Werkcollege, Sterrenstelsels, Week 7

These are the assignments for week 7 of the course *Sterrenstelsels*.
Every week, one of the problems provides credit towards the final exam. If at least **five** of these problems are handed in and approved, one question on the final exam may be skipped. The hand-in assignment for this week is **Problem 7.3** below.

7.1 Chemical evolution with inflow

Adapted from Sparke & Gallagher, problem 4.10

Suppose that the inflow of metal-poor gas is proportional to the rate at which new stars form, so that

$$\delta M_s + \delta M_g = \nu \delta M_s \tag{7.1.1}$$

for some constant $\nu > 0$.

- *Verify Eq. (7.1.1)*
- *Show that S&G Equation 4.14*,

$$\delta Z = \delta\left(\frac{M_h}{M_g}\right) = \frac{p\delta M_s - Z\left[\delta M_s + \delta M_g\right]}{M_g} \tag{7.1.2}$$

becomes

$$\delta Z = \frac{p - \nu Z}{\nu - 1}\frac{\delta M_g}{M_g} \tag{7.1.3}$$

so that the metallicity of the gas is

$$Z = \frac{p}{\nu}\left[1 - \left(\frac{M_g(t)}{M_g(0)}\right)^{\nu/(1-\nu)}\right] \tag{7.1.4}$$

7.2 Two-body relaxation

- Chapter 3.2.2: Fill in the missing steps leading up to (3.50) and (3.51). You may find the following integral useful:

$$\int \frac{1}{(A + Bx^2)^{3/2}}\mathrm{d}x = \frac{x}{A\sqrt{A + Bx^2}} \tag{7.2.1}$$

We saw that the two-body relaxation time scale can be written as

$$t_{\mathrm{rel}} = \frac{V^3}{8\pi G^2 m^2 n \ln\Lambda} \tag{7.2.2}$$

for relative velocities V, particle mass m and particle density n. Being slightly more precise about the meaning of V, we rewrite this as

$$t_{\rm rel} = \frac{\langle V^2 \rangle^{3/2}}{8\pi G^2 m^2 n \ln \Lambda} \tag{7.2.3}$$

For a system in *virial equilibrium*, the velocity dispersion $\langle V^2 \rangle$, mass M and radius R of the system are related as

$$M = \eta \frac{\langle V^2 \rangle R}{G} \tag{7.2.4}$$

The constant η depends on the radial structure of the system and how exactly the radius R is defined (note the similarity of this relation to the corresponding relation for a circular orbit).

- For a system of uniform density, total mass M and radius R, show that the two-body relaxation time scale for a virialized system can be expressed as

$$t_{\rm rel} = \frac{M^{1/2} R^{3/2}}{6 G^{1/2} \eta^{3/2} m \ln \Lambda} \tag{7.2.5}$$

- With M and R defined as above, the constant $\eta = 5/3$. Now calculate $t_{\rm rel}$ for the following systems (you can assume a mean stellar mass of $m = 1 M_\odot$ and $\ln \Lambda = 10$ in all cases):
  - An open cluster of stars ($M = 10^3\, M_\odot$, $R = 3$ pc)
  - A globular cluster ($M = 10^6\, M_\odot$, $R = 10$ pc)
  - A giant elliptical galaxy ($M = 10^{12}\, M_\odot$, $R = 10$ kpc)

7.3 Spiral structure: Lindblad resonances

In the theory of spiral structure, the *global pattern speed* Ω_{GP} is the overall pattern speed of the spiral density wave. In general, this may differ from the *local pattern speed* for a two-armed spiral pattern, Ω_{LP}, that follows from the condition $\Omega_{\mathrm{LP}} = \Omega - \frac{1}{2}\kappa$. However, where the two are equal, we talk about an *Inner Lindblad Resonance*.

A spiral galaxy is observed to have a flat rotation curve with circular speed $v_c = 250$ km s^{-1} from near the centre of the galaxy to very large radii. The co-rotation radius of the spiral pattern is at $R = 5$ kpc.

- Calculate the location of the Inner ($m = 2$) Lindblad Resonance. To do this, you will need to know how to compute the epicyclic frequency from the rotation curve. This can be done using the Oort constants:

$$\kappa^2 = -4B(A - B) \tag{7.3.1}$$

  for Oort constants A and B (see chapter 3.3 in S&G).

- Assuming that the mass distribution is dominated by a spherical dark matter halo, calculate the density profile $\rho(R)$ of the halo.