4}{3}\frac{u_1 - u_2}{v_i} = \frac{4}{3}\left(u_1 - u_2\right)\Sigma_{i=1}^{n_{\rm sc}}\frac{1}{v_i}$$

Similarly, the probability of not leaving the acceleration process is

$$\mathcal{P}_n \sim \Pi_{i=1}^{n_{\rm sc}}\left(1 - \frac{4u_2}{v_i}\right)$$

$$\to \ln\mathcal{P}_n \sim -4u_2\Sigma_{i=1}^{n_{\rm sc}}\frac{1}{v_i}$$

Inserting the above derived relation for the acceleration (Σv_i^{-1}) we get

$$\ln\mathcal{P}_n \sim -4u_2\frac{\ln\left(\frac{p_n}{p_0}\right)}{\frac{4}{3}u_1 - u_2}$$

$$\Rightarrow \ln\mathcal{P}_n \sim \frac{-3u_2}{u_1 - u_2}\ln\left(\frac{p_n}{p_0}\right)$$

$$\Rightarrow \mathcal{P}_n \sim \left(\frac{p_n}{p_0}\right)^{\frac{-3u_2}{u_1-u_2}}$$

The number density per momentum at a given momentum p_n is then simply the original number density times the probability to reach such a momentum divided by the momentum.

$$\frac{dN(p_n)}{dp} \sim N_0\mathcal{P}_n$$

Hence the particle distribution is a power law with functional form

$$\frac{dN}{dp} \sim \left(\frac{p_n}{p_0}\right)^{-p}$$

where

$$p = \frac{3u_2}{u_1 - u_2} + 1.$$

The powerlaw is a consequence of the fact that with every cycle the energy is increased by a fixed factor (a fixed step in the log-plane) and number of particles is reduced by a fixed factor (a fixed step in the log-plane). Hence in a log-log plane one gets a linear functional dependence!

To insert realistic numbers, we can express the power-law index by the compression ratio:

$$p = \frac{3}{R-1} + 1$$

and for a monoatomic gas with $\Gamma = 5/3$ and $R = 4$ we obtain $p = 2$ and a spectral index for synchrotron radiation of

$$\alpha = -(p-1)/2 = -0.5.$$

This is very close to what is observed in the hotspots of radio galaxies. The coincidence is surprising since this is a non-relativistic treatment for a non-relativistic gas!

Radiation losses and inverse Compton losses will steepen the power-law index by 1 since the energy loss is proportional to the energy. The spectral index will steepen by 1/2.

