Werkcollege, Sterrenstelsels, Week 4

These are the assignments for the fourth week of the course *Sterrenstelsels*. Every week, one of the problems provides credit towards the final exam. If at least 5 of these problems are handed in and approved, one question on the final exam may be skipped. The hand-in assignment for this week is **Problem 4.2** below.

4.1 Oort constants

The two Oort constants are defined as

$$A = -\frac{1}{2}\left[\left(\frac{\mathrm{d}v}{\mathrm{d}R}\right)_{R_0} - \frac{v_0}{R_0}\right] \tag{4.1.1}$$

$$B = -\frac{1}{2}\left[\left(\frac{\mathrm{d}v}{\mathrm{d}R}\right)_{R_0} + \frac{v_0}{R_0}\right] \tag{4.1.2}$$

for circular velocity $v(R)$. The subscript 0 denotes the corresponding quantities for a circular orbit with the same radius as the Sun's current distance from the Galactic centre, i.e. the "Local System of Rest" (LSR).

In the lecture we saw that the radial velocity of a star on a circular orbit with radius R, relative to the LSR, is given by

$$v_r = [\Omega(R) - \Omega_0]\, R_0 \sin l \tag{4.1.3}$$

where Ω is the angular frequency, $\Omega = v/R$ and l is the Galactic longitude. Near the Sun (i.e., for distance $d \ll R_0$) this can be approximated as

$$v_r \approx -\frac{1}{2}\left[\left(\frac{\mathrm{d}v}{\mathrm{d}R}\right)_{R_0} - \frac{v_0}{R_0}\right] d\,\sin 2l \tag{4.1.4}$$

and hence

$$v_r \approx A\, d\, \sin 2l \tag{4.1.5}$$

- Show that the equivalent expression for the *tangential* velocity (i.e. the velocity in the plane of the sky) is

$$v_t = [\Omega(R) - \Omega_0]\, R_0 \cos l - \Omega(R) d \tag{4.1.6}$$

- Show that the corresponding approximate expression for small d is

$$\begin{aligned} v_t &\approx -\left[\left(\frac{\mathrm{d}v}{\mathrm{d}R}\right)_{R_0} - \frac{v_0}{R_0}\right] d\,\cos^2 l - \frac{v}{R} d \\ &\approx -\left[\left(\frac{\mathrm{d}v}{\mathrm{d}R}\right)_{R_0} - \frac{v_0}{R_0}\right] d\,\cos^2 l - \frac{v_0}{R_0} d \end{aligned}$$

- Then show that, in terms of the Oort constants, this can be written as

$$v_t \approx Ad\cos 2l + Bd \tag{4.1.7}$$

  Hint: you may find the following trigonometric identities useful:
  $\cos^2 l + \sin^2 l = 1$
  $\cos^2 l - \sin^2 l = \cos 2l$.

4.2 Kinematic distances (***Homework***)

An observer is located in the disc of a spiral galaxy, 10 kpc from the centre of the galaxy. The galaxy has a flat rotation curve with $v_{\text{circ}} = 200$ km s^{-1}. At a galactic longitude of $l = 150°$, a star cluster with a radial velocity of -20 km s^{-1} is observed.

- Assuming that both the cluster and the observer are on circular orbits, calculate the distance from the centre of the galaxy to the cluster.
- Calculate the distance from the observer to the cluster