Population synthesis of common-envelope mergers on the giant branches

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Stellar mergers

Occurrence:

- Collisions: $\tau \sim \text{day}?$ (Sills et al. 2001)
- Binary mergers: convective envelope: $\tau \sim \tau_{\text{dyn}}; \text{yr} - \text{kyr}?$
- Binary mergers: radiative envelope: $\tau \sim \tau_{\text{th}} \rightarrow \tau_{\text{dyn}}$

A significant fraction of stars ($\sim 10\%?$) may be involved in mergers
- Luminous red novae?
- V 838 Mon?
Merger products

Physics:

- Angular momentum!
- Rapid, differential rotation
- Enhanced mixing
- Magnetic fields
- Enhanced mass loss
Merger products

Physics:

- Angular momentum!
- Rapid, differential rotation
- Enhanced mixing
- Magnetic fields
- Enhanced mass loss
Merger products

Observability:
- Rapid rotation?
- Abundance anomalies?
- Circumstellar material
- Blue stragglers
- Cluster dynamics

“Weird” binaries
- B[e] stars?
- Hot subdwarfs?
- Asymmetric PNe
- IMBHs?
Detailed collisions

Use:
- 1D stellar models
- collide them in hydro
- bring remnant in hydrostatic equilibrium
- evolve in 1D
- for low-mass stars: “Entropy” “sorting”

Differences in:
- Timescales
- Luminosities
- Core masses
- Mixing

1.75 \( M_\odot \): Collision product  Normal star
(dashes): Fully mixed model

Glebbeek & Pols, 2008
Input models

Stellar-evolution code \texttt{ev} (Eggleton, 1971, 2, etc.):

- 116: single-star models: 0.5, 0.6, \ldots, 10.0, 10.5, \ldots, 20.0 \, M_\odot \text{ (primary, merger remnant)}

- 28 brown-dwarf models: 0.01 – 0.60 \, M_\odot \text{ (secondary)}

- Solar composition; \(X=0.70, \; Y=0.28, \; Z=0.02\)
Input models

Stellar-evolution code $\text{ev}$:

- Core mass: $M_c \equiv$ central region where $X < 0.1$

- Envelope binding energy: $E_{\text{bind}} \equiv \int_{M_c}^{M_\odot} \left( E_{\text{int}}(m) - \frac{Gm}{r(m)} \right) \, dm$

- Convective mixing: $l/H_P = 2.0$

- Overshooting: none for $M < 1.2 \, M_\odot$, $\delta_{\text{ov}} = 0.12$ for $M \geq 1.2 \, M_\odot$

- Stellar wind: “Reimers” (1975), De Jager et al. (1988)

- Helium-flash-avoidance routine $\text{FGB2HB}$
Treatment of evolution

**Stars**

- Constant star-formation rate
- Randomly select $10^7$ binaries:
  - $M_p$: Miller-Scalo IMF
  - $q \equiv M_s / M_p$:
    $$g(q) \, dq = \{ q^{-0.9}, 1, q \} \, dq$$
- Follow the evolution of track closest in mass to primary
- When mass comes closer to next track, jump with conservation of $M_c$
Treatment of evolution

Stars

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- Randomly select $10^7$ binaries:
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**Treatment of evolution**

**Orbit**

- Assume synchronous rotation on RGB, AGB: $\omega_p = \omega_{orb}$
- Mass and AM loss from stellar wind
  - If $v_{rot} > v_{crit}$: lose additional mass and AM until $v_{rot} \leq v_{crit}$
  - Redistribute AM, so that $J_{tot} = (I_p + I_{orb}) \omega_{orb}$
- $v_{crit} \equiv \{0.1, \frac{1}{3}, 1.0\} v_{br}$
Common envelope and spiral-in

- CE occurs when:
  - $R_p > R_{RL,p}$ and $q > q_{\text{crit}}(M_p, M_c)$ (Hurley et al. 2002)
  - $J_{\text{prim}} > \frac{1}{3} J_{\text{orb}}$ (Darwin 1879)

- Classical energy formalism to determine post-CE orbit (Webbink 1984):
  \[
  E_{\text{bind}} = \alpha_{CE} \left( \frac{G M_p M_s}{2 a_i} - \frac{G M_c M_s}{2 a_f} \right)
  \]
  - $\alpha_{CE} = \{0.1, 0.5, 1.0\}$

- Merger occurs if after CE: $R_{RL,s} < R_s$
The merger product has:

- the core mass of the original primary
- the maximum mass for which:
  - the star is spinning (sub-)critically \( v_{\text{rot}} \leq v_{\text{crit}} \)
  - \( M_{\text{mrg}} \leq M_p + M_s \)
- the evolutionary state of the primary, or later

