ASSIGNMENTS Week 1 (F. Saueressig)
Cosmology 16/17 (NWI-NM026C)

Dr. S. Larsen and Dr. F. Saueressig

Exercise 1: The age of the universe

Consider a flat Friedman-Robertson-Walker universe populated by non-relativistic matter and a cosmological constant. In this case the evolution of the scale-factor $a(t)$ is governed by the equation

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \frac{1-\Omega_\Lambda}{a^3} \, . \tag{1}$$

Here $H \equiv a^{-1}\frac{da}{dt}$ is the Hubble parameter, H_0 denotes its value today, and Ω_Λ is the relative energy density of the cosmological constant.

a) By evaluating (1) today, corresponding to the time t_0, show that the scale factor as the canonical normalization $a_0 \equiv a(t_0) = 1$.

b) Rewrite eq. (1) as

$$dt = H_0^{-1} \frac{da}{a} \left[\Omega_\Lambda + \frac{1-\Omega_\Lambda}{a^3}\right]^{-1/2} \, . \tag{2}$$

c) Integrate (2) from $a = 0$ (when $t = 0$) until today at $a = 1$ to get the age of the universe today. Note that the integral can be done analytically.

d) Use the measured value of the Hubble parameter today,

$$H_0 = \frac{h}{9.8 \times 10^9 \, \text{years}} \tag{3}$$

with $h \approx 0.72$ to determine the age of a universe with $\Omega_\Lambda = 0$. Compare your result with the age of our milky way 13.2×10^9 years and the oldest observed star, SM0313, 13.6×10^9 years. What conclusion can be drawn from this comparison?

e) Compute the age of the universe for $\Omega_\Lambda = 0.7$. How does the inclusion of a cosmological constant change the conclusion drawn in part d)?

Exercise 2: Galaxy rotation curves

a) Use Newtonian dynamics to derive the circular velocity $v_c(r)$ of a star orbiting the center of a (spherical) galaxy at distance r

$$v_c(r) = \sqrt{\frac{G\,M(r)}{r}} \, . \tag{4}$$

Here $M(r)$ is the total mass sitting inside the sphere of radius r.

b) The Navarro-Frenk-White profile for dark matter halo's describes the mass-density distribution

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2} \,. \tag{5}$$

where ρ_0 and R_s are fit-parameters of the model. Construct $M(r)$ by integrating (5). Plot the resulting $v_c(r)$ for $\rho_0 = 1$, $R_s = 10$.

c) Repeat the construction for the Einasto-profile

$$\rho(r) = \rho_0 \, e^{-Ar^\alpha} \tag{6}$$

with $\rho_0 = 1$, $A = 2$ and $\alpha = 1$.

Exercise 3: Lyman α lines in deuterium

Using the fact that the reduced mass of the electron-nucleus system in deuterium (D) is larger than in hydrogen, and the fact that the Lyman α ($n = 2 \to n = 1$) transition in hydrogen (H) has a wavelength 1215.67 A, find the wave length of the photon emitted in the corresponding transition in deuterium. Astronomers often define

$$v \equiv c \, \frac{\Delta\lambda}{\lambda} \tag{7}$$

to characterize the splitting of two nearby lines. What is v for the H-D-pair?

Hints: This question should be addressed within the Bohr model. You may use the following numbers:

$$\begin{aligned} m_p &= 938.272 \, \mathrm{MeV\,c^{-2}} \\ m_D &= 1875.613 \, \mathrm{MeV\,c^{-2}} \\ m_e &= 0.511 \, \mathrm{MeV\,c^{-2}} \\ c &= 299792 \, \mathrm{km\,s^{-1}} \end{aligned} \tag{8}$$