Gravitational Waves Data Analysis

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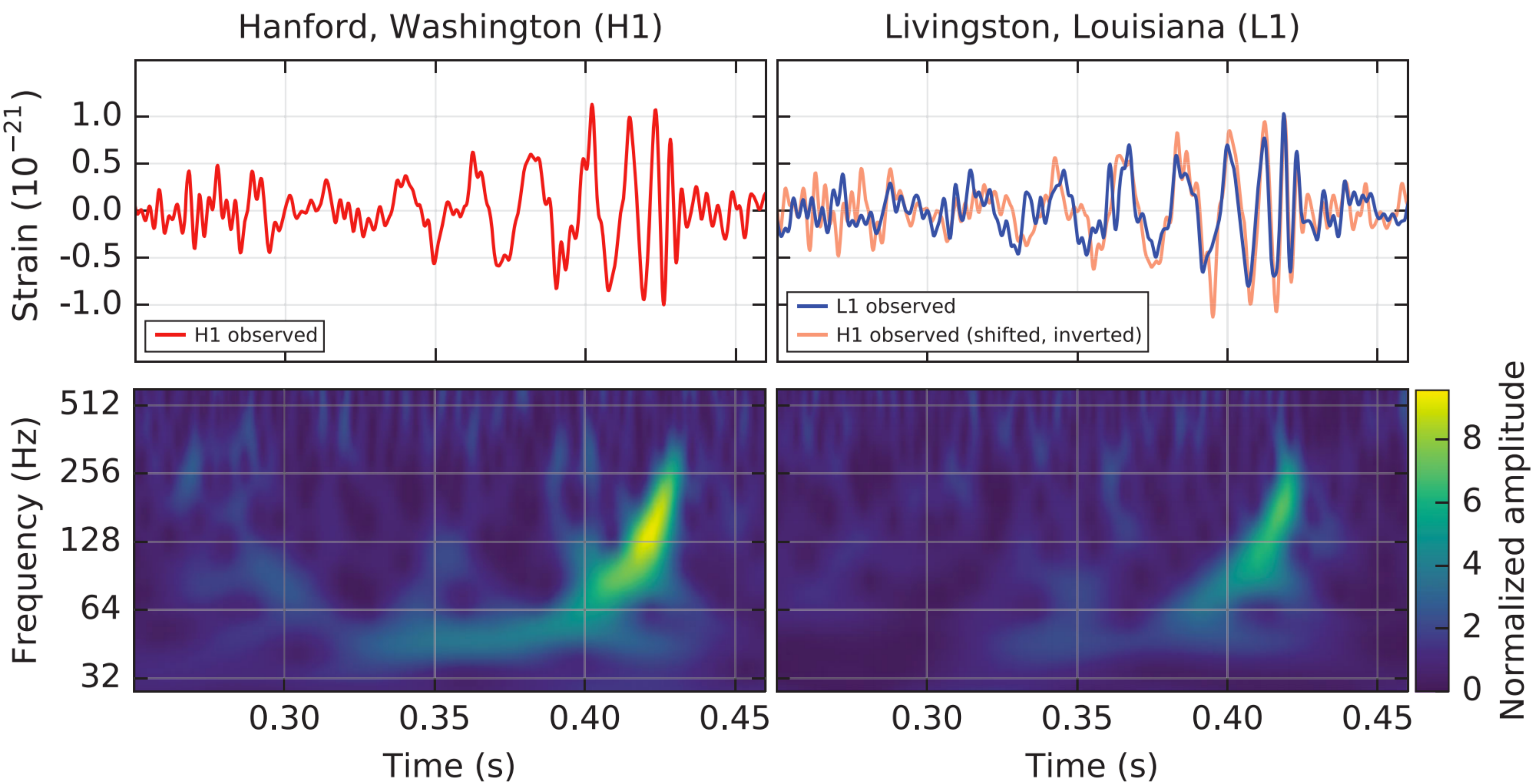
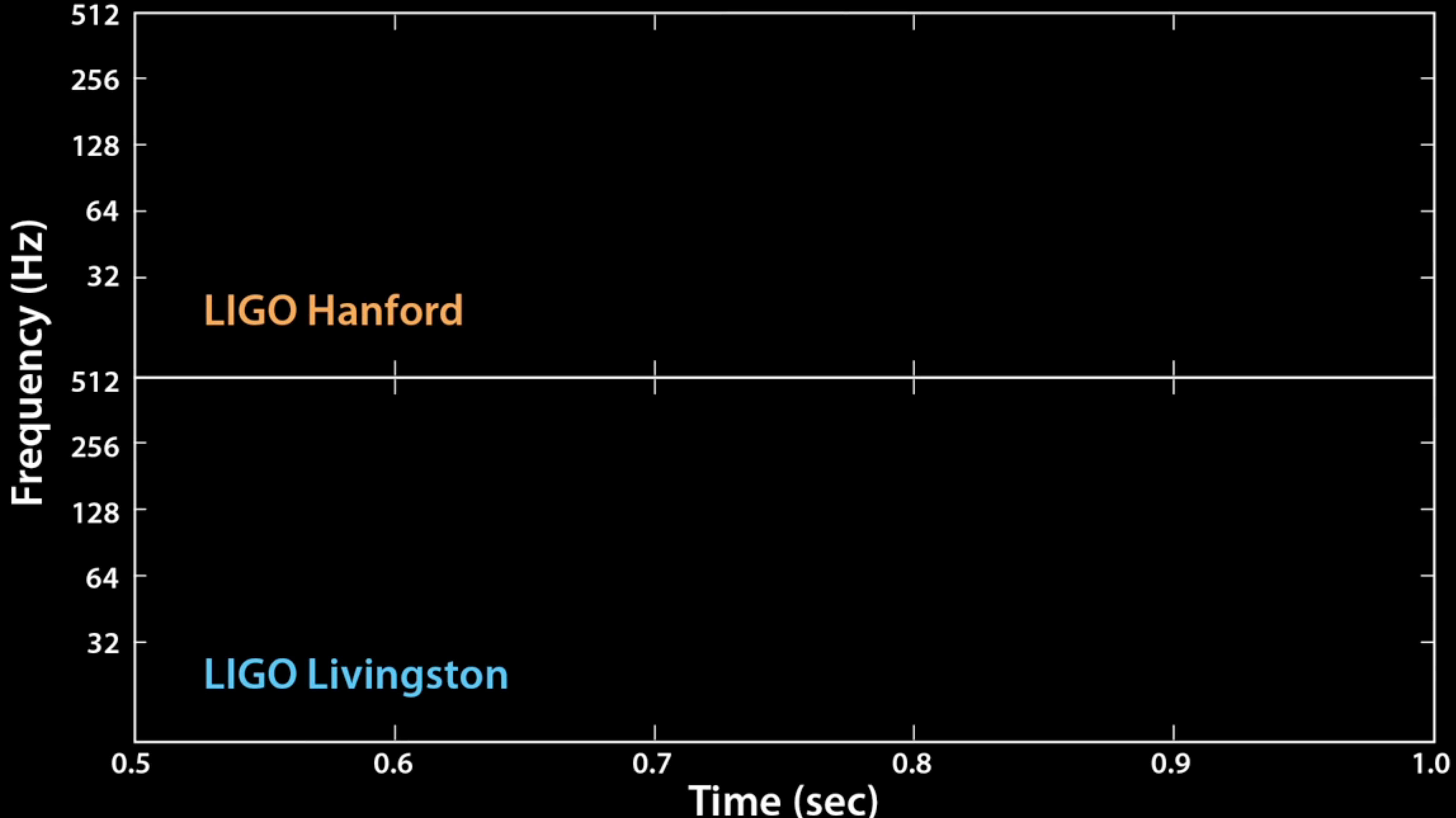


GW150914

[LVC 2016]



GW150914: GW strain to sound

[LVC 2016]

Have we detected anything?

In the case of Compact Binary Coalescences, we know:

$$h(t) \equiv \frac{1}{D} A(t) \cos(\phi(t) - \theta)$$

where $A(t) \propto f^{2/3}$ and ϕ(t) is known from post-Newtonian or Effective-one-body models

$\Rightarrow$ we know the form of the signal (compared to burst searches)

Have we detected anything II?

Our job is to know if the detector output:

$$s(t) = \begin{cases} n(t) & \text{noise} \\ n(t) + h(t) & \text{detection!!!} \end{cases}$$

What does the noise n(t) look like?

