6. The physics of inflation

last time :

- derived FRW equations from GR
- discussed solutions when the universe contains matter, cosmological constant : ΛCDM - model
- models discussed last time have shortcoming that they do not naturally explain smallness of perturbations (e.g in cosmic microwave background)

$\Rightarrow$ points at missing ingredient

- goal of this lecture is to show that inflation provides an elegant mechanism for creating a universe that is homogeneous and isotropic to a very good approximation

6.1. Shortcomings of the Big Bang model

- reconsider flat ($k = 0$) FRW universe filled with non-relativistic matter ($\rho_0 = \rho_{crit}$, $w = 0$). Evolution of the scale-factor :

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\rho(a) = \rho_0 \, a^{-3}$$

Conclusion :

- there is a point in time ($t = 0$) where

$$a(t=0) = 0 \qquad \rho(t=0) = \infty$$

- Scale factor vanishes
- universe has infinite energy density

This point corresponds to the beginning of the universe and is called the "Big Bang".

remark:

- the occurrence of a Big Bang singularity is rather generic for all FRW solutions containing matter or radiation.

This leads us to the following puzzles:

1) <u>The horizon problem:</u>

We need to explain the observed homogenity of the universe
Heuristically: a patch that had causal contact at the time the CMB was formed ($\sim 3 \times 10^5$ years after the big bang) is covering an angle of about $1°$ of todays sky. Thus the CMB is formed by ~ 40.000 causally disconnected regions. Why is the observed CMB so homogeneous?

This problem can be made precise by analyzing the comoving particle horizon:
(recall the comoving particle horizon measures the size of the coordinate patch that we can see looking backward in time:)

$$\eta \equiv \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da}{a^2 H}$$

Since we have the explicit solution for $a(t)$ for general equation of state parameter w, it is easy to find the comoving Hubble radius $(aH)^{-1}$ in terms of a:

$$(aH) = \dot{a}$$

$$= \frac{2}{3(1+w)} \frac{1}{t_0} \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)} - 1}$$

$$= \frac{2}{3(1+w)} \frac{1}{t_0} a^{-\frac{1}{2}(1+3w)}$$

Evaluating at t_0 the prefactor is seen to be H_0:

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}$$

Compute η:

$$\eta = \int_0^a \frac{da}{a} \frac{1}{(aH)}$$

$$= \frac{1}{H_0} \int_0^a da \, a^{\frac{1}{2}(3w-1)}$$

$$= H_0^{-1} \frac{2}{(1+3w)} a^{\frac{1}{2}(1+3w)} \qquad w > -\frac{1}{3}$$

Conclusion:

$$\eta \propto \begin{cases} a & \text{radiation dominated} \\ a^{1/2} & \text{matter dominated} \end{cases}$$

$\Rightarrow$ comoving horizon increases monotonically.

$\Rightarrow$ Patches that were out of causal contact in the past move into the horizon at later times.

$\Rightarrow$ Homogeneity problem!

2. Flatness problem

- measurements : $\Omega_k \simeq 0$:
  - spacial slices appear to be flat
  - relative energy densities Ω_i in "matter sector" sum up to the critical density

  $\Rightarrow$ initial energy density must be extremly finetuned!

precise formulation :

- introduce a-dependent relative energy densities :

$$\Omega_i(a) = \frac{\rho_i(a)}{\rho_{crit}(a)} \qquad \rho_{crit}(a) = \frac{3}{8\pi G} H^2$$

  - where H is function of a :

- Friedmann equation (compare to last time) :

$$1 - \sum_i \Omega_i(a) = -\frac{k}{(aH)^2}$$

- showed above : $(aH)^{-1} \propto a^{1/2(1+3w)}$ grows in time

Thus if $k \neq 0$:

$\Rightarrow |\sum_i \Omega_i(a) - 1|$ grows in time and $\sum_i \Omega_i = 1$ is an unstable fixed point

Consequence:

$\Omega(a_0) \simeq 1$ today requires extreme finetuning of initial conditions in the past:

Nucleosynthesis : $|\Omega(a_{BBN}) - 1| \leq 10^{-16}$

Grand unified scale: $|\Omega(a_{GUT}) - 1| \leq 10^{-55}$

Planck scale : $|\Omega(a_{Pl}) - 1| \leq 10^{-61}$

Strong hint that we are missing some important physics!