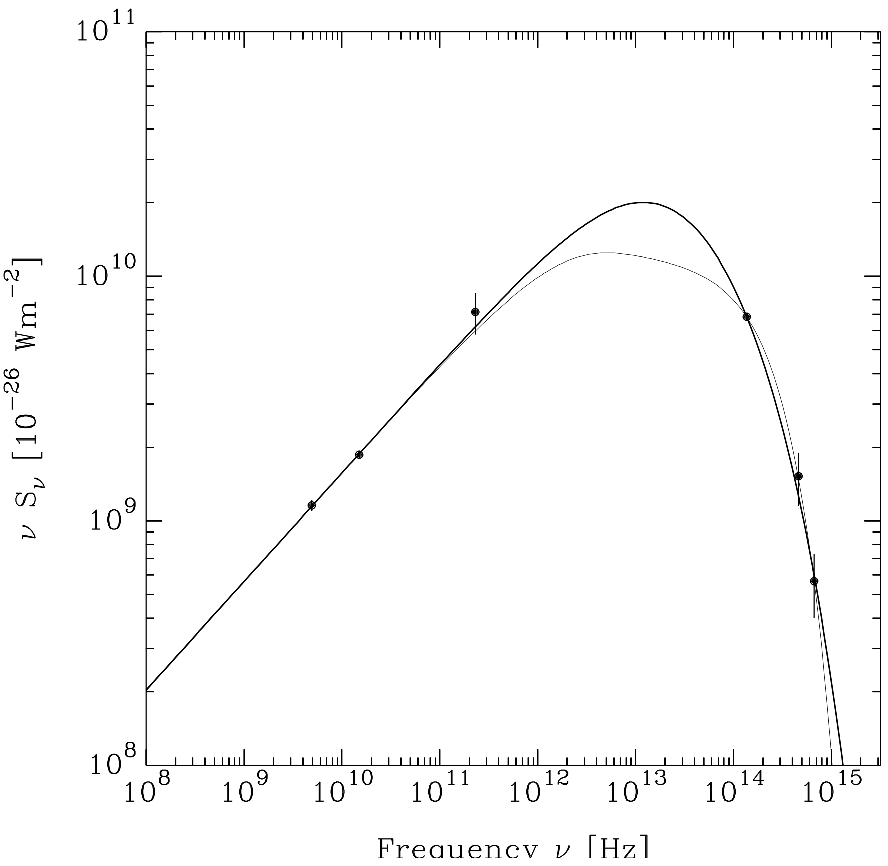
Hotspots of Jets

K. Meisenheimer et al.: The synchrotron spectra of radio hot spots. II

Table 5. Parameters from the synchrotron model fits.

Hotspot	Model	Spectral index α_0	Cutoff frequency ν_c [10^{14} Hz]	Energy ratio γ_c/γ_b	S_ν(5 GHz) [mJy]	$\chi^2/N_{\rm free}$	Data points
3C 20 west (B)	(i)	-0.56 ± 0.03	1.16 ± 0.04	10 ± 3	231 ± 6	0.55	6
	(ii)	-0.56 ± 0.03	1.83 ± 0.08	9.5 ± 1.2	231 ± 6	0.40	6
3C 33 south	(i)	-0.750 ± 0.010	2.99 ± 0.09	4.4 ± 0.8	611 ± 13	0.90	10
	(ii)	-0.745 ± 0.007	3.06 ± 0.08	10 ± 3	609 ± 13	2.3	10
3C 111 east	(i)	-0.520 ± 0.013	0.46 ± 0.11	< 2	267 ± 6	1.28	8
	(ii)	-0.527 ± 0.010	1.27 ± 0.03	4.5 ± 0.5	269 ± 6	0.90	8
3C 123 east	(i)	-0.45 ± 0.07	0.0060 ± 0.0040	6 ± 1	6100 ± 145	2.8	8
	(ii)	-0.42 ± 0.07	0.0026 ± 0.0005	5 ± 1	6130 ± 145	2.6	8
3C 303 west	(i)	-0.84 ± 0.08	18.2 ± 2.5	$< 2^a$	266 ± 8	0.31	9
	(ii)	-0.84 ± 0.07	11.5 ± 1.3	$< 2^a$	265 ± 8	0.37	9
3C 405 A	(i)	-0.51 ± 0.05	$0.09^{+0.10}_{-0.05}$	$40^{+20\ b}_{-15}$	37300 ± 300	0.62	14
3C 405 D	(i)	-0.45 ± 0.05	$0.08^{+0.04}_{-0.02}$	$25^{+11\ c}_{-9}$	49600 ± 500	0.46	14
3C 273 A	(i)	-0.60 ± 0.04	4.2 ± 0.3	290 ± 50	2110 ± 60	1.27	7
Pictor A west[d]	(i)	-0.740 ± 0.015	9.2 ± 0.2	< 2	2200 ± 20	0.66	10

[a] Deliberatly chosen since the solution is not unique, see text.
[b] The break frequency is well determined: ν_b = (5.5 ± 0.5) GHz.
[c] ν_b = (10.8 ± 1.0) GHz.
[d] Preliminary result, based on unpublished radio and K-band data.



The spectrum of the hotspot in 3C20 (west)

■ Maximum Energy

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Shock Acceleration

■ Complications

This was an extremely simplified treatment. Obvious things that need to be considered and which will change the shock acceleration process:

- oblique shocks
- back reaction of particles on shock
- non-planar geometry
- radiation losses
- injection process
- relativistic formulation
- excitation of MHD waves
- two fluids: relativistic plasma and background plasma
- 2nd order Fermi acceleration

Further applications of this theory:

- Supernovae
- Cosmic rays
- Gamma-ray bursts