At a first approx, the simplest is to assume the noise is Gaussian, stationary, with zero mean, so the probability of n(t) is:

$$p(n) = K \exp^{-\frac{1}{2}(n|n)}$$

Noise properties

We define $S_n(f)$ as:

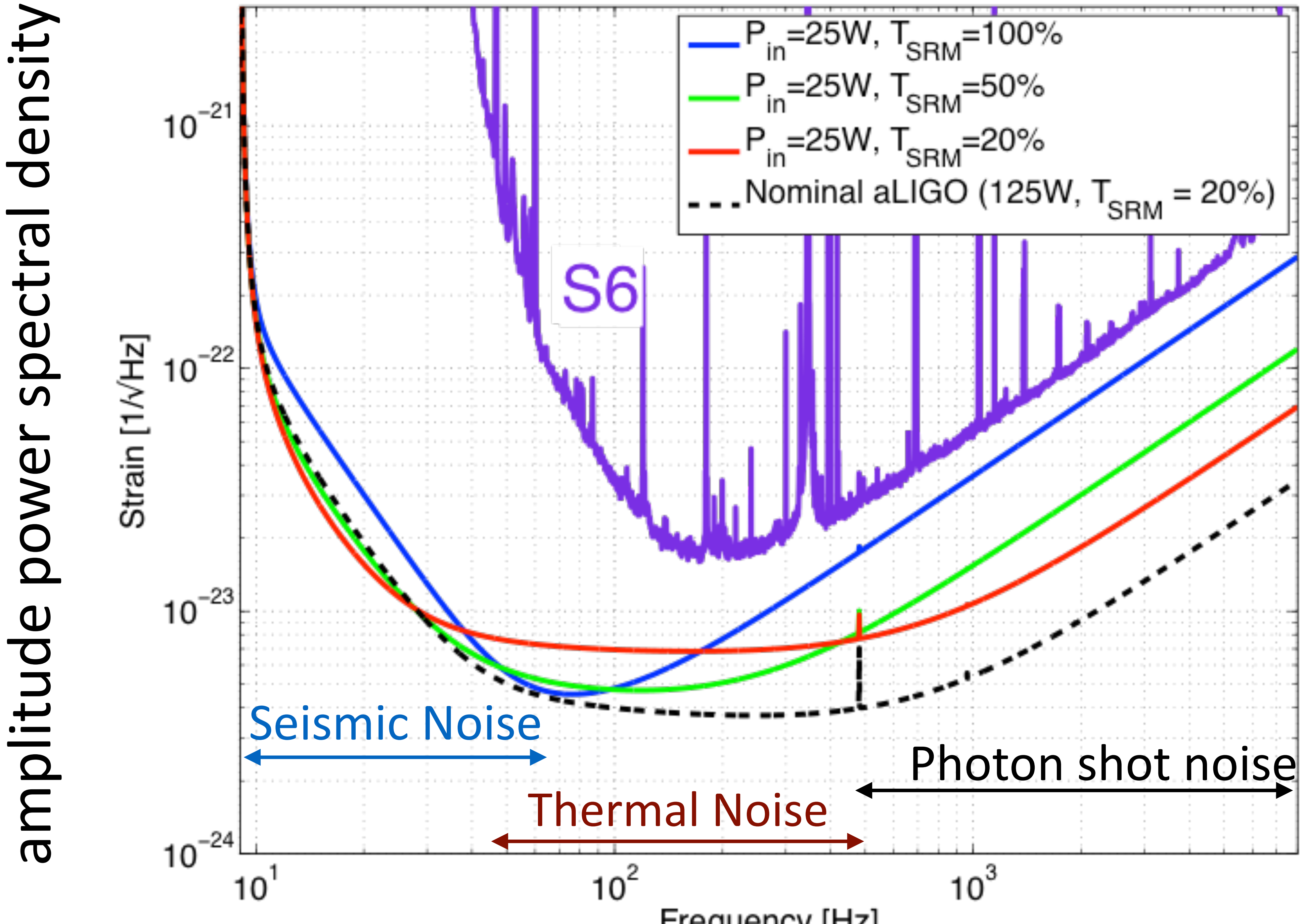
$$< \tilde{n}(f)\tilde{n}(f') >= \frac{1}{2}S_n(f)\delta(f - f')$$

and is called the (one-sided) power spectral density of the noise.

~ how much noise power is in a given frequency bin?
(see notes by Blandford and Thorne)

Understand the instrument noise very well

projected advanced LIGO noise curve (2017?)



courtesy of LIGO labs

Lets go back to Bayes' theorem …

$$p(h|s) = \frac{p(h)p(s|h)}{p(s)}$$

where

- p(h|s) is the probability that the data contains a signal h(t) given the data output s(t)
- <u>LIKELIHOOD</u>: p(s|h) is the probability of obtaining a signal s(t) given that the signal is present

signal   no signal

where:

$$p(s) = p(h)p(s|h) + p(0)p(s|0)$$

$$\Rightarrow \qquad p(h|s) = \frac{p(s|h)/p(s|0)}{p(s|h)/p(s|0) + p(0)p(s|0)}$$

Λ = Likelihood ratio - depends on the data

$$\Rightarrow \qquad p(h|s) = \frac{\Lambda}{\Lambda + p(0)/p(h)}$$

where $$p(0|s) = 1 - p(h|s)$$

$$\Rightarrow \qquad \frac{p(h|s)}{p(0|s)} = \Lambda \frac{p(h)}{p(0)}$$

priors that are independent of the data

Likelihood function I

$$\Lambda = \frac{p(s|h)}{p(s|0)} = \exp[(s|h) - \frac{1}{2}(h|h)]$$

constant

so we can threshold off (s|h):

$$\Lambda(\theta) = p(\theta)\exp[\frac{1}{D}\,(s|A(t)\cos(\phi(t)-\theta)) - \frac{1}{2}\frac{1}{D^2}(h|h)]$$

Integrating out the phase:

Likelihood function II

$$(s|A(t)\cos(\phi(t)-\theta)) = \cos\theta(s|A\cos 2\phi) + \sin\theta(s|A\sin 2\phi)$$

$$|z|\cos\Phi = x \qquad\qquad |z|\sin\Phi = y$$

$$|z| = \sqrt{x^2+y^2}$$

$$\Lambda = \frac{1}{2\pi}\int_0^{2\pi} \exp[\frac{1}{D}|z|\cos(\Phi-\theta) - \frac{1}{2D^2}(h|h)\ d\theta$$

$$\Lambda = I_0(D^{-1}|z|)\exp^{\frac{1}{2r^2}(h|h)}$$

monotonic in z

modified Bessel fn. of the 1st kind

Threshold off z!

$$|z| = \sqrt{(s|h_c)^2 + (s|h_s)^2}$$

Define an effective signal-to-noise ratio

$$\rho^2 = \frac{|z|^2}{\sigma^2}$$

where $\sigma^2 = (h_c|h_c) = (h_s|h_s)$

Template search!

Define the MATCH:

$$M = \frac{(h_1|h_2)}{\sqrt{(h_1|h_1)}\sqrt{(h_2|h_2)}}$$

max
phase and time

where λ represents our set of parameters

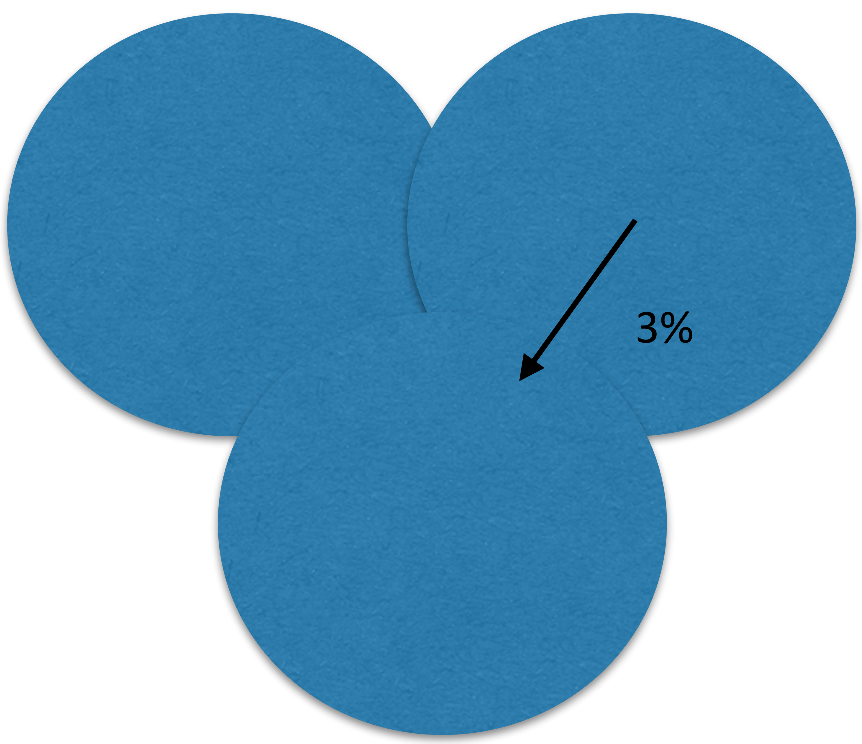
Define the metric for templates:

$$g_{ij}\delta\lambda^i\delta\lambda^j = 1 - (h(\lambda)|h(\lambda + \delta\lambda))$$

$$g_{ij} = -\frac{1}{2}\frac{\delta^2(h(\lambda|h(\lambda))}{\delta\lambda^i\delta\lambda^j}$$

& use this metric to place a template bank

such that $1 - M \leq 0.03$ $\Rightarrow$ 3% loss in SNR



use hexagonal placement to grid up space

for NS-NS, NS-BH systems, we need 10^5 templates!

