THE UNIVERSITY OF SYDNEY

What do plasma astrophysicists ignore by using only half of Maxwell's equations?

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Maxwell's equations

The terms in red are neglected (explicitly or implicitly) in astrophysical electrodynamics when a plasma is included:

$$\mathrm{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\mathrm{curl}\,\mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t},$$

$$\mathrm{div}\,\mathbf{B} = 0, \qquad \mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0}.$$

I refer to these as the inductive electric field $\mathbf{E}_{\mathrm{ind}}$,

& (its time derivative) the displacement current $\mathbf{J}_{\mathrm{disp}}$.

When is this justified?

What physical effects are being ignored?

Example: pulsar electrodynamics

Pulsar electrodynamics is based on two incompatible models.

Vacuum dipole model

Rotating magnetized neutron star in vacuo
dipole axis at oblique angle to rotation axis.

"dipole" term: $\mathbf{B}_{\rm dip} \propto 1/r^3$
"inductive" terms: $\mathbf{E}_{\rm ind}, \mathbf{B}_{\rm ind} \propto 1/r^2$
"radiative" terms: $\mathbf{E}_{\rm rad}, \mathbf{B}_{\rm rad} \propto 1/r$

$\mathbf{E}_{\rm ind}$ is not negligible: similar order of magnitude as $\mathbf{E}_{\rm cor} \propto 1/r^2$

Torque causing slow-down due to $\mathbf{E}_{\rm rad}, \mathbf{B}_{\rm rad}$

Another torque causes alignment on same time scale

Plasma is specifically excluded

Corotating magnetosphere model

Plasma assumed to corotate with the star

Electrodynamics reduced to electrostatics

either, magnetic and rotation axes assumed aligned.

or, assumed stationary in corotating frame

"In a noninertial frame rotating with the neutron star ...
$\nabla \times \mathbf{E} = 0$, hence $\mathbf{E} = -\nabla\Phi$"

(Scharlemann, Arons & Fawley ApJ 222, 297, 1978)

Thus: $\mathbf{E}_{\rm ind}$ & $\mathbf{J}_{\rm disp}$ are effectively assumed to be zero.

This assumption is wrong

$\mathbf{E}_{\rm ind}$ is not negligible & it is not screened by plasma

Example: solar flares

Explosive magnetic energy release => $\partial \mathbf{B}/\partial t \neq 0$
=> $\mathbf{E}_{\rm ind} \neq 0$ should be an essential feature.

Magnetic reconnection essential to magnetic energy release.

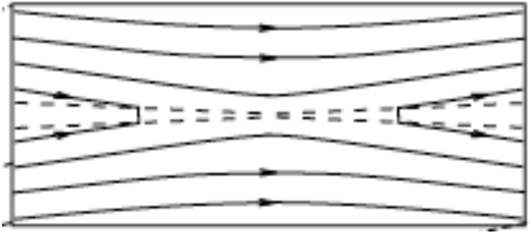


Figure: Petschek reconnection model

Reconnection models are steady-state => $\mathbf{E}_{\rm ind} = 0$, $\mathbf{J}_{\rm disp} = 0$.

Potential $\mathbf{E} \neq 0$ due to $\rho \neq 0$ & charges on boundaries.

Implications

Does it matter that we ignore $\mathbf{E}_{\rm ind}$ in astrophysical plasmas?

Yes: I identify 7 implications: 3 for pulsars, 3 for flares + 1 other

Pulsars:

1. Inclusion of $\mathbf{J}_{\rm disp}$ => quasi-static models violently unstable
   (Levinson, Melrose, Judge & Luo, ApJ 631, 456, 2005)
2. $\mathbf{E}_{\rm ind} \neq 0$ => magnetosphere not corotating
3. Acceleration by inductive $E_{\parallel}$ in outer gap

Flares:

4. EMF that drives a flare (Swann, Phys. Rev. 43, 217, 1933)
5. Inductively induced inflow in