Werkcollege, Cosmology 2016/2017, Week 7

These are the exercises and hand-in assignment for the 8th week of the course *Cosmology*. Every week, one of the problems provides credit towards the final exam. If at least **10** of these problems are handed in and approved, one problem on the final exam may be skipped. The hand-in assignment for this week is **Problem 7.2** below.

7.1 Rotation of "Spiral Nebulae"

In 1914, V. M. Slipher deduced from spectroscopic observations of the Sombrero galaxy (NGC 4594) a rotational velocity of about 100 km/s (at $20''$ from the nucleus). Slipher had also measured positive radial velocities for many spiral "nebulae", often several hundred km/s.

Around the same time, Adriaan van Maanen compared several images of M101 taken over a period of about 15 years and measured an annual rotation of $0.022''$ at a distance of $5'$ from the centre (meaning that, according to van Maanen's measurement, a point located $5'$ from the centre would move $0.022''$ in a year). Van Maanen's measurement was used by Harlow Shapley in the "great debate" as one argument against the idea that spiral nebulae are external galaxies similar to the Milky Way.

Let us now explore some of the implications of these measurements:

1. Based on van Maanen's measurement, what is the rotation period of M101 (in years)?

2. Shapley had estimated that the Sun is located about 15 kpc from the centre of the Milky Way. If the Sun is orbiting around the centre of the Milky Way with the same period as van Maanen's measurement implied for M101, what would be the speed of the Sun? In km/s? In units of c, the speed of light? Would you agree with Shapley that this is unreasonable?

3. If, on the other hand, M101 rotates as fast as NGC 4594 (100 km/s), what would be the distance of M101? Does this seem more reasonable? Why / why not?

Both Slipher's and van Maanen's observations were extremely challenging at the time. An angle of $0.022''$ is tiny. G. W. Ritchie had already measured two of van Maanen's plates before and found no rotation. The spectroscopic measurements were based on exposures that had to extend over many hours, and not everybody believed Slipher's radial velcities, either.

4. The "plate scale" on the photographs used by van Maanen was about $30''\ \mathrm{mm}^{-1}$. For two observations made 15 years apart, what is the shift measured by van Maanen in mm?

5. If you had been attending the debate and knew what was known then, what would you have concluded about the galactic or extragalactic nature of spiral nebulae?

7.2 Radiation Pressure and Radial Velocities

In the "great debate", neither Shapley nor Curtis had a good explanation for the positive radial velocities of the nebulae. Today we know that this is due to the expansion of the Universe itself, but cosmology was still in its infancy in the 1920s and most people believed in a static Universe. Shapley suggested, somewhat hand-wavingly, that the nebulae might be accelerated by radiation pressure from the Milky Way. However, Henry Norris Russell was quick to demonstrate this cannot plausibly work. In this assignment we examine some of Russell's arguments.

Russell made a few simple assumptions:

1. Masses of the nebulae can be estimated from their rotation, assuming the standard Newtonian formula for circular rotation (but note that, strictly speaking, this assumes a spherically symmetric mass distribution). In 1921, such measurements were available for two nebulae: M31 and NGC 4594.

2. The plane of a nebula is perpendicular to the line-of-sight towards the Milky Way.

3. A nebula absorbs all the radiation from the Milky Way that falls upon it.

4. As seen from a nebula, the Milky Way occupies half the sky.

5. Seen from a nebula, the intensity of the light from the Milky Way is similar to that seen from Earth.

6. The intensity of the Milky Way corresponds to 3.5% of the flux from a 1st magnitude star per square degree (this number came from measurements by the Dutch astronomer Pieter van Rhijn, a student of Kapteyn). Such a star is a factor of $10^{0.4\times(1+26.7)} = 1.2\times10^{11}$ times fainter than the Sun.

7. Two measures of the "radius" of a nebula were considered: 1) an "inner" radius r, containing the majority of the mass, and 2) an "outer" radius R that represents the maximum area on which the radiation pressure would act.

The momentum of a photon (or a collection of photons) with energy E is $p = E/c$. Also, recall that pressure is force per area.