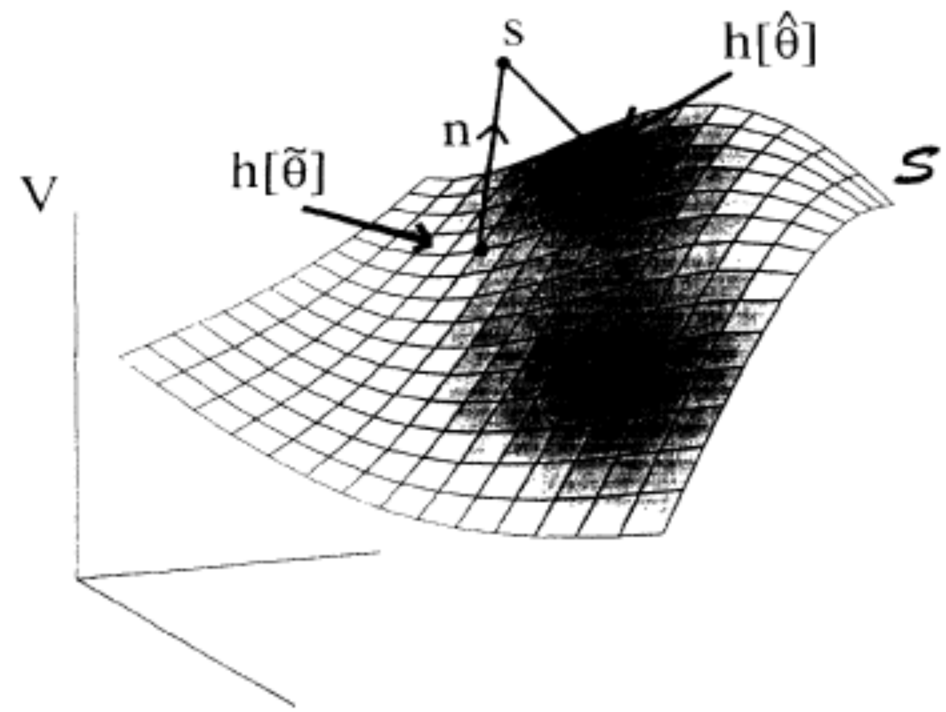


FIG. 1. Gravitational waveforms from coalescing compact binaries are completely specified by a finite number of parameters $\theta = (\theta^1, \ldots, \theta^k)$, and so form a surface $\mathcal{S}$ in the vector space V of all possible measured detector outputs $s = s(t)$. The statistical properties of the detector noise endow V with the structure of an infinite-dimensional Euclidean space. This figure illustrates the relationships between the true gravitational wave signal $h(\tilde{\theta})$, the measured signal s, and the "best-fit" signal $h(\hat{\theta})$. Given a measured detector output $s = h(\tilde{\theta}) + n$, where $n = n(t)$ is the detector noise, the most likely values $\hat{\theta}$ of the binaries parameters are just those that correspond to the point $h(\hat{\theta})$ on the surface $\mathcal{S}$ which is closest [in the Euclidean distance $(s - h \,|\, s - h)$] to y.

Gaussian noise: distribution of SNR

$$p(|\rho|) = \frac{1}{\sqrt{2\pi}} \exp^{(-\rho^2/2)}$$

We are interested in the probability of getting $|\rho| > \rho^*$ in a single trial, which is obtained from integrating $p(|\rho|)$ from $|\rho|^* \rightarrow \infty$

$$p(|\rho| > \rho^*) = erfc(\rho^*/\sqrt{2})$$

$$|\rho|^* = \begin{cases} 5 \rightarrow 10^{-7} \\ 6 \rightarrow 10^{-9} \\ 7 \rightarrow 10^{-12} \end{cases}$$

How many trials in a year?

For an inspiral signal, the autocorrelation time is ~3 ms, so 1010 independent trials in a year of data. Need 104 templates to cover mass space.

So, for a false alarm rate of 1/100 years, we need

$$p(|\rho| > \rho^*) = 10^{-17}$$

or

$$\rho^* > 8.6$$

But, we have multiple detectors! $\rho = \sqrt{\rho_H^2 + \rho_L^2}$

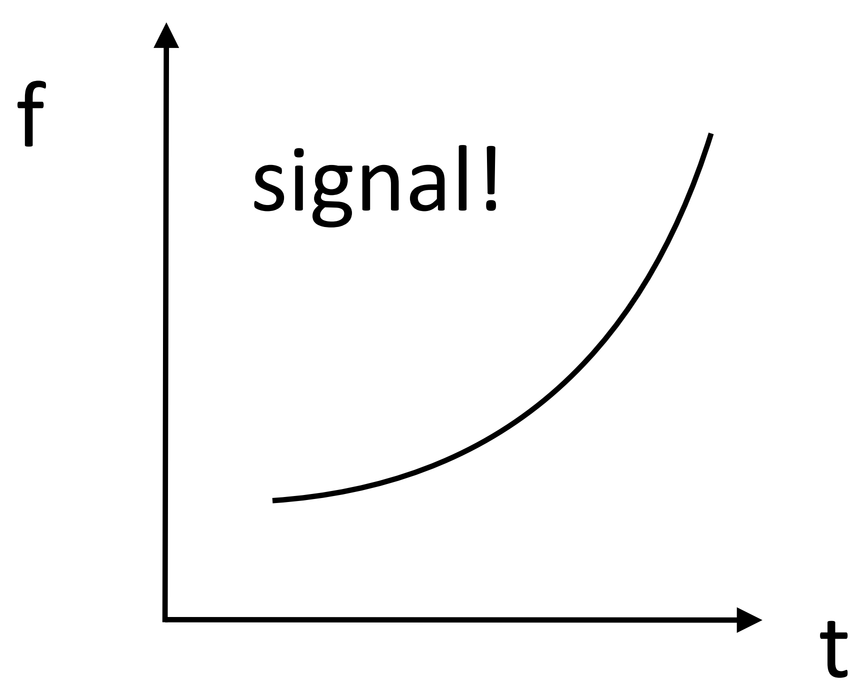
Question in homework!

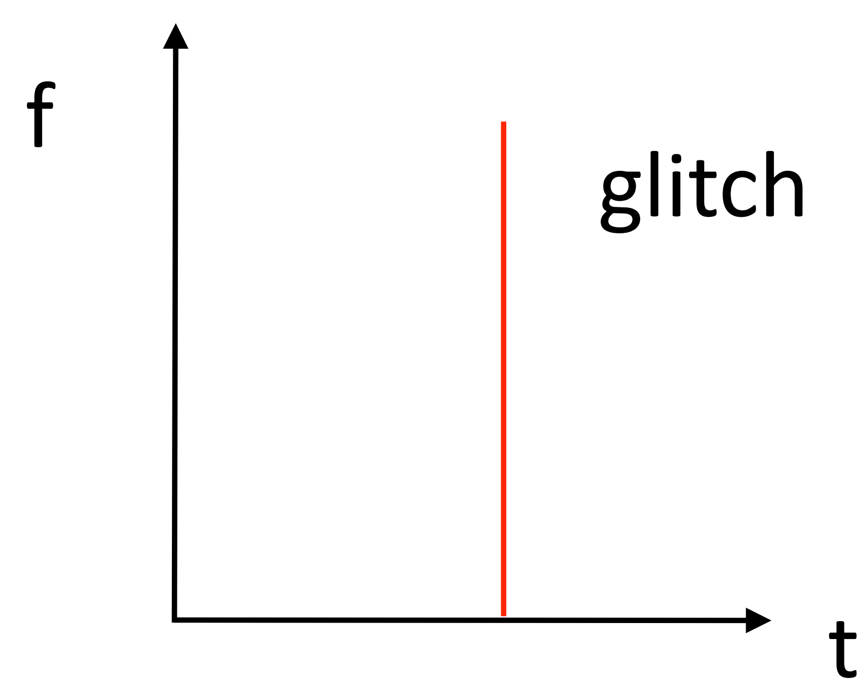
Noise is reality in non-Gaussian & non-stationary .. glitchy!

Imagine impulse response for some matched filter:

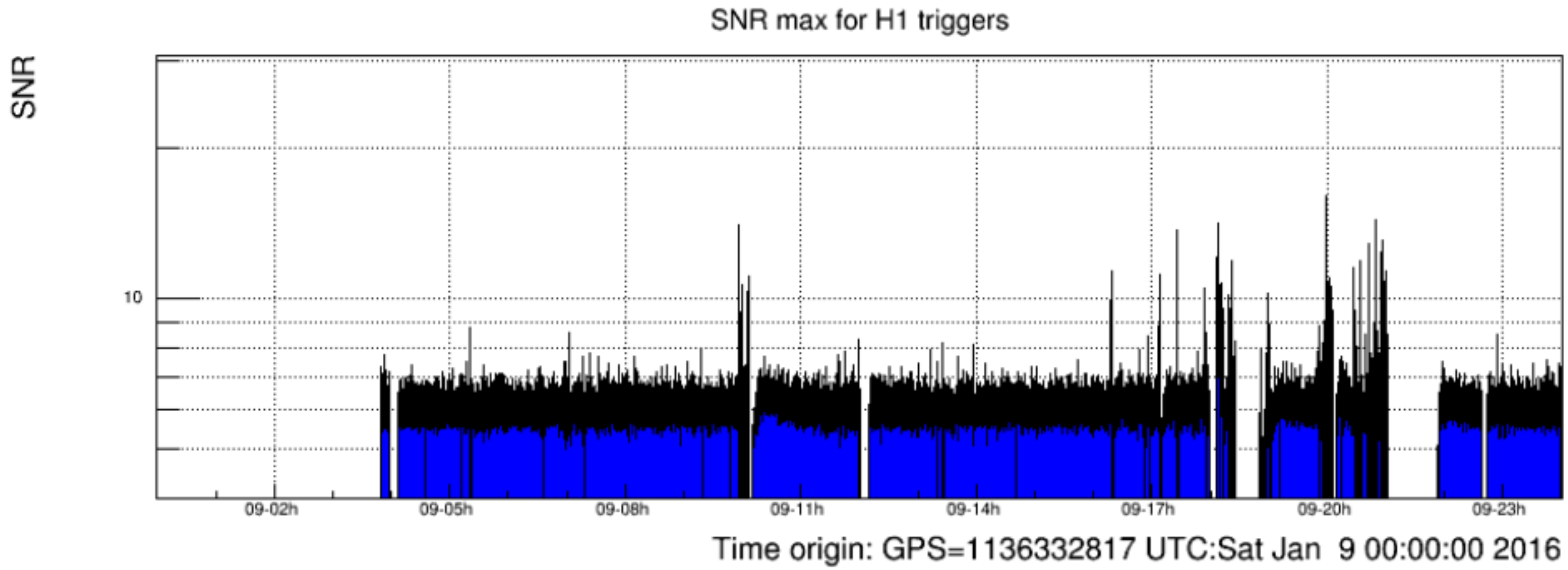
$$s(t) = n(t) + \delta(t - t_d)$$

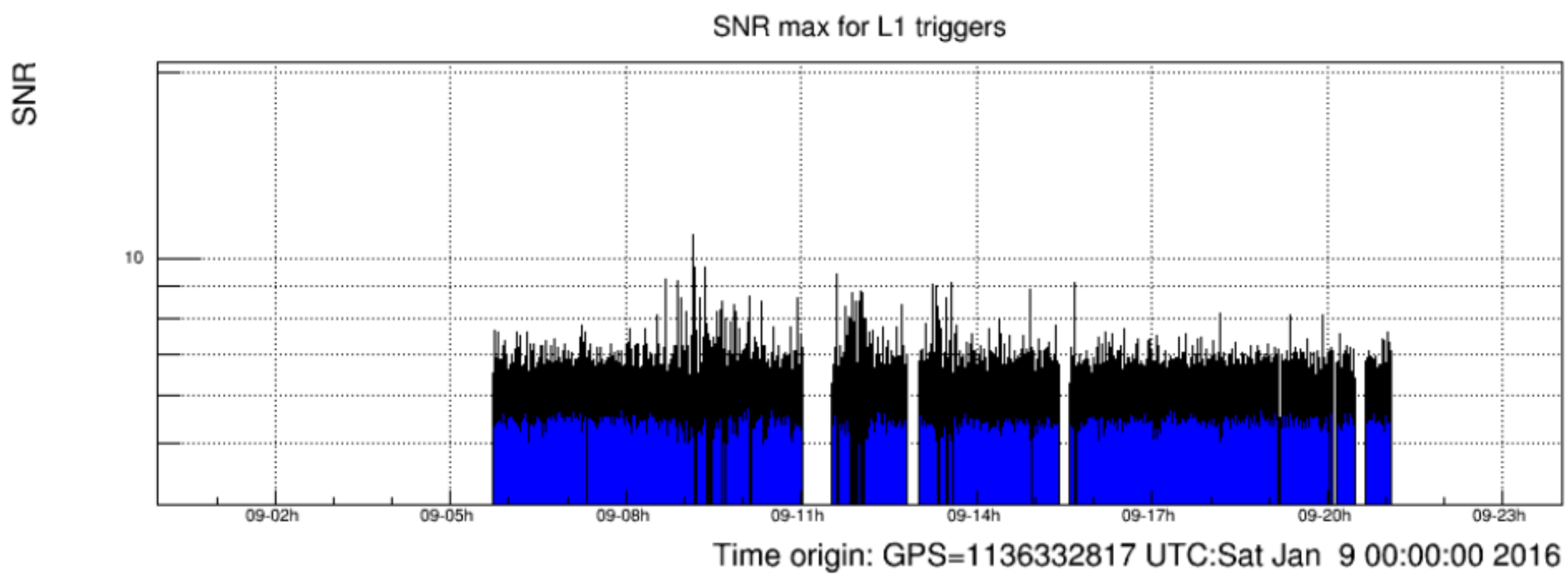
… causes peaks of SNR not due to signal!





Very noisy data, many “outliers”





How to discriminate?

Divide h(t) into p bins of equal power … ask whether we get 1/p th power from the pth bin?

Define:

$$\chi^2 = \frac{1}{p}\sum_{l=1}^{p}\left[\frac{\rho}{p} - \rho_2\right]^2$$

χ^2 has 2p-2 degrees of freedom

χ^2 small -> signal

large-> noise

…finally our detection statistic!

Weight SNR by χ^2

$$\rho_{new} = \begin{cases} \rho \ \text{ if } \ \chi^2 \leq n_{dof} \\ \rho \left[\frac{1}{2}(1 + (\frac{\chi^2}{n_{dot}})^3 \right]^{-1/6} \quad \chi^2 > n_{dof} \end{cases}$$

Challenges: complicated detection statistic, no good model for non-Gaussian noise, template cancellation….

…so instead we have to measure the false alarm rate (FAR) …

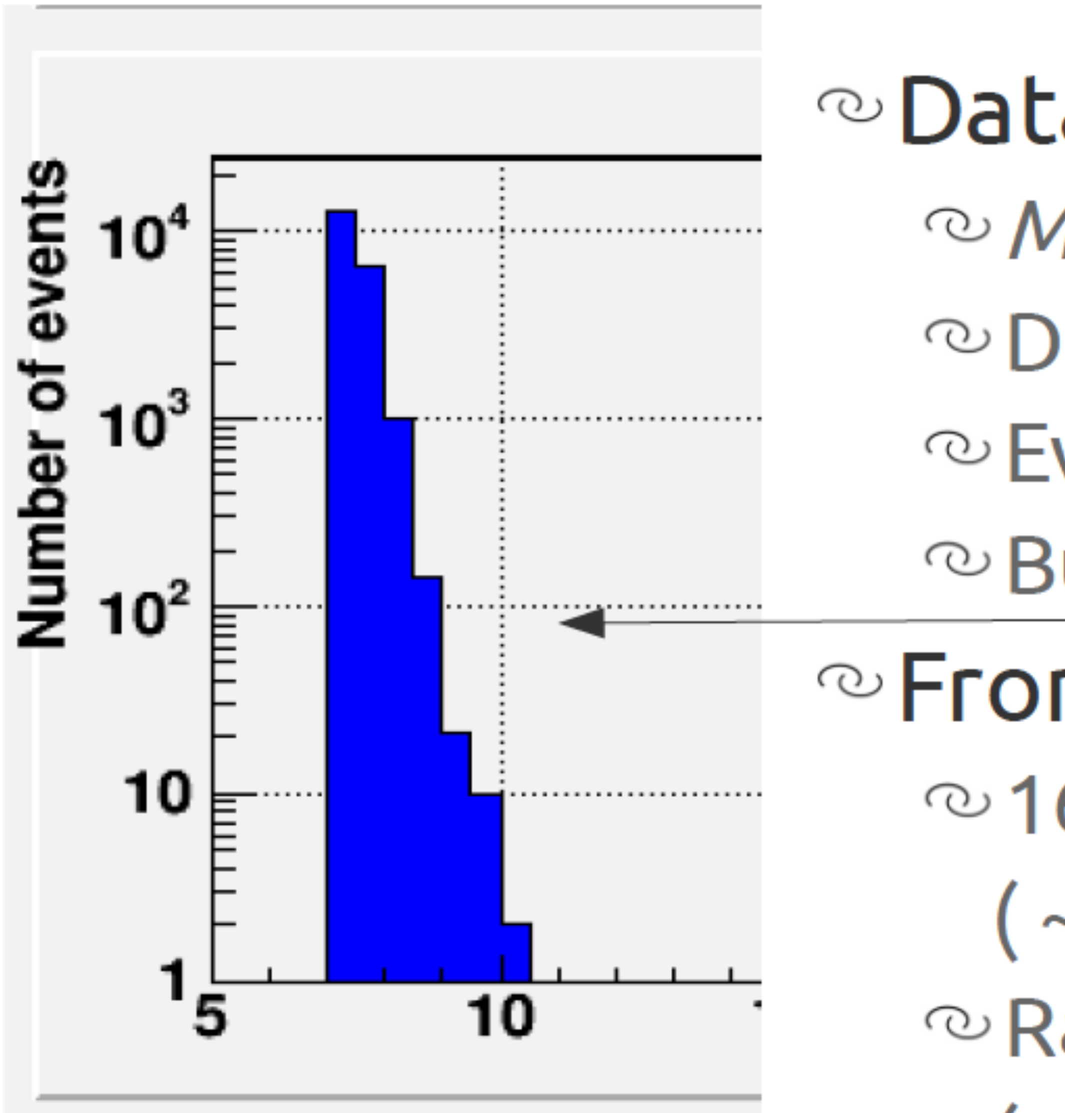
… calculating our background rate.

Signal is coincident if it has the “same” mass parameters and arrives with the light travel time + measurement error.

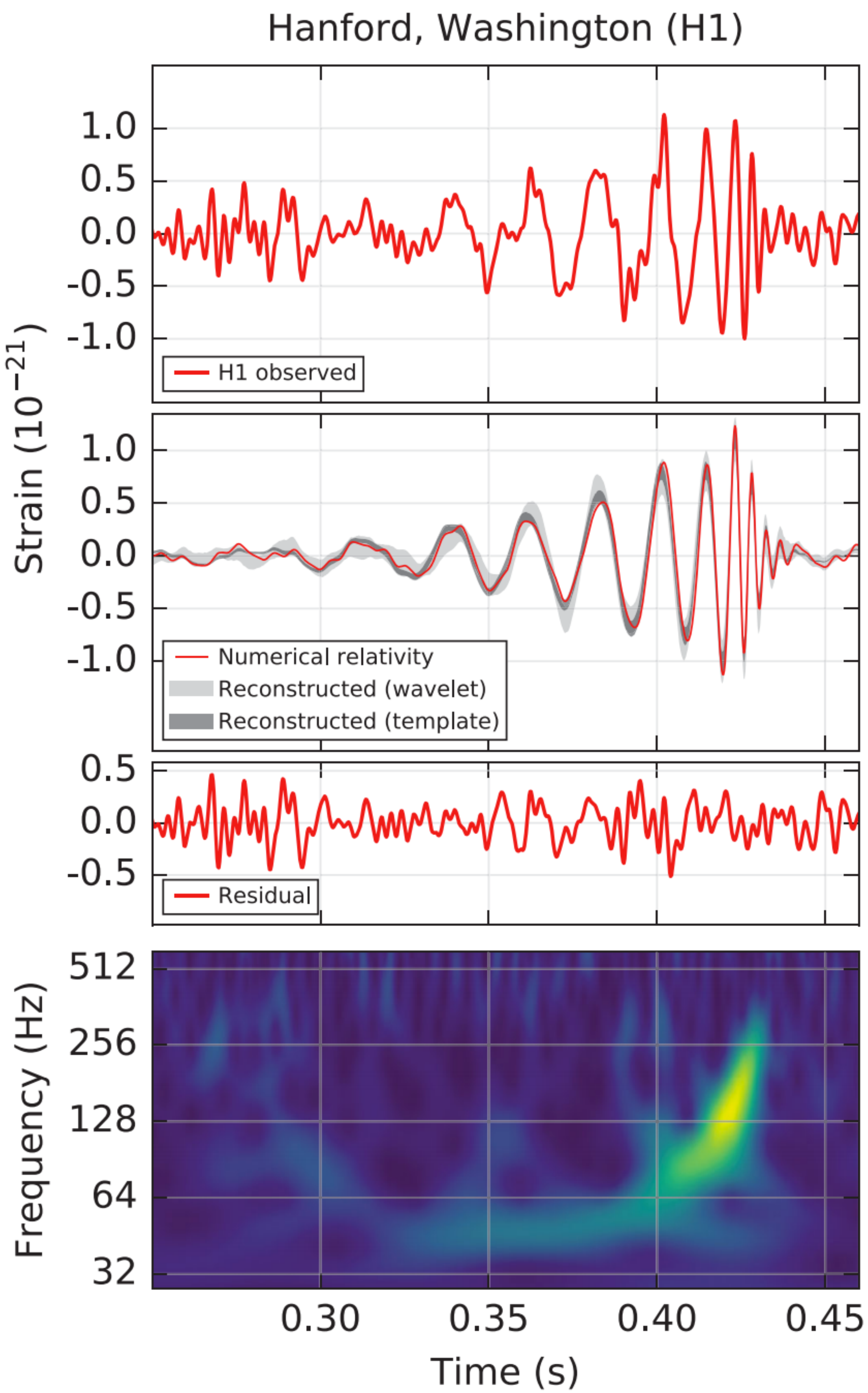
Shift the data by moving the light travel time
⇒any coincidences are due to noise alone