6. 2. Inflation:

- observation:

  The horizon and flatness problems are essentially caused by the monotonic increase of the comoving Hubble radius $(aH)^{-1}$

- basic idea for solving the big bang puzzles:

  have a period in the early universe where the comoving Hubble radius <u>decreases</u> sufficiently $\leftrightarrow$ inflationary phase

Need that today:

- comoving horizon $\eta \gg (aH)^{-1}$ comoving Hubble radius of observable universe

Consequence :

patches in the sky that are out of causal contact today had causal contact in the past !

Solving the big bang puzzles by inflation

- assume phase where $(aH)^{-1}$ decreases

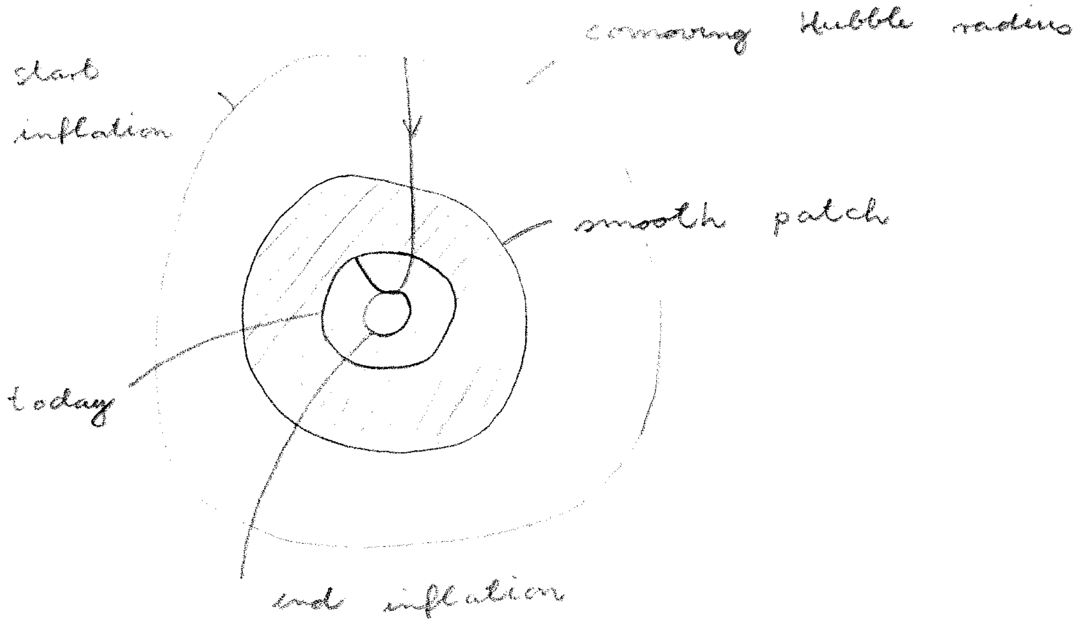
a) Flatness problem :

$$\left| 1 - \sum_i \Omega_i(a) \right| = \frac{1}{(aH)^2}$$

shows : a decreasing comov. Hubble radius drives the universe towards flatness !

b) Horizon problem :

large scales becoming observable today have been inside the horizon in the past :



- uniform CMB is not a mystery !