In addition,

- the surplus mass from the binary does not interact with the star (accretion, tides)
Evolution of the merger product

After the merger:

- the merger product evolves mostly in the same way as a normal single star
  - e.g. $L$, $R$, etc. are identical to those for a star with the same $M$, $M_c$
  - difference: $v_{\text{rot}}$, hence $\dot{M}$

- whenever $v_{\text{rot}} \geq v_{\text{crit}}$, the star undergoes enhanced mass loss, to ensure that it remains spinning sub-critically
  - this is especially important around core helium ignition
<table>
<thead>
<tr>
<th>Category</th>
<th>Number</th>
<th>Fraction of previous group</th>
<th>Fraction of initial population</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total binary population:</strong></td>
<td>10,000,000</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>No MT</td>
<td>7,094,523</td>
<td>71%</td>
<td>71%</td>
</tr>
<tr>
<td>Stable MT</td>
<td>1,267,854</td>
<td>13%</td>
<td>13%</td>
</tr>
<tr>
<td>Unstable MT:</td>
<td>1,637,623</td>
<td>16%</td>
<td>16%</td>
</tr>
<tr>
<td>CE Survivors:</td>
<td>789,807</td>
<td>48%</td>
<td>7.9%</td>
</tr>
<tr>
<td>Mergers:</td>
<td>847,816</td>
<td>52%</td>
<td>8.5%</td>
</tr>
<tr>
<td>Mergers due to RLOF</td>
<td>689,815</td>
<td>81%</td>
<td>6.9%</td>
</tr>
<tr>
<td>Mergers due to tidal capture</td>
<td>158,001</td>
<td>19%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Mergers on RGB</td>
<td>738,385</td>
<td>87%</td>
<td>7.4%</td>
</tr>
<tr>
<td>Mergers on AGB</td>
<td>109,431</td>
<td>13%</td>
<td>1.1%</td>
</tr>
<tr>
<td>WDs</td>
<td>822,773</td>
<td>97%</td>
<td>8.2%</td>
</tr>
<tr>
<td>GB/HB stars:</td>
<td>25,041</td>
<td>3%</td>
<td>0.25%</td>
</tr>
<tr>
<td>RGB</td>
<td>9,301</td>
<td>37%</td>
<td>0.09%</td>
</tr>
<tr>
<td>HB</td>
<td>14,305</td>
<td>57%</td>
<td>0.14%</td>
</tr>
<tr>
<td>AGB</td>
<td>1,435</td>
<td>6%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Critically rotating RGB stars</td>
<td>297</td>
<td>3.2%</td>
<td>0.003%</td>
</tr>
<tr>
<td>Critically rotating HB stars</td>
<td>4,504</td>
<td>31%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Critically rotating AGB stars</td>
<td>1</td>
<td>0.1%</td>
<td>0.000001%</td>
</tr>
</tbody>
</table>
Merger properties

Total mass:

Luminosity:

\[ \nu_{\text{crit}} = \frac{1}{3} \nu_{\text{br}} \]
Merger population

All merger products:

Merger products on HB:

\[ \nu_{\text{crit}} = \frac{1}{3} \nu_{\text{br}} \]
Rotational velocities

\[
v_{\text{rot}} / v_{\text{crit}}:
\]