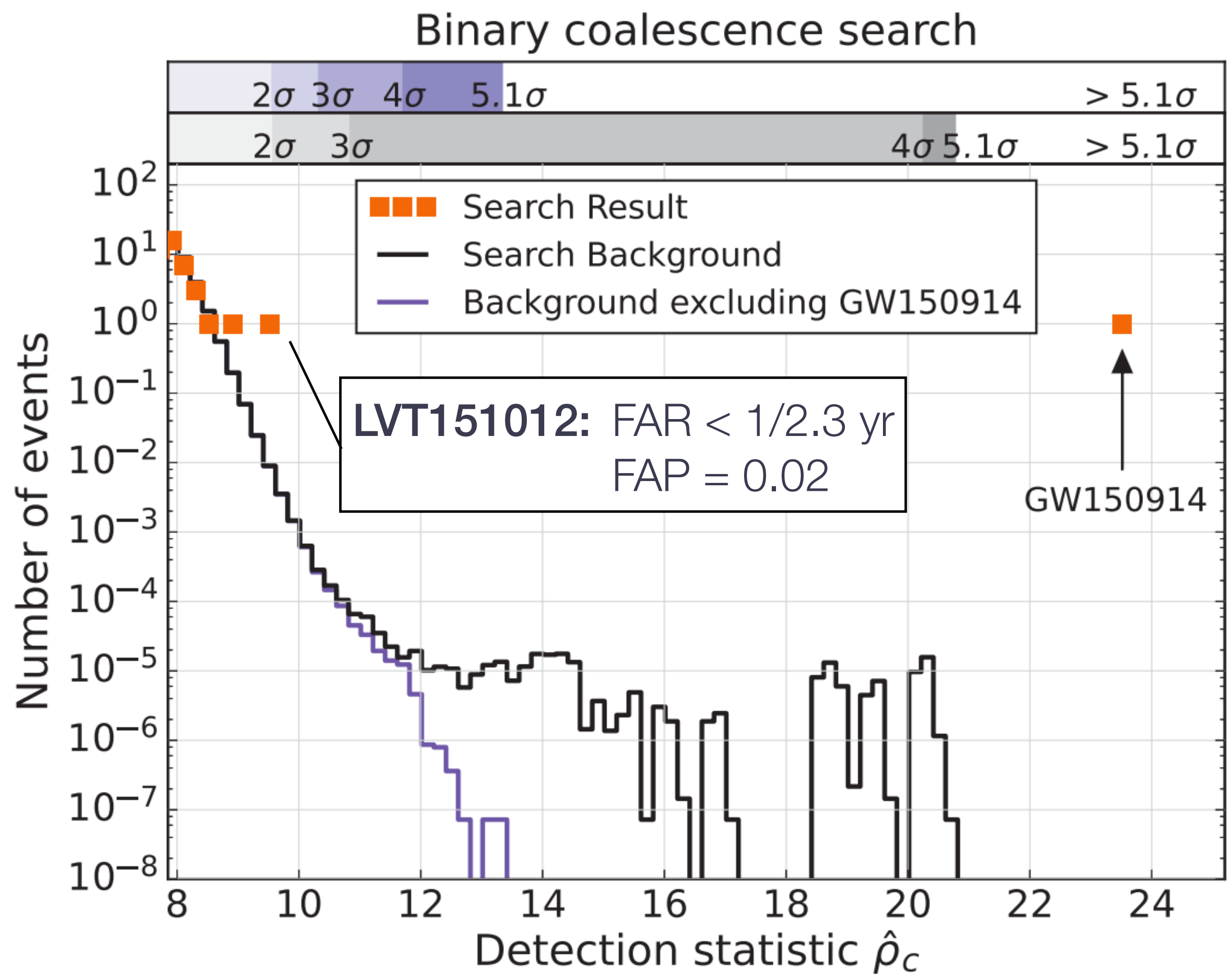
Background rate of noise coincidences; repeat many times with many different shifts to build up statistics.

Determine false alarm probability



- Data properties:
  - *Many* outliers in H1, L1
  - Do coincidence test
  - Even then, trigger rate >100Hz
  - But: extremely steep fall-off with S/N
- From 16 days to >200,000 yrs?
  - 16 day data cut in 0.5s segments (~3 million)
  - Random combine segments! (~9 10^{12} 0.5 s "time slides")
  - @S/N ~13: N in 16 days = 2 10^-7 i.e 5 σ
  - No stronger noise triggers & cannot cut data below 0.5 s



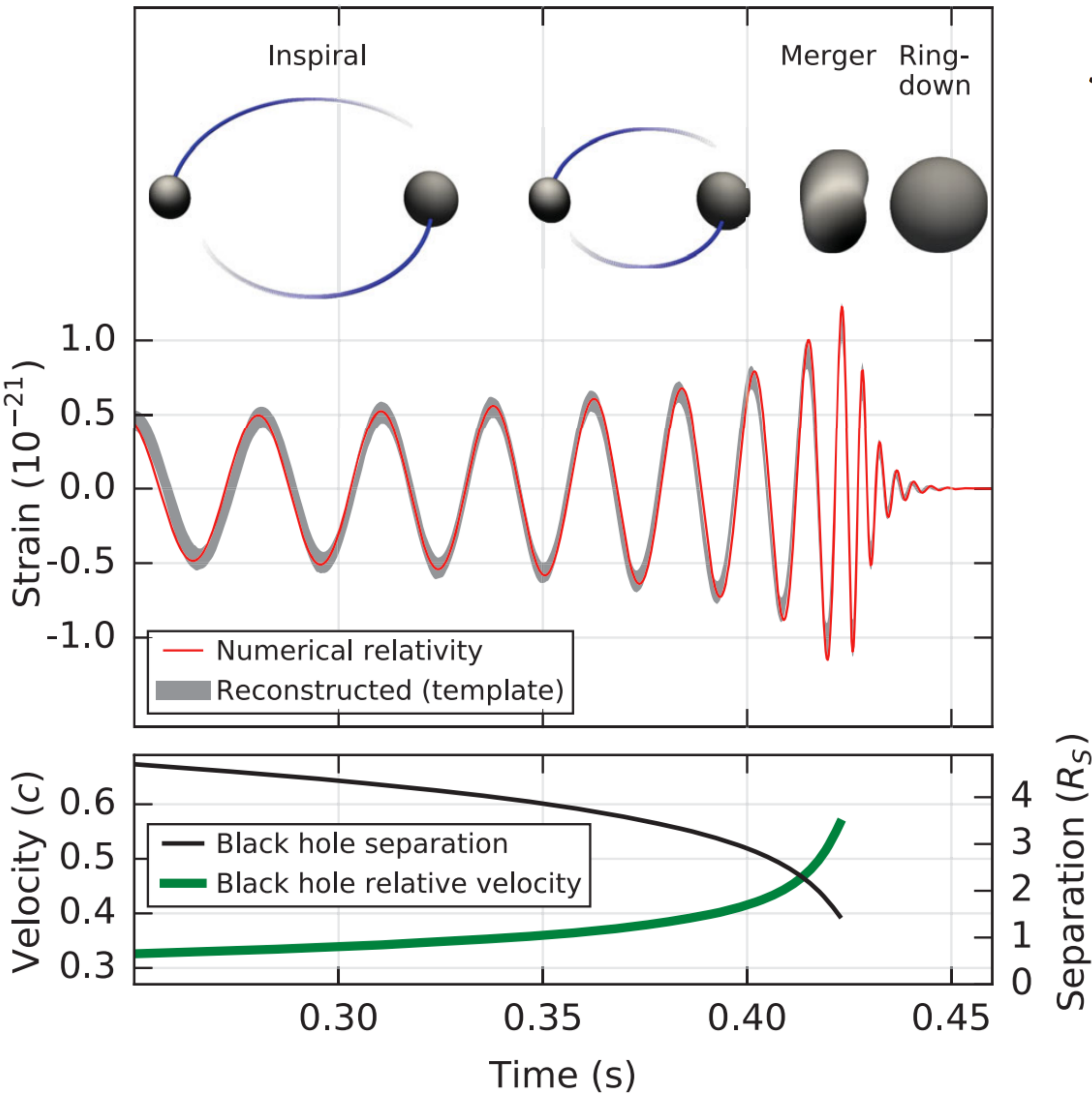


- Binaries with masses 1–99 M⊙, total mass < 100 M⊙, dim.less spin < 0.99
- 250,000 PN and EOB signal templates. Matched-filter SNR + χ^2 statistic.
- Measured on 608,000-yr background, false-alarm rate < 1 in 203,000 yr (2×10^{-7} false alarm = 5.1σ)