Conditions for inflation:

- We can formulate 3 equivalent condition for an inflationary phase:

1) <u>Decreasing comoving Hubble radius (fundamental definition)</u>

$$\frac{d}{dt}(aH)^{-1} < 0$$

2) <u>accelerated expansion:</u>

$$\frac{d^2a}{dt^2} > 0$$

equivalence to (1) follows from:

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}\frac{1}{\dot{a}} = -\frac{1}{\dot{a}^2}\ddot{a} \Rightarrow \ddot{a} > 0$$

3) <u>matter with negative pressure $p < -\frac{1}{3}\rho$</u>

- follows from 2nd Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p)$$

Indicates that the contributions to the stress-energy tensor causing inflation must violate the strong energy condition

SEC: $\rho + 3p > 0$

$\Rightarrow$ neither pressureless dust nor radiation can trigger inflation.

Slow - roll parameters as indicators for inflation :

- the slow - roll parameter ε relates inflation to the evolution of the Hubble parameter :

$$\varepsilon_H \equiv - \frac{\dot{H}}{H^2}$$

from

$$\frac{\ddot{a}}{a} = H^2 (1 - \varepsilon_H)$$

one finds accelerated expansion requires $\varepsilon_H < 1$

Introducing $dN = H\, dt$ where $N = \ln \frac{a_{end}(t)}{a_{start}}$

measures the e - folds of the expansion, $\varepsilon < 1$ corresponds to the statement that the change of the Hubble parameter is small (H approx. constant, as in de Sitter case) :

$$\varepsilon_H = - \frac{d \ln H}{dN}$$

- the second slow - roll parameter η_H is defined as

$$\eta_H \equiv \varepsilon_H - \frac{1}{2\varepsilon_H} \frac{d\varepsilon_H}{dN}$$

$|\eta_H| < 1$ ensures that the change of ε_H per e - fold is small .

6.3. Scalar field inflation

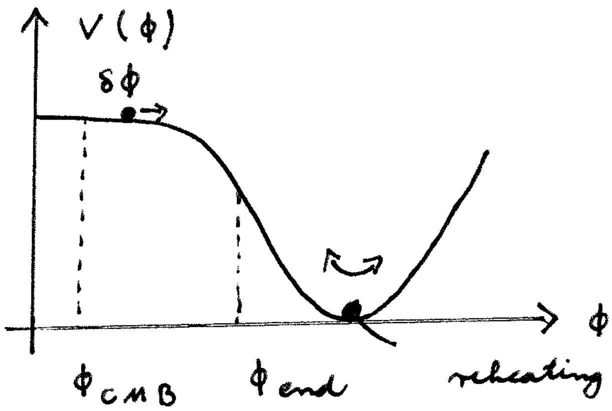
observations:

- ordinary matter obeying the strong energy condition cannot trigger inflation
- a positive cosmological constant leads to eternal inflation (de Sitter universe)

Idea: trigger inflation by scalar field ϕ starting uphill:

- ϕ : inflaton field

  $V(\phi)$: scalar potential (may act like a cosmol. const.)



Evolution:

- fluctuations $\delta\phi$ are created at ϕ_{CMB}
- universe inflates while scalar evolves from ϕ_{CMB} to ϕ_{end}
- inflation ends once kin. energy and pot. energy of ϕ are approximately equal
- inflated universe is repopulated by particles (reheating)

The dynamics of single field inflation

setup:

- consider a single scalar field ϕ minimally coupled to gravity

remarks:

- this is arguably the simplest setting realising inflation
- more complicated multi-field inflationary models do exist and you are invited to explore them in exercise 6.
- the class of single-field models discussed in this chapter is (surprisingly) successful in explaining observations (nature seems to prefer simplicity in terms of inflationary models)

Equations of motion: follow from the action principle

- recall: action of a scalar field ϕ in a spacetime with metric $g_{\mu\nu}$ is given by:

$$S^{\phi} = - \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + V(\phi) \right)$$

