Werkcollege, Cosmology 2016/2017, Week 5

These are the exercises and hand-in assignment for the 5th week of the course *Cosmology*. Every week, one of the problems provides credit towards the final exam. If at least **10** of these problems are handed in and approved, one problem on the final exam may be skipped. The hand-in assignment for this week is **Problem 5.5** below.

5.1 Distance and distance modulus

Show that an error or uncertainty of 0.1 magnitudes in the distance modulus, $m - M$, is roughly equivalent to a 5% error in the distance, D

5.2 Moving cluster method

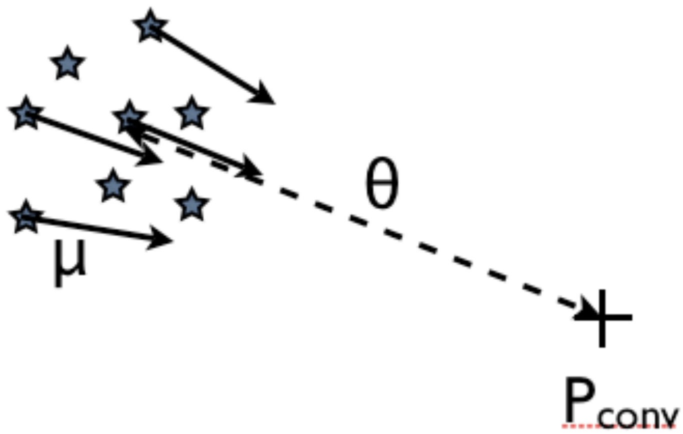

Fig. 1: The star cluster discussed in Problem 5.2.

A star cluster is observed to have a proper motion $\mu = 0.110'' \text{ yr}^{-1}$ and radial velocity $v_r = 40$ km s^{-1}. The proper motions of stars in the cluster appear to be converging towards the point $P_{\rm conv}$, located at an angle of $\theta = 30°$ from the centre of the cluster on the sky.

1. Calculate the distance to the cluster
2. What was the smallest distance between the Sun and the cluster, relative to the current distance?
3. When did the closest passage occur? Show that this can be calculated without knowing the distance of the cluster!
4. Assuming effects of stellar evolution and extinction are negligible, when will the apparent brightness of the cluster have decreased by 1 magnitude?

5.3 Cepheids

The relation between the mean apparent visual magnitude m_V and the period P (in days) for Cepheids in the Large Magellanic Cloud (LMC) is observed to be

$$m_V = -2.5 \log_{10} P + 17.0 \tag{5.3.1}$$

For the Galactic Cepheid δ Cep, a trigonometric parallax of 3.8×10^{-3} arcseconds is observed. δ Cep has $\log_{10} P = 0.73$ and a mean apparent magnitude $m_V = 3.8$.
In the following, assume that Cepheids everywhere follow a universal period-luminosity relation. You can ignore the effects of interstellar extinction (or, to put it differently, assume that all measurements have been corrected for this effect).

1. Find the distance to the LMC.
2. A Cepheid in the galaxy M100 has apparent mean magnitude $m_V = 27.1$ and period $P = 10$ days. Find the distance to M100.

5.4 Baade-Wesselink method

This exercise is taken from the book "Galactic Dynamics", J. Binney & M. Merrifield

A star expands in a spherically-symmetric manner with radial velocity v_r. Defining a spherical coordinate system on the surface of the star with the polar axis aligned along the line of sight, show that the measurable flux-weighted mean line-of-sight velocity will be

$$v_{\rm los} = v_r \frac{\int_0^{\pi/2} I(\theta) \cos^2\theta \sin\theta \, \mathrm{d}\theta}{\int_0^{\pi/2} I(\theta) \cos\theta \sin\theta \, \mathrm{d}\theta} \tag{5.4.1}$$

Hence show that, for a star of uniform brightness, $p = v_r/v_{\rm los} = 1.5$. In reality, a star will not appear uniformly bright: its opacity means that near the edge of the star (its "limb") one cannot peer so far into its atmosphere, so one sees the less bright outer layers. A reasonable analytic approximation to this **limb darkening** is given by $I(\theta) = I(0)(0.4 + 0.6\cos\theta)$. In this approximation, show that $p = 24/17$.

5.5 *K*-corrections

The K-correction is the difference between the observed magnitude $m_{\rm obs}(z)$ for a source at redshift z and the magnitude that would be observed if the source were at rest, $m_{\rm rest}$:

$$m_{\rm rest} = -2.5 \log_{10} \int f(\lambda) S(\lambda) \, \mathrm{d}\lambda + \text{const} \tag{5.5.1}$$

$$m_{\rm obs}(z) = -2.5 \log_{10} \int f(\lambda') S[\lambda'(1+z)] \, \mathrm{d}\lambda' + \text{const} \tag{5.5.2}$$

$$\tag{5.5.3}$$

In addition to the redshift z, the K-correction depends on the spectrum of the source (here expressed as a function of wavelength, $f[\lambda]$) and the spectral response of the system used for the observations, $S(\lambda)$. The K-correction is a purely instrumental effect that simply accounts for the fact that light emitted at wavelength λ' is observed at wavelength λ. It does *not* take into account the cosmological effects of the redshift due to the expansion of the Universe.

1. Show that (5.5.2) and (5.5.1) lead to the following expression for the K-correction:

$$K = m_{\rm obs} - m_{\rm rest} \tag{5.5.4}$$

$$= 2.5 \log_{10} \frac{\int f(\lambda) S(\lambda)\,\mathrm{d}\lambda}{\int f[\lambda/(1+z)] S(\lambda)\,\mathrm{d}\lambda} + 2.5 \log_{10}(1+z) \tag{5.5.5}$$

2. Find and write down the equivalent expression for the K-correction in terms of the spectrum as a function of *frequency*, $f(\nu)$

3. Calculate the K-correction for a source with a power-law spectrum, $f(\lambda) \propto \lambda^{\beta}$. To simplify the calculations, you can approximate the bandpass transmission curve $S(\lambda)$ as a box function, i.e., assume that $S(\lambda)$ is a (positive) constant for $\lambda_1 < \lambda < \lambda_2$ and zero elsewhere.

Formulae and constants

Distance modulus (D in pc):

$$m - M = 5 \log_{10} D - 5$$

Black-body radiation:

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Radius of the Sun: $R_\odot = 7 \times 10^8$ m

1 pc = 3.09×10^{16} m

Planck's constant: $h = 6.626 \times 10^{-34}$ m^2 kg s^{-1}

Boltzmann's constant: $k = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}