Background drop by 100 per unit SNR

GW150914: matched-filter inspiral search

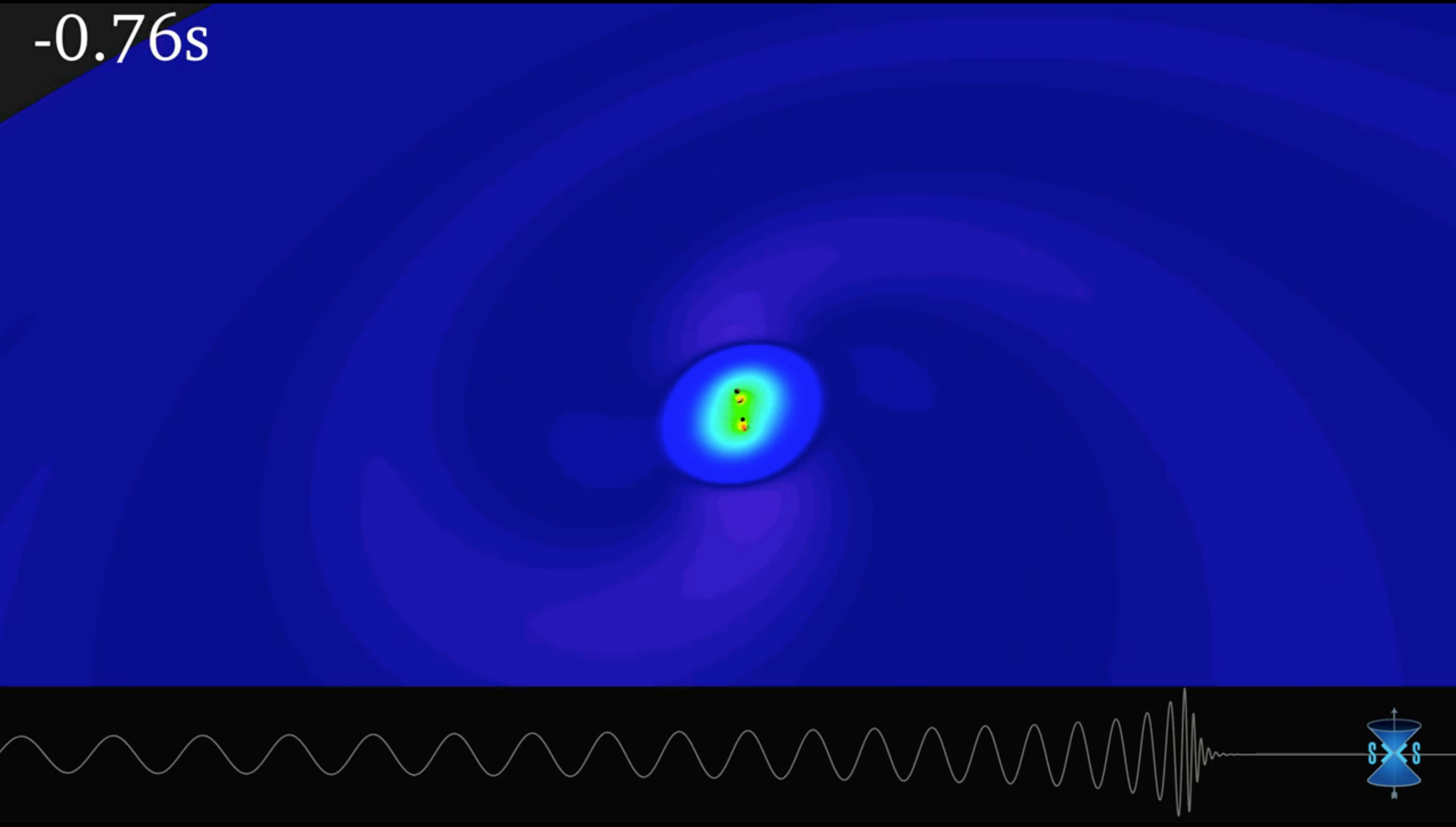
[LVC 2016]



$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$
$$= \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

GW150914: inspiral, merger, and ringdown

[LVC 2016]



GW150914: numerical relativity simulation

[SXS collaboration 2016]

Inference of Source Parameters

Explicitly map out:

$$p(\boldsymbol{\theta}|s) \propto p(\boldsymbol{\theta})\mathcal{L}_{\rm total}(s|\boldsymbol{\theta})$$

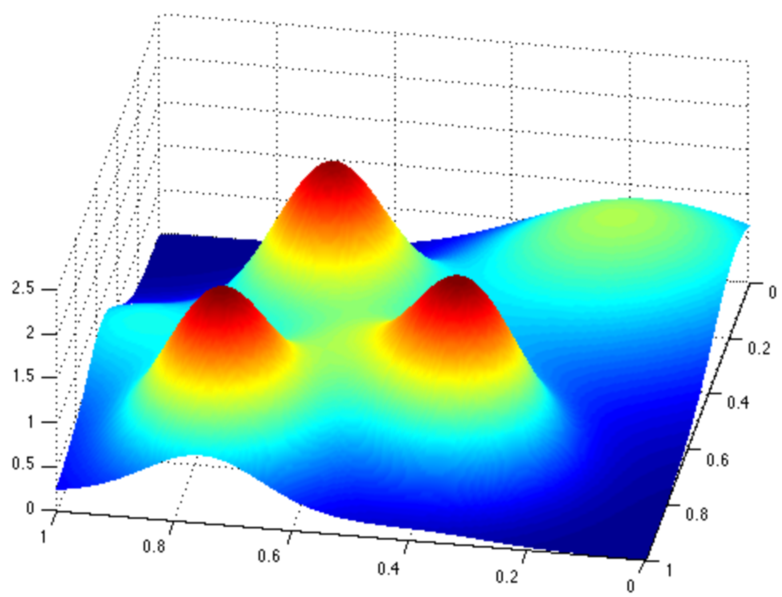
h(t): 30-40 dimensions

+ Masses (initial + final)

+ Spin (initial + final)

+ Geometric properties:
  - Inclination angle
  - Source Position
  - Luminosity distance

[+ 10 Instrument Calibration per IFO]

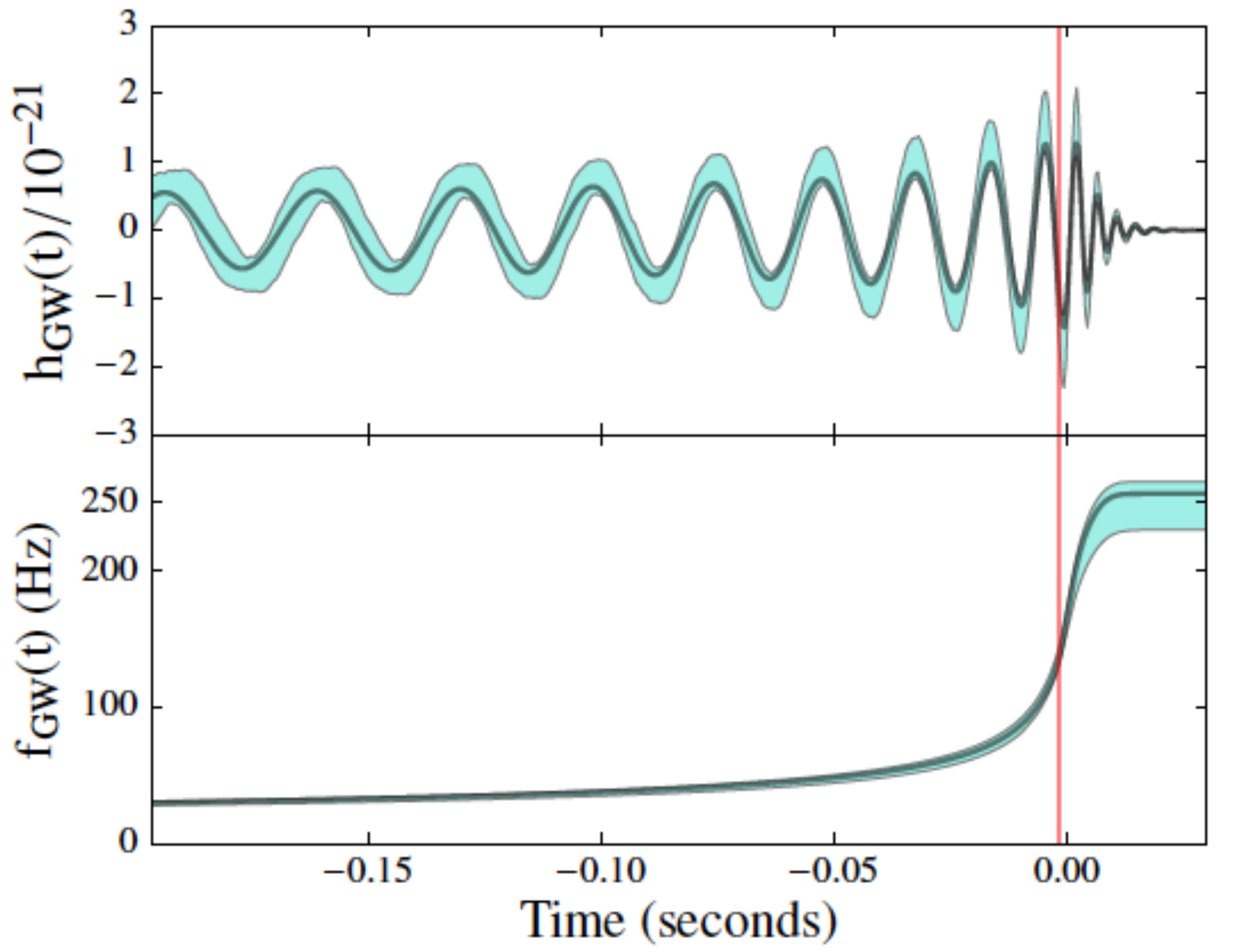


a) Markov Chain Monte Carlo
b) Nested Sampling

Model Waveforms used in GW150914

$$h_k = F_k^{(+)} h_+ + F_k^{(\times)} h_\times$$

$$h_+(t) = A_{\rm GW}(t)\left(1+\cos^2\iota\right)\cos\phi_{\rm GW}(t)\,,$$
$$h_\times(t) = -2A_{\rm GW}(t)\cos\iota\sin\phi_{\rm GW}(t)\,,$$