$V(\phi)$ is the scalar potential

e.g.: $V(\phi) = \frac{1}{2} m^2 \phi^2$ describes a mass-term

In exercise 1 (Tutorial week 6) we derived the stress-energy tensor

$$T_{\mu\nu} \equiv - \frac{2}{\sqrt{-g}} \frac{\delta S^{\phi}}{\delta g^{\mu\nu}}$$

resulting from this action:

$$T^{\phi}_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} (\partial_\alpha \phi)(\partial^\alpha \phi) + V(\phi) \right)$$

impose homogeneity: scalar field depends on t only: $\phi = \phi(t)$

- denoting time-derivatives by $\dot{}$:

$$T^{\phi}_{\mu\nu} = \delta_{\mu 0} \delta_{\nu 0} \dot{\phi}^2 + g_{\mu\nu} \left(\frac{1}{2} \dot{\phi}^2 - V(\phi) \right)$$

structure of a stress-energy tensor of a perfect fluid

$$T_\mu{}^\nu = \mathrm{diag}\left(-\rho(t), p(t), p(t), p(t) \right)$$

extract energy density $\rho(t)$, pressure $p(t)$ by raising one index of $T^{\phi}_{\mu\nu}$:

$$T^{\phi}{}_\mu{}^\nu = - \delta^0_\mu \delta^\nu_0 \dot{\phi}^2 + \delta_\mu{}^\nu \left(\frac{1}{2} \dot{\phi}^2 - V(\phi) \right)$$

comparison yields:

$$\rho^\phi(t) = - T^{\phi}{}_0{}^0 = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

$$p^\phi(t) = T^{\phi}{}_1{}^1 = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

remarks :

- The expression for $\rho(t)$ is intuitive : it is the kinetic energy of the scalar ($\frac{1}{2}\dot{\phi}^2$) plus the potential energy ($V(\phi)$)
- Friedmann's equations for the scalar coupled to gravity are obtained by substituting $p^\phi(t)$ and $\rho^\phi(t)$ into the general expressions derived in the previous chapter.
- The equation of state parameter w^ϕ is given by :

$$w^\phi = \frac{p^\phi}{\rho^\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

limiting cases :

- if the kinetic energy is negligible $w = -1$

  $\Rightarrow$ scalar works like a cosmological constant
- if kin. energy equals pot. energy : $w = 0$

  $\Rightarrow$ dynamics like pressureless dust.

The complete dynamics also requires the

<u>equation of motion for the scalar field</u>

- Varying S^ϕ with respect to the scalar yields (see exercise 1, week 6) :

$$\frac{1}{\sqrt{-g}}\,\partial_\mu \sqrt{-g}\; g^{\mu\nu}\,\partial_\nu \phi - \frac{\partial V}{\partial \phi} = 0$$

Restrict to the special case where

- $g_{\mu\nu}$ is the FRW metric
- scalar field is homogeneous, i.e., $\phi = \phi(t)$

Interlude :

compute $\sqrt{-g}$ for the FRW metric

since the metric is diagonal, the determinant g is the product of the diagonal entries :

$$\sqrt{-g} = \Big(- \underbrace{(-1)}_{g_{00}} \cdot \underbrace{\frac{a^2}{1 - k r^2}}_{g_{rr}} \cdot \underbrace{r^2 a^2}_{g_{\theta\theta}} \cdot \underbrace{r^2 a^2 \sin^2\theta}_{g_{\phi\phi}} \Big)^{1/2}$$

$$= a^3 \left(\frac{r^4 \sin^2\theta}{1 - k r^2} \right)^{1/2}$$

Plug into general formula :

$$\frac{1}{a^3 \left(\frac{r^4 \sin^2\theta}{1 - k r^2} \right)^{1/2}} \partial_t \left(a^3 \left(\frac{r^4 \sin^2\theta}{1 - k r^2} \right)^{1/2} (-1) \right) \dot{\phi} - \frac{\partial V}{\partial \phi} = 0$$

- a - independent parts of determinant cancel to give

$$- \ddot{\phi} - 3 H \dot{\phi} - \frac{\partial V}{\partial \phi} = 0$$

Combining all results yields the e.o.m. of a scalar field $\phi(t)$ in a FRW universe :

$$\boxed{\begin{aligned} H^2 &= \frac{1}{3 M_{Pl}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \\ \ddot{\phi} + 3 H \dot{\phi} &= - \frac{\partial V}{\partial \phi} \end{aligned}}$$

Here $M_{Pl} = \sqrt{8 \pi G}^{-1}$ is the reduced Planck scale

Question :

- when does the system exhibit accelerated expansion ?