\[
v_{\text{rot}} = \frac{1}{3} v_{\text{br}}
\]
Rotational velocities

$\frac{v_{\text{rot}}}{v_{\text{crit}}} :$

$\frac{v_{\text{rot}} \sin i}{(\text{km/s})} :$

$v_{\text{crit}} = \frac{1}{3} v_{\text{br}}$
# Sub-populations

<table>
<thead>
<tr>
<th>Population</th>
<th>N</th>
<th>N_{rot}^N</th>
<th>M (M_☉)</th>
<th>v sin i (km/s)</th>
<th>v_{rot} ≤ 0.1 v_{crit}</th>
<th>v_{rot} = v_{crit} (M_☉)</th>
<th>M_{rej}</th>
<th>M_{rej}/M_{bin}</th>
<th>ΔM_{mrg}</th>
<th>M_{mrg,i}</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGB</td>
<td>9301</td>
<td>0.37</td>
<td>1.20</td>
<td>18.4</td>
<td>(0.001)</td>
<td>0.0319</td>
<td>0.63</td>
<td>0.34</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>HB</td>
<td>14305</td>
<td>0.57</td>
<td>1.35</td>
<td>16.1</td>
<td>(0.0000)</td>
<td>0.3149</td>
<td>0.93</td>
<td>0.40</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>AGB</td>
<td>1435</td>
<td>0.06</td>
<td>1.34</td>
<td>6.0</td>
<td>0.0683</td>
<td>(0.0007)</td>
<td>0.94</td>
<td>0.42</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>25041</td>
<td>1.00</td>
<td>1.28</td>
<td>16.2</td>
<td>0.0043</td>
<td>0.1918</td>
<td>0.81</td>
<td>0.38</td>
<td>0.07</td>
<td></td>
</tr>
</tbody>
</table>
## Dependence on input parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>$N$</th>
<th>$N_{\text{RGB}} / N_{\text{rot}}$</th>
<th>$M$</th>
<th>$v \sin i$</th>
<th>$v_{\text{rot}} \leq 0.1 v_{\text{crit}}$</th>
<th>$v_{\text{rot}} = v_{\text{crit}}$</th>
<th>$M_{\text{rej}} / M_{\text{bin}}$</th>
<th>$\Delta M_{\text{mrg}} / M_{\text{mrg}, i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\text{CE}} = 0.1$</td>
<td>32882</td>
<td>0.29</td>
<td>1.23</td>
<td>16.5</td>
<td>0.0054</td>
<td>0.2726</td>
<td>0.83</td>
<td>0.40</td>
</tr>
<tr>
<td>$\alpha_{\text{CE}} = 0.5$</td>
<td>28269</td>
<td>0.34</td>
<td>1.23</td>
<td>16.2</td>
<td>0.0048</td>
<td>0.2201</td>
<td>0.81</td>
<td>0.38</td>
</tr>
<tr>
<td>$\alpha_{\text{CE}} = 1.0$</td>
<td>25041</td>
<td>0.37</td>
<td>1.28</td>
<td>16.2</td>
<td>0.0043</td>
<td>0.1918</td>
<td>0.81</td>
<td>0.38</td>
</tr>
</tbody>
</table>

**Common-envelope parameter**

- for a *larger* $\alpha_{\text{CE}}$, a smaller fraction of all CEs leads to merger
- for a *smaller* $\alpha_{\text{CE}}$, wider binaries can merge
  - merger remnants have more angular momentum
Dependence on input parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>N</th>
<th>$\frac{N_{RGB}}{N_{tot}}$</th>
<th>M</th>
<th>$v\sin i$</th>
<th>$v_{rot} \leq v_{crit}$</th>
<th>$v_{rot} = v_{crit}$</th>
<th>$M_{rej}$</th>
<th>$\frac{M_{rej}}{M_{bin}}$</th>
<th>$\Delta M_{mrg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(q) = q^{-0.9}$</td>
<td>25343</td>
<td>0.35</td>
<td>1.29</td>
<td>16.3</td>
<td>0.0045</td>
<td>0.2012</td>
<td>0.28</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>$g(q) = 1$</td>
<td>25041</td>
<td>0.37</td>
<td>1.28</td>
<td>16.2</td>
<td>0.0043</td>
<td>0.1918</td>
<td>0.81</td>
<td>0.38</td>
<td>0.07</td>
</tr>
<tr>
<td>$g(q) = q$</td>
<td>24853</td>
<td>0.36</td>
<td>1.29</td>
<td>16.0</td>
<td>0.0049</td>
<td>0.2015</td>
<td>1.10</td>
<td>0.46</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Initial-mass-ratio distribution

- $g(q) = q$ favours equal-mass binaries, $g(q) = q^{-0.9}$ favours extreme mass ratios
- For $g(q) = q$, secondary masses are larger and more mass is rejected during the merger
Dependence on input parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>N</th>
<th>( \frac{N_{\text{RGB}}}{N_{\text{tot}}} )</th>
<th>( M ) (( M_\odot ))</th>
<th>( v \sin i ) (km/s)</th>
<th>Fraction with ( v_{\text{rot}} )</th>
<th>( M_{\text{rej}} ) (( M_\odot ))</th>
<th>( \frac{M_{\text{rej}}}{M_{\text{bin}}} )</th>
<th>( \frac{\Delta M_{\text{mrg}}}{M_{\text{mrg,i}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{\text{crit}} = 0.1 v_{\text{br}} )</td>
<td>25490</td>
<td>0.38</td>
<td>1.20</td>
<td>4.6</td>
<td>0.0058</td>
<td>0.1974</td>
<td>0.90</td>
<td>0.41</td>
</tr>
<tr>
<td>( v_{\text{crit}} = \frac{1}{3} v_{\text{br}} )</td>
<td>25041</td>
<td>0.37</td>
<td>1.28</td>
<td>16.2</td>
<td>0.0043</td>
<td>0.1918</td>
<td>0.81</td>
<td>0.38</td>
</tr>
<tr>
<td>( v_{\text{crit}} = v_{\text{br}} )</td>
<td>24414</td>
<td>0.33</td>
<td>1.46</td>
<td>47.7</td>
<td>0.0051</td>
<td>0.1343</td>
<td>0.63</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Critical rotational velocity