- Start by calculating the radiation pressure from a square degree of the Milky Way, seen from a nebula. Show that this pressure is

$$\mathcal{P} = 2.3\times10^{-14}\frac{L_\odot}{c(1\mathrm{AU})^2}$$

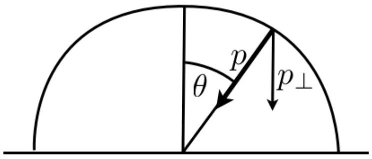
  where 1 AU = 1 astronomical unit = the distance from the Sun to the Earth, $L_\odot$ is the luminosity of the Sun, and c is the speed of light.

- Next, show that the force on the nebula due to radiation pressure from by a whole hemisphere is

$$\mathcal{F} = 7.5\times10^{-10}\frac{D^2R^2L_\odot}{c(1\mathrm{AU})^2}$$

  for distance D.

Hint: For radiation originating somewhere on the hemisphere, only the component of the momentum vector perpendicular to the surface of the nebula ($p_\perp$ in the figure below) contributes to the acceleration. The integral $\int_0^{\pi/2} \sin\theta\cos\theta\,\mathrm{d}\theta = \frac{1}{2}$.



- Finally, show that the acceleration produced by radiation pressure is then

$$A = 7.5 \times 10^{-10} \frac{L_\odot G}{c(1\mathrm{AU})^2} \frac{DR^2}{rv^2}$$

for "inner" radius r, "outer" radius R, circular velocity v at r. G is the gravitational constant.

Some of the assumptions made here (e.g. #4) may seem very unrealistic today, but it is important to keep the context of this calculation in mind. Russell's aim was to examine whether radiation pressure could significantly affect the kinematics of nebulae, given Shapley's view that the Milky Way was very large, and the nebulae all essentially part of the Milky Way.

One of the few nebulae for which the necessary observations were available in 1921 was the "Sombrero galaxy", NGC 4594. NGC 4594 has a radial velocity of +1000 km/s. For r and R, values of $r = 150''$ and $R = 210''$ may be assumed, as well as a rotational velocity of $v = 415$ km/s. The distance was very uncertain, but Russell assumed a distance of 1.43 Mpc or 4.4×10^{22} m.

- Under the above assumptions, calculate the current acceleration of the Sombrero galaxy due to radiation pressure
- If the acceleration had remained constant, and the Sombrero were initially at rest, how long would it have taken to accelerate to the current radial velocity?
- How far would the Sombrero have moved in this time?

Of course, the calculation above is extremely simplified. Which effects have been ignored? How would the calculation change (qualitatively) if these were included?

7.3 Hot gas in dark matter halos

In the classical picture, gas is shock-heated as it falls into dark matter halos and must cool before it can form stars. The rate at which the gas can cool is very sensitive to the composition, because gas that is enriched in heavy elements can cool more efficiently via a large number of atomic line transitions.

Recall that the r.m.s. velocity of particles in a gas with temperature T is given by

$$v_{\rm rms} = \sqrt{\frac{3kT}{\mu}} \tag{7.3.1}$$

where μ is the mean molecular weight, $\mu \approx 10^{-27}$ kg for a highly ionized plasma of typical composition and k is Boltzmann's constant, $k = 1.38 \times 10^{-23}\ \mathrm{m^2\ kg\ s^{-2}\ K^{-1}}$.

- Show that we may expect the temperature of the hot gas to be related to the observed line-of-sight velocity dispersion as

$$T = 72 \times 10^6 \left(\frac{\sigma_{1D}}{1000\,\mathrm{km\,s^{-1}}}\right)^2 \mathrm{K} \tag{7.3.2}$$

Formulae and constants

Distance modulus (D in pc):

$$m - M = 5\log_{10} D - 5$$

Black-body radiation:

$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

$$I_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

Radius of the Sun: $R_\odot = 7 \times 10^8$ m

Mass of the Sun: $M_\odot = 2 \times 10^{30}$ kg

1 pc = 3.09×10^{16} m

Planck's constant: $h = 6.626 \times 10^{-34}$ m^2 kg s^{-1}

Boltzmann's constant: $k = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}

Gravitational constant: $G = 6.673 \times 10^{-11}$ m^3 kg^{-1} s^{-2}