  From
  $$\frac{\ddot{a}}{a} = - \frac{4}{3} \pi G (\rho + 3 p)$$
  we require $\rho + 3 p < 0$

  substituting ρ^{ϕ}, p^{ϕ} this translates into $(\dot{\phi}^2 - V) < 0$

  Thus accelerated expansion needs
  $$V > \dot{\phi}^2$$
  inflation occurs if the potential energy dominates over the kinetic energy.

remark :

- once inflation is under way the Hubble radius shrinks and spatial curvature contributions are diluted away :

  $\Rightarrow$ solution to the flatness problem

  $\Rightarrow$ renders the flat FRW metric a good approximation for spacetime after inflation occured.

Slow-roll inflation

The slow-roll conditions implement that:

$$\dot{\phi}^2 \ll V, \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|, \left|\frac{\partial V}{\partial \phi}\right|$$

- kin. energy is negligible compared to the potential energy of the scalar
- the acceleration of the scalar field is negligible (slow)

The slow-roll conditions are the standard approximation for analyzing inflation

Implementing the slow-roll approximation the e.o.m describing the dynamics of the system are

$$H^2 \simeq \frac{V}{3 M_{Pl}^2}$$

$$3H\dot{\phi} \simeq -\frac{\partial V}{\partial \phi}$$

remark:

- we indicate the use of the slow-roll approximation by $\simeq$

At this stage, it is useful to introduce an alternative set of slow-roll parameters relating inflation to properties of the scalar potential:

$$\varepsilon_V \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V \equiv M_{Pl}^2 \frac{V''}{V}$$

remarks:

- in the exercise we will show that these potential slow-roll parameters are related to the initial ones by:

$$\varepsilon \simeq \varepsilon_V \qquad \eta = \eta_V - \varepsilon_V$$

- inflation requires $\varepsilon_V(\phi) \ll 1$, $|\eta_V(\phi)| \ll 1$

  note that these are necessary but not sufficient conditions. since ε_V, η_V do not contain $\dot\phi$, taking a large initial value for $\dot\phi$ may prohibit inflation even though ε_V, η_V are small. In this sense, $\varepsilon_V, |\eta_V|$ are useful probes for which values of ϕ inflation may occur.

- conventionally the end of the inflation era is set at the point where $\varepsilon_V = 1$ or $|\eta_V| = 1$, i.e., the slow-roll conditions are violated.

The e-folds of expansion obtained during the inflation can be related to the slow-roll parameter:

$$N \equiv \ln \frac{a(t_{end})}{a(t)} = \int_t^{t_{end}} H \, dt$$

$$\simeq \frac{1}{M} \int_{\phi_{end}}^{\phi} \frac{d\phi}{\sqrt{2\varepsilon_V}}$$

Proof: (uses monotonicity as implicit assumption):

$$\int_t^{t_{end}} H \, dt = - \int_{\phi_{end}}^{\phi} H \frac{dt}{d\phi} \, d\phi$$

use $\dot{\phi} = - \sqrt{-2M^2 \dot{H}}$

$$= + \int_{\phi_{end}}^{\phi} \frac{1}{M} \sqrt{- \frac{1}{2} \frac{H^2}{\dot{H}}} \, d\phi$$

$$\simeq + \int_{\phi_{end}}^{\phi} \frac{1}{M} \frac{d\phi}{\sqrt{2\varepsilon_V}}$$

where we used $\varepsilon_H = - \frac{\dot{H}}{H^2} \simeq \varepsilon_V$ in the last step.

