Werkcollege, Sterrenstelsels, Week 5

These are the assignments for the fifth week of the course *Sterrenstelsels*. Every week, one of the problems provides credit towards the final exam. If at least 5 of these problems are handed in and approved, one question on the final exam may be skipped. The hand-in assignment for this week is **Problem 5.3** below.

5.1 Motions of stars near the Sun

A star is currently observed at distance R_0 from the Galactic centre. Let $R_0 + \rho$ be the radial distance at which the centre of the star's epicycle is orbiting, where $\rho \ll R_0$.

- Show that the difference between the velocity of the star in the azimuthal direction and that of a star orbiting on a circular orbit at R_0, is

$$v_\chi(\rho) \approx -2B\rho$$

where B is the second Oort constant,

$$B = -\frac{1}{2}\left(\frac{\partial v_c}{\partial R} + \frac{v_c}{R}\right)$$

Hint: Use the conservation of angular momentum to calculate the azimuthal velocity of the star at R_0. Then use a first-order Taylor expansion of the rotation curve to express the circular velocity at $R_0 + \rho$ in terms of that at R_0.

5.2 Epicycle motion of the Sun

By analysing the motion of the Sun with respect to other stars, one finds that the Sun currently has an *inwards* velocity of 9 km/s towards the Galactic centre (i.e., $v_\rho = -9$ km/s) and a *forwards* velocity of 12 km/s with respect to the Local System of Rest ($v_\chi = +12$ km/s). The Sun completes one epicyclic oscillation in 170×10^6 years (this follows from the Oort constants: the epicyclic frequency is $\kappa^2 = -4B(A - B)$).

- Assume that the Sun is currently at the midpoint of its epicyclic oscillation. Estimate the amplitude of the Sun's excursion in the radial direction relative to a circular orbit. Does your result represent a minimum or maximum estimate of the actual deviation?

5.3 Chemical evolution

Recall that, in the closed-box model for chemical evolution, the change in metallicity δZ during a small time step is

$$\delta Z = -p\delta M_g/M_g \tag{5.3.1}$$

for yield p and gas mass M_g. If the yield is assumed to be constant and the initial metallicity of the system (when all the mass is in the form of gas) is $Z(t=0)=0$, then Eq. (5.3.1) implies

$$Z = -p \ln f_{\text{gas}} \tag{5.3.2}$$

for gas fraction f_{gas}.
In the more general case, we may define the *effective yield* as:

$$p_{\text{eff}} \mathrel{\hat{=}} -\, Z/\ln f_{\text{gas}} \tag{5.3.3}$$

Suppose the yield depends on metallicity in the following way:

$$p(Z) = 0.002 + 0.6\, Z \tag{5.3.4}$$

- Show that for $p(Z)$ given by Eq. (5.3.4), the effective yield is

$$p_{\text{eff}} = \frac{0.6Z}{\ln(1+300Z)} \tag{5.3.5}$$

  (assuming, as before, that $Z=0$ initially)