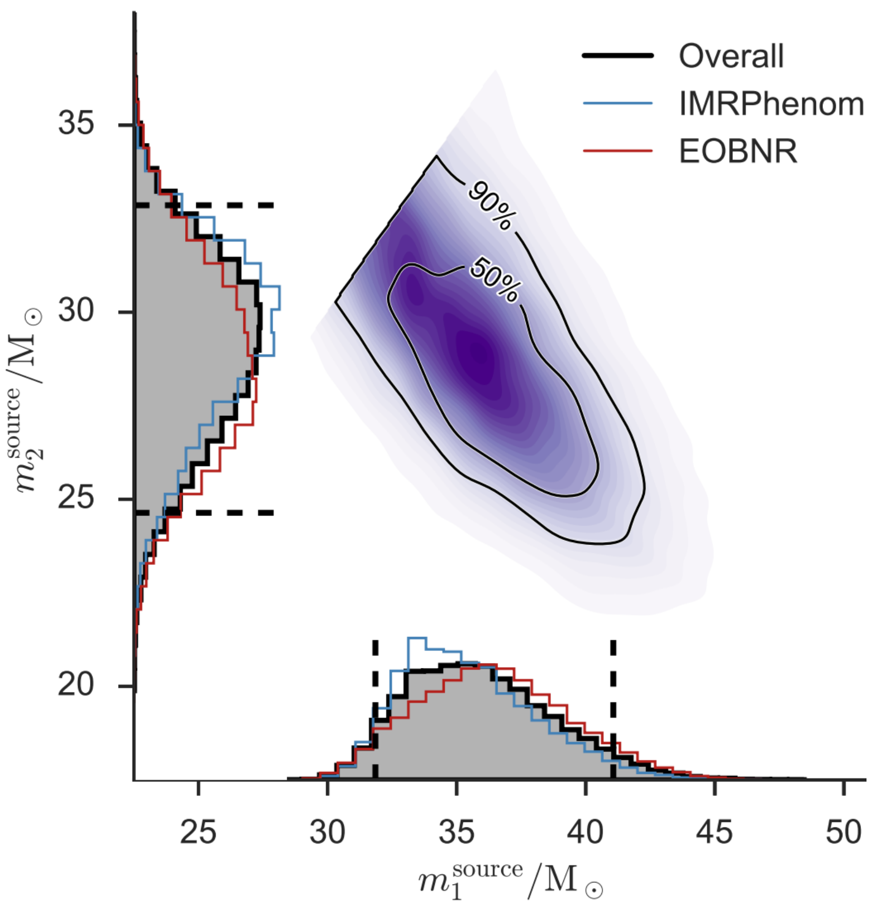


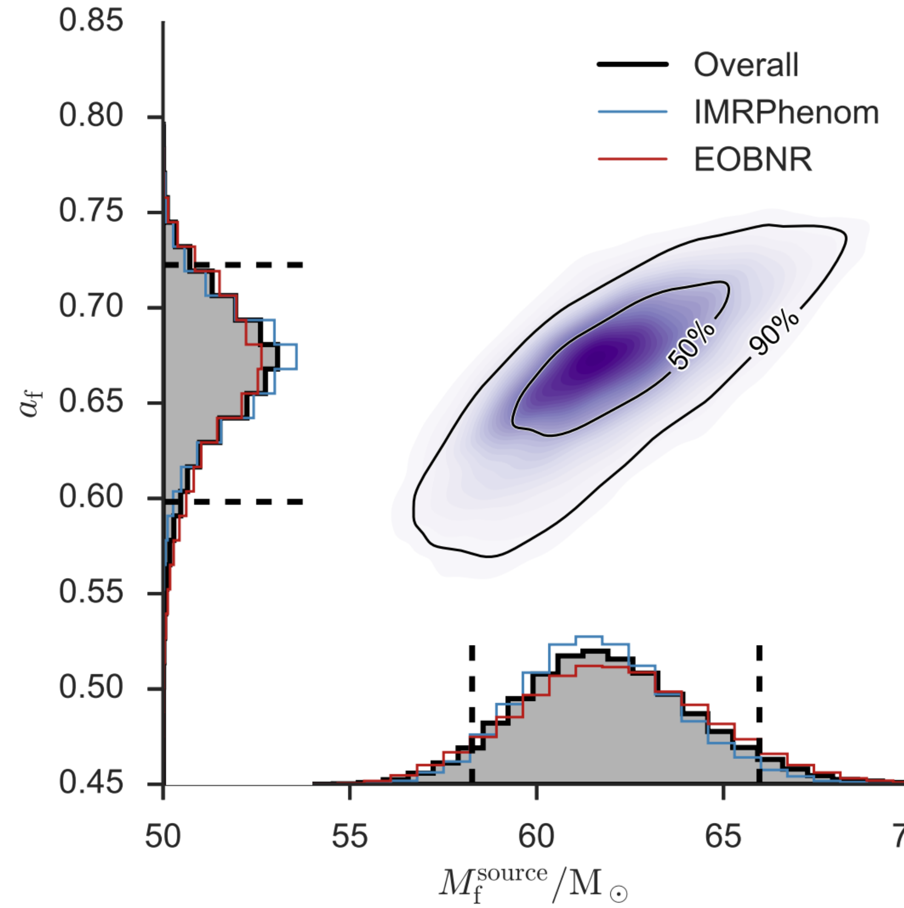
2 Classes of Waveforms Used:
a) Spinning Effective One Body Numerical Relativity

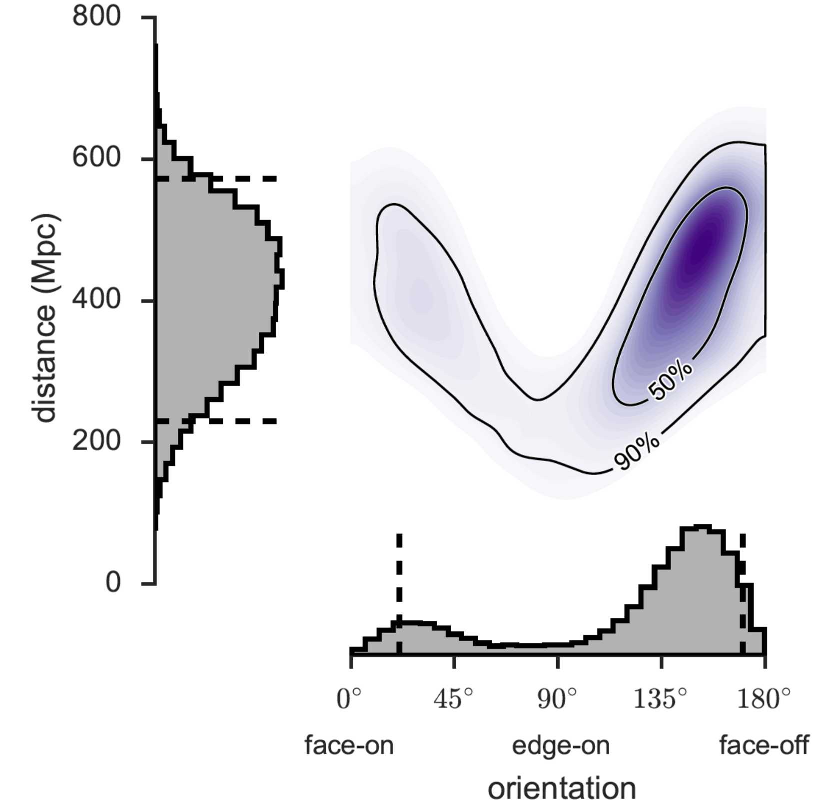
weak field PN + test particle resummed Hamiltonian, RR

b) Inspiral-Merger-Ringdown Phenomenological Fit.