Hamilton - Jacobi formulation of the Friedmann equations

- up to now we specify our model of the early universe by giving the scalar potential $V(\phi)$

Idea underlying the Hamilton - Jacobi formulation

- construct models by giving $H(\phi)$

remarks:

- advantage:

  giving $H(\phi)$ instead of $V(\phi)$ allows to construct exact solutions more easily

- implicit assumption: the resulting $\phi(t)$ is strictly monotonic, i.e., $\dot{\phi} \neq 0$

  note:

  if $\phi(t)$ is not monotonic, e.g. when undergoing oscillations at the end of inflation, the complete cosmic history may be obtained by patching together monotonic solutions

recover the complete cosmic solution:

Step 1: need $\phi(t)$

- based on $\phi(t)$ one obtains $H(t)$ which can be integrated to obtain the scale factor $a(t)$.

Strategy :

· Take t - derivative of

$$H^2 = \frac{1}{3 M_{Pl}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \qquad (1)$$

which yields

$$2 H \dot{H} = \frac{1}{3 M_{Pl}^2} \left(\dot{\phi} \ddot{\phi} + V' \dot{\phi} \right)$$

$$\underset{\text{e.o.m } \phi}{=} \frac{1}{3 M_{Pl}^2} \dot{\phi} \left(- 3H\dot{\phi} - V' + V' \right)$$

Thus :

$$2 \dot{H} = - \frac{\dot{\phi}^2}{M_{Pl}^2} \qquad (2)$$

Divide by $\dot{\phi}$ (monotonicity assumption)

$$\dot{\phi} = - 2 M_{Pl}^2 \frac{\dot{H}}{\dot{\phi}}$$

or

$$\dot{\phi} = - 2 M_{Pl}^2 \frac{\partial H}{\partial \phi} \qquad (3)$$

$\Rightarrow$ differential equation determining $\phi(t)$!

Step 2 :

· determine scalar potential $V(\phi)$ supporting the evolution $H(\phi)$

Substituting eq. (3) back into (1) yields the Hamilton-Jacobi (first order) form of Friedmanns equations :

$$\left(\frac{\partial H}{\partial \phi} \right)^2 - \frac{3}{2 M_{Pl}^2} H^2 = - \frac{1}{2 M_{Pl}^4} V(\phi) \qquad (4)$$

- given $H(\phi)$ this eqn determines $V(\phi)$

The Hamilton - Jacobi formalism allows to construct exact inflationary solutions in a rather straightforward way.

<u>The inflationary attractor</u>

problem :

- the initial conditions before inflation, e.g. the initial value of $\dot{\phi}$, is unknown.

$\Rightarrow$ in order to give robust predictions, these predictions must be insensitive to the initial conditions how inflation started.

Realization :

- Attractor mechanism washes out information on initial data.

remark :

- Idea is quite similar to the stability analysis of Einstein's static universe. In this case a small change in initial conditions magnifies in time, signalling an instability

Goal: show that inflationary solutions are attractors in the sense that differences in the initial conditions decrease exponentially fast.

Strategy:

- use Hamilton - Jacobi formalism supplemented by the working assumptions:
  - ϕ increases with time $\Leftrightarrow$ $\dot{\phi} > 0$
  - perturbations are assumed to be homogeneous and linear

Ansatz for perturbed solution:

$$H(\phi) = H_0(\phi) + \delta H(\phi)$$

- $H_0(\phi)$: solution to unperturbed e.o.m.
- $\delta H(\phi)$: small perturbation

Substituting ansatz into the Hamilton - Jacobi eq. (4) and retaining terms up to linear order in $\delta H(\phi)$ only yields e.o.m. capturing the dynamics of the perturbation

$$2 H_0' \, \delta H' = \frac{3}{M_{Pl}^2} H_0 \, \delta H$$

- separable first order differential equation for $\delta H(\phi)$
- prime denotes derivative w.r.t. ϕ.