- The observed (projected) rotational velocity scales with our definition of \( v_{\text{crit}} \)
- For smaller \( v_{\text{crit}} \), more mass is ejected during and after merger
Comparison to single stars

Merger remnants:  

Single stars:

\[ v_{\text{crit}} = \frac{1}{3} v_{\text{br}} \]
Comparison to single stars

<table>
<thead>
<tr>
<th>Ev. phase</th>
<th>population</th>
<th>N</th>
<th>$\frac{N}{N_{\text{tot}}}$</th>
<th>M</th>
<th>$v \sin i$</th>
<th>$v_{\text{rot}} \leq 0.1 v_{\text{crit}}$</th>
<th>$v_{\text{rot}} = v_{\text{crit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGB</td>
<td>mergers</td>
<td>9301</td>
<td>0.37</td>
<td>1.20</td>
<td>18.4</td>
<td>(0.001)</td>
<td>0.0319</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td>178651</td>
<td>0.61</td>
<td>1.20</td>
<td>1.9</td>
<td>0.9627</td>
<td>0.000</td>
</tr>
<tr>
<td>HB</td>
<td>mergers</td>
<td>14305</td>
<td>0.57</td>
<td>1.35</td>
<td>16.1</td>
<td>(0.0000)</td>
<td>0.3149</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td>104979</td>
<td>0.36</td>
<td>1.58</td>
<td>3.2</td>
<td>0.0886</td>
<td>0.0021</td>
</tr>
<tr>
<td>AGB</td>
<td>mergers</td>
<td>1435</td>
<td>0.06</td>
<td>1.34</td>
<td>6.0</td>
<td>0.0683</td>
<td>(0.0007)</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td>10487</td>
<td>0.04</td>
<td>1.45</td>
<td>1.3</td>
<td>0.5657</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Total</td>
<td>mergers</td>
<td>25041</td>
<td>1.00</td>
<td>1.28</td>
<td>16.2</td>
<td>0.0043</td>
<td>0.1918</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td>294117</td>
<td>1.00</td>
<td>1.23</td>
<td>2.3</td>
<td>0.6366</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Critical rotational velocity

- The observed (projected) rotational velocity is roughly an order of magnitude larger for merger products
- Most merger products on the GBs have ignited helium, most normal single stars have not
**sdB stars**

**Basic properties:**

- Core helium burning stars with very thin ($\lesssim 0.02 M_\odot$) hydrogen-rich envelope

- In the field $\sim 40$–$70\%$ are found in binaries

- In GCs mostly observed as **single** sdB stars

- Masses observed $\sim 0.39 M_\odot - 0.7 M_\odot$ (e.g. asteroseismology)
sdB stars

Possible formation channels:

**In wide binaries:**
- One or two phases of stable Roche-lobe overflow

**In close binaries:**
- One or two CE/spiral-in phases

**Single sdB stars:**
- He-WD–He-WD mergers ($M \gtrsim 0.4 \, M_\odot$)
- Strong mass loss at tip of RGB (e.g. capture of planet; Soker & Harpaz, 2000, 2007; Livio & Siess, 1999a,b)
- **CE merger on the RGB** (Soker 1998, Soker & Harpaz 2000, 2007)
Rotational velocities for merged HB stars

All merger products:

Merger products on HB:

\[ \nu_{\text{crit}} = \frac{1}{3} \nu_{\text{br}} \]
Rotational velocities

\( \frac{v_{\text{rot}}}{v_{\text{crit}}} : \)

\( v_{\text{rot}}(\text{km/s}): \)

Merger products

Single stars

\( v_{\text{crit}} = \frac{1}{3} v_{\text{br}} \)
Core and envelope masses

**Helium-core masses:**

- Fraction of objects as a function of helium-core mass at present epoch ($M_\odot$)

**Envelope masses:**

- Fraction of objects as a function of envelope mass at present epoch ($M_\odot$)

Merger products

Single stars
Losing the envelope

**Detailed model of an HB star with initial parameters** $M \approx 0.59 \, M_\odot$, $M_{\text{env}} \approx 0.11 \, M_\odot$ and $v_{\text{rot}} \approx 25 \, \text{km/s}$:

- $M_{\text{env}}$ vs. $\log R$:
- $M_{\text{env}}$ vs. $T_{\text{eff}}$:
- $M_{\text{env}}$ vs. $v_{\text{rot}}$:
Lithium-rich giants

Reddy & Lambert 2005; Kumar & Reddy 2009:

<table>
<thead>
<tr>
<th>Star</th>
<th>[Fe/H]</th>
<th>$T_{\text{eff}}$</th>
<th>$M^*/M_\odot$</th>
<th>$\log L/L_\odot$</th>
<th>$\log \epsilon$(Li)</th>
<th>$^{12}\text{C}/^{13}\text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 77361</td>
<td>$-0.02 \pm 0.1$</td>
<td>4580 ± 75</td>
<td>1.5 ± 0.2</td>
<td>1.66 ± 0.1</td>
<td>3.82 ± 0.10</td>
<td>4.3 ± 0.5</td>
</tr>
<tr>
<td>HD 233517</td>
<td>$-0.37$</td>
<td>4475 ± 70</td>
<td>1.7 ± 0.2</td>
<td>2.0$^c$</td>
<td>4.22 ± 0.11</td>
<td>...</td>
</tr>
<tr>
<td>IRAS 13339-4153</td>
<td>$-0.13$</td>
<td>4300 ± 100</td>
<td>0.8 ± 0.7</td>
<td>1.60$^a$</td>
<td>4.05 ± 0.15</td>
<td>20</td>
</tr>
<tr>
<td>HD 9746</td>
<td>$-0.06$</td>
<td>4400 ± 100</td>
<td>1.92 ± 0.3</td>
<td>2.02</td>
<td>3.75 ± 0.16</td>
<td>28 ± 4</td>
</tr>
<tr>
<td>HD 19745</td>
<td>$-0.05$</td>
<td>4700 ± 100</td>
<td>2.2 ± 0.6</td>
<td>1.90$^c$</td>
<td>3.70 ± 0.30</td>
<td>16 ± 2</td>
</tr>
<tr>
<td>IRAS 13331-5838</td>
<td>$-0.09$</td>
<td>4540 ± 150</td>
<td>1.1</td>
<td>1.85$^a$</td>
<td>3.3 ± 0.20</td>
<td>12 ± 2</td>
</tr>
</tbody>
</table>
Oblateness

Zhao et al. 2009

\[ e = 0.60 \quad e = 0.55 \]

\[ e \equiv \sqrt{1 - \left( \frac{R_{\text{pol}}}{R_{\text{eq}}} \right)^2} \]

MacLaurin spheroids (1742):

\[
\frac{\omega}{\sqrt{2\pi G \rho}} = \sqrt{\frac{1 - e^2}{e^3} \left( 3 - 2e^2 \right) \sin(e) - \frac{3}{e^2} (1 - e^2)}
\]
Oblateness

Single stars

Merger products

Oblateness of single stars at present epoch

Oblateness (e) of merged objects at present epoch
Asymmetric planetary nebulae?

Planetary Nebula M2-9

Butterfly nebula (HST)

Planetary Nebula M2-9

HST • WFPC2

PRC97-38a • ST ScI OPO • December 17, 1997
B. Balick (University of Washington) and NASA
Conclusions

Population-synthesis code:
- We produced an initial version of a code with which we can study large populations of merger remnants, albeit with simplified assumptions.

Results:
- Common-envelope mergers on the giant branches lead to rapidly rotating merger products.
- Merger products through this channel rotate roughly $10 \times$ faster than normal single stars.
- Roughly 60% of merger products have ignited helium; ~ 40% of normal single stars have not.
- In a population with 50% initial binaries, ~ 3.4% of the single stars would be a GB merger remnant.
Conclusions

**sdB stars:**

- Contraction of a merger product due to helium ignition provides a natural way to create rapidly rotating HB stars.
- A small fraction of these HB stars have thin envelopes; these stars are close to becoming single sdB stars.

**Other observables:**

- Telltales of (former) rapid rotation may include abundance anomalies, small envelope mass, oblate stars, IR excess and asymmetric nebulae.
Future work

To-do list

- Use more flexible implementation for mass loss due to winds and rotation
- Include magnetic braking for merger product
- Look for mechanism to remove last bit of HB-star envelope (perhaps on RGB?)
- Combine population synthesis and “entropy” “sorting”:
  - do population synthesis to get the mergers
  - use entropy sorting to get a merger product
  - interpolate to create an evolution model
  - evolve it with a detailed stellar-evolution code (including rotation)
The End