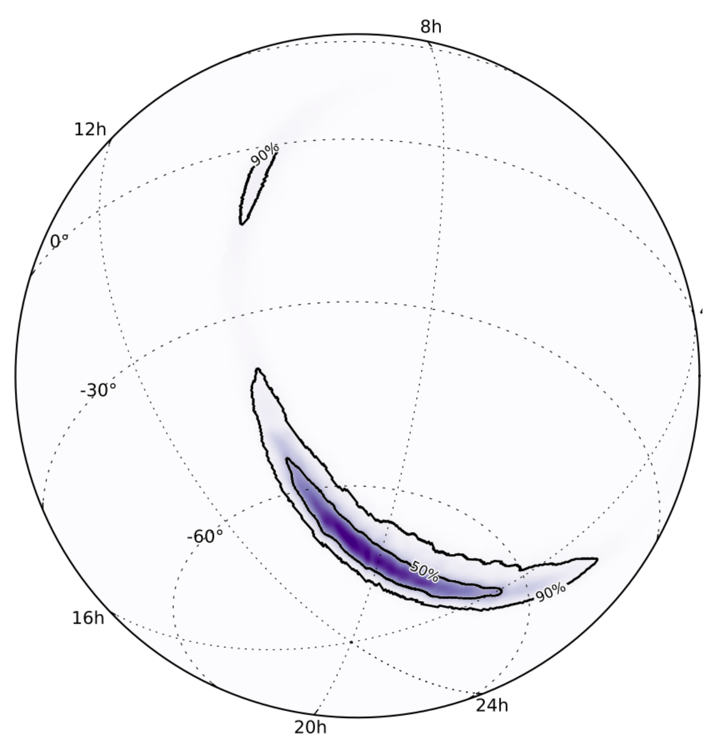
=> Consistency & Systematic Error Analysis

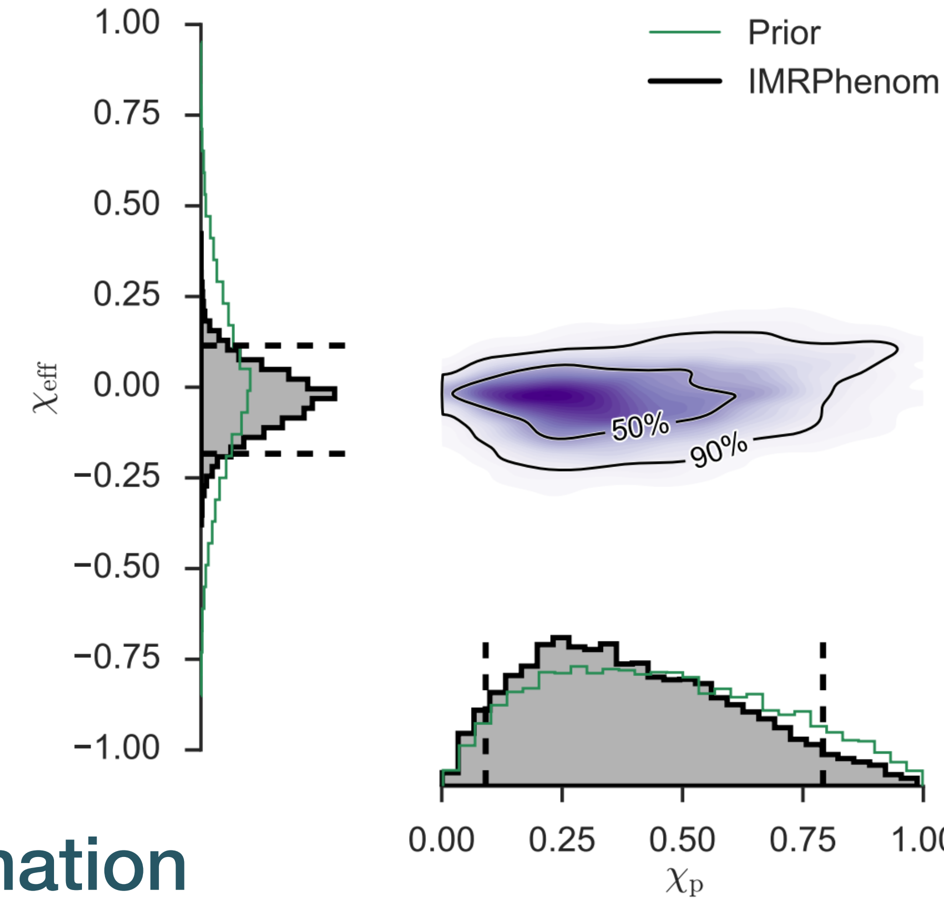






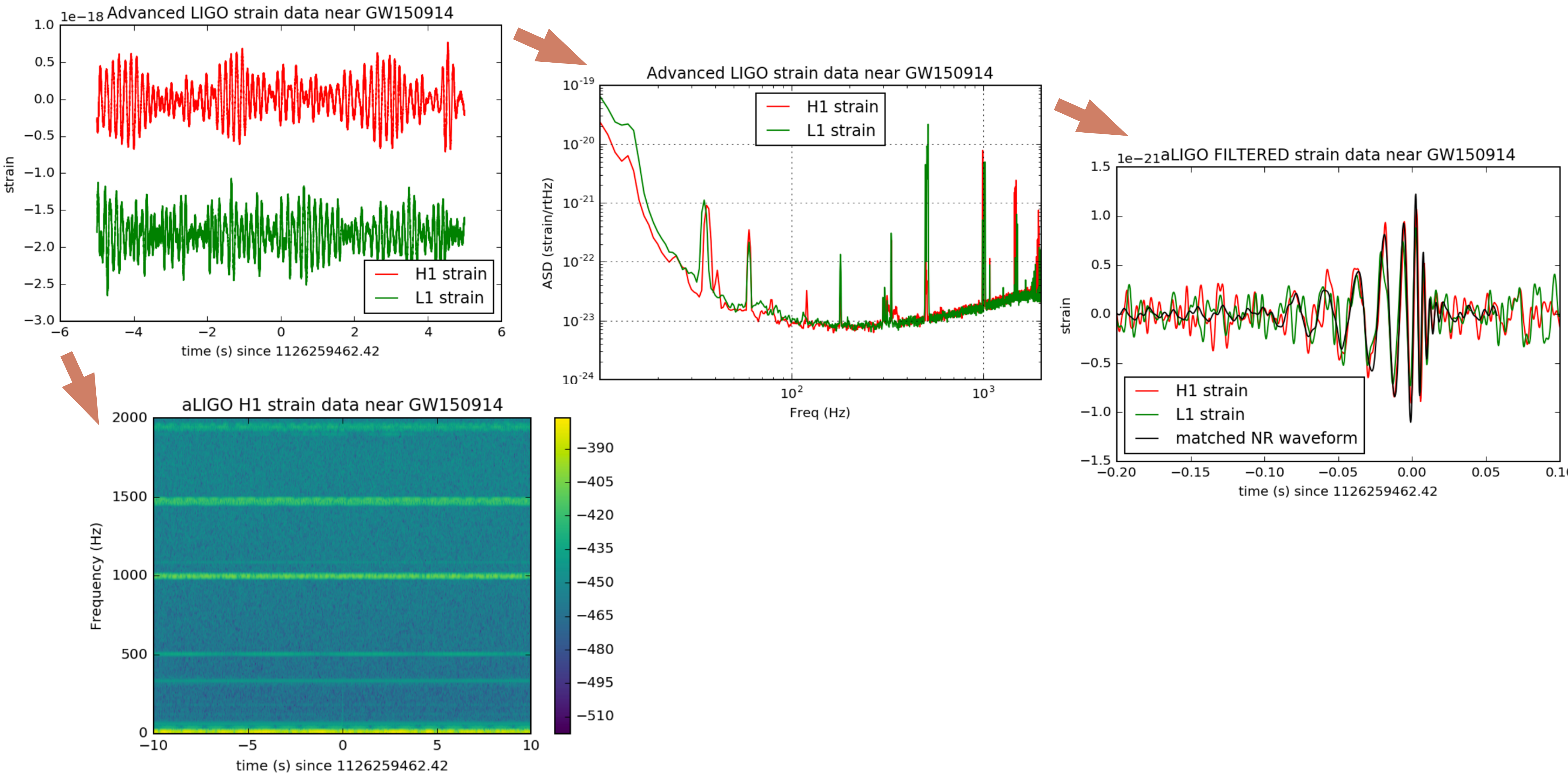
Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

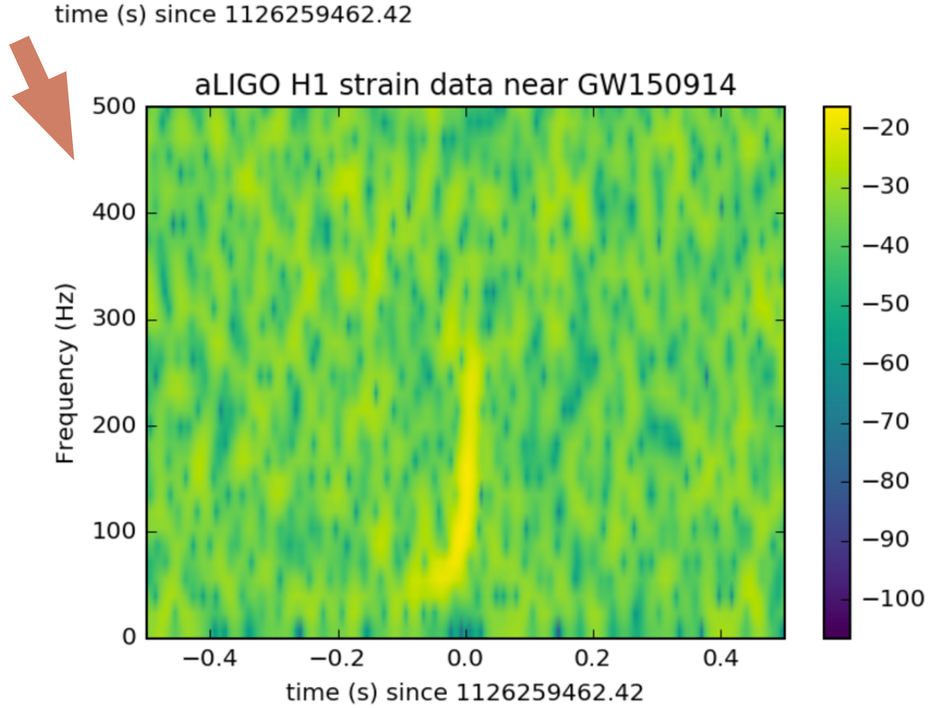




GW150914: parameter estimation

[LVC 2016]





GW150914 data release

[LVC 2016]