ODE can be integrated yielding:

$$\delta H(\phi) = \delta H(\phi_i) \exp\left(\frac{3}{2 M_{Pl}^2} \int_{\phi_i}^{\phi} \frac{H_0(\phi)}{H_0'(\phi)} d\phi\right)$$

where ϕ_i is the initial value of the field configuration

Observation:

Integrand is negative definite

- as a consequence of eq (3) H' and $d\phi$ must have opposite signs.

Conclusion:

- perturbation decreases with time

  $\Rightarrow$ $H_0(\phi)$ acts as an attractor

Interesting special case: $H_0(\phi)$ is inflationary solution satisfying the slow-roll condition

$$\varepsilon_H = \frac{1}{2 M_{Pl}^2} \left(\frac{V'}{V}\right)^2 \simeq 2 M_{Pl}^2 \left(\frac{H'}{H}\right)^2 < 1$$

In this case the integral is bounded away from zero:

$$\int_{\phi_i}^{\phi} \frac{H_0}{H_0'} d\phi \simeq - \int_{\phi_i}^{\phi} \sqrt{\frac{2 M_{Pl}^2}{\varepsilon_H}} \, d\phi$$

$$\leq - \int_{\phi_i}^{\phi} \sqrt{2} M_{Pl} d\phi = - \sqrt{2} M_{Pl} (\phi - \phi_i)$$

- sign in first step fixed by taking correct branch of the square root
- second step approximates $\varepsilon_H = 1$ to get a bound on the slow-roll parameter

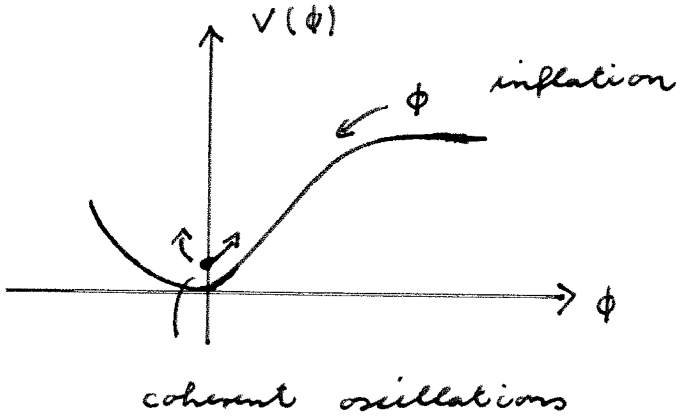
Hence linear perturbations die exponentially fast in ϕ:

$$\delta H(\phi) < \delta H(\phi_i) \exp\left(-\frac{3}{\sqrt{2}} \frac{\phi - \phi_i}{M_{Pl}}\right)$$

This attractor underlies the predictivity of inflation

End of inflation: reheating

- during inflation: scalar field rolls downhill
- end of inflation: ϕ starts oscillating at a minimum of the scalar potential



connect inflation era to nucleosynthesis requires "reheating"

This process consists of 3 parts:

1) non-inflationary scalar field dynamics

2) decay of inflaton particles

(needs a specific particle physics model describing the coupling of the inflaton field to other (e.g. standard model) matter)

3) Thermalization of decay products

Decay of inflaton particles:

- typical assumptions: oscillations are fast compared to the change in the Hubble parameter (i.e. $H \sim$ const.)

- define the average energy density

$$\bar{\rho}_\phi \equiv \langle \dot{\phi}^2 \rangle_t$$

where the average is over the time-scale of the oscillations

Energy density $\bar{\rho}_\phi$ dilutes (see exercise) as

$$\dot{\bar{\rho}}_\phi + (3H + \Gamma_\phi) \bar{\rho}_\phi = 0$$

H - term : Hubble friction

$\Rightarrow$ dilution of energy density due to expansion of the universe

Γ_ϕ : dilution due to particle decays

(particle physics model dependent)

Evolution of scales

- outstanding success of inflation :

  $\Rightarrow$ prediction of density fluctuations

  $\Rightarrow$ seeds for galaxies and galaxy clusters observable today

"Label" perturbation by their comoving wavenumber k

$$k \equiv \frac{2\pi}{\lambda}$$

$\lambda \equiv$ comoving wave - length

remark :

