Werkcollege, Cosmology 2016/2017, Week 4

These are the exercises and hand-in assignment for the 4th week of the course *Cosmology*. Every week, one of the problems provides credit towards the final exam. If at least **10** of these problems are handed in and approved, one problem on the final exam may be skipped. The hand-in assignment for this week is **Problem 4.2** below.

Flux and intensity

In the lecture we discussed the fundamental concepts *flux* (the energy passing through a unit surface per unit time) and *intensity* (flux per solid angle). In S.I. units, flux is expressed in units of [$\mathrm{W\,m^{-2}}$] and intensity in [$\mathrm{W\,m^{-2}\,sr^{-1}}$]. In many practical applications, it is more useful to specify the *flux density* (or *specific flux*), i.e., the flux per unit wavelength (F_λ) or frequency (F_ν) interval, and the corresponding quantities for the intensity (I_λ, I_ν). The flux density is often expressed in Jy, where 1 Jy = $10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$. The total flux is then, in principle, found by integration over all wavelengths (or frequencies):

$$F = \int F_\lambda \,\mathrm{d}\lambda \tag{4.0.1}$$

or

$$F = \int F_\nu \,\mathrm{d}\nu \tag{4.0.2}$$

In practice, measuring the specific flux at all wavelengths is difficult, and we sometimes also refer to the flux integrated over a specific wavelength region, such as a particular photometric band.

4.1 F_λ and F_ν

Show that F_ν and F_λ are related as

$$\frac{F_\nu}{F_\lambda} = \frac{c}{\nu^2} = \frac{\lambda^2}{c} \tag{4.1.3}$$

4.2 Flux of astronomical objects

The brightest star in the sky, Sirius, has a radius of about 1.75 $\mathrm{R}_\odot$ and a temperature of 9900 K. Its distance is 2.6 pc.

1. Approximating the spectrum of Sirius by a black-body with $T_{\mathrm{eff}} = 9900$ K, calculate the specific intensity I_λ of light emitted at 5500 Å (i.e. at the centre of the V-band)

2. Making the approximation that I_λ is constant over the wavelength range covered by the V-band, and assuming that the bandwidth is $\Delta\lambda_V = 900$ Å, what is the V-band intensity I_V of the light emitted by Sirius?

3. Calculate the V-band flux from Sirius measured above the Earth's atmosphere. (*Hint:* you may make use of the fact that the intensity of black-body radiation is independent of the viewing angle. The integral $\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = \frac{1}{2}$ might be useful).

4. How many V-band photons would enter the aperture of the Hubble Space Telescope per second if it were pointed at Sirius? Assume that HST has a circular aperture with a diameter of 2.4 m. You can also assume that all photons have the same energy, corresponding to $\lambda = 5500$ Å.

4.3 Flux, Magnitude and Surface Brightness

The flux density received from the star Vega (above the Earth's atmosphere) is $F_\nu = 3.6 \times 10^{-23}$ W m^{-2} Hz^{-1} in the visual region of the spectrum. Vega has a visual magnitude $m_V = 0$.

1. Calculate the visual magnitude of a source with a flux density of $F = 1$ Jy
2. The faintest stars visible to the unaided eye under a dark sky have visual magnitudes $V \approx 6$. Calculate the limiting sensitivity of the eye in Jy.
3. In astronomy, the term *surface brightness* is sometimes used instead of intensity. The natural night sky has an average visual surface brightness of about 22 mag arcsec^{-2} at new Moon (meaning that the flux received from one square arcsecond of blank sky is the same as that received from a 22nd magnitude star). Over how large an area of the sky does one need to integrate to get a flux similar to that of the faintest naked-eye stars?

Formulae and constants

Black-body radiation:

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Radius of the Sun: $R_\odot = 7 \times 10^8$ m

1 pc = 3.09×10^{16} m

Planck's constant: $h = 6.626 \times 10^{-34}$ m^2 kg s^{-1}

Boltzmann's constant: $k = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}