- given a function $g(\vec{x})$ we can construct its Fourier transform via

$$g(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \; g(\vec{k}) \, e^{i\vec{k}\vec{x}} \quad ; \quad g(k) = \int d^3x \; g(\vec{x}) \; e^{-i\vec{k}\vec{x}}$$

where $g(k)$ is the amplitude associated with "frequency" k, i.e. the function is decomposed in a frequency spectrum recall : definition of comoving Hubble scale $d_c^H \equiv (aH)^{-1}$

special point :

- comoving wave number equals horizon scale

$$k = aH$$

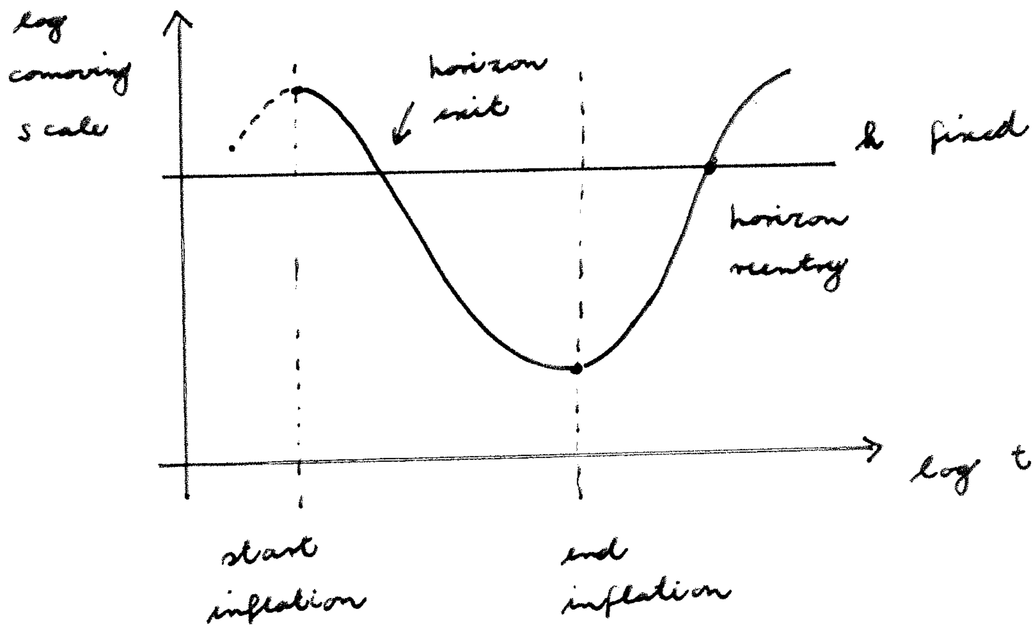
History of perturbations :

- before inflation perturbations with wave-number smaller then the comoving Hubble radius can be created

- during inflation the Hubble length decreases

  $\Rightarrow$ perturbations may exit the horizon

- inflation ends and the Hubble length starts to increase again $\Rightarrow$ perturbations reenter the Hubble horizon at later times



Tracing the evolution of perturbations requires a model for the cosmic evolution. It is useful to splitt this evolution into the following eras:

1) From k^{-1} equalling the Hubble radius (horizon exit) to the end of inflation

2) From the end of inflation to thermal equilibrium (common assumption: this is a matter-dominated era)

3) Radiation dominated era between the end of reheating and matter-radiation equality at t_{eq}

4) matter-dominated era from t_{eq} until today

Concluding remark:

- size of observable universe $\simeq$ 28 GPc

  resolution of large-scale structure surveys $\simeq$ 1 MPc

Consequence:

- fluctuations visible at 1 MPc cross the Hubble horizon approximately 10 e-folds after a fluctuation at scale $a_0 H_0$ (size of todays universe) does.

$\Rightarrow$ Fluctuations responsible for observed large scale structures cross horizon during a small number of e-folds well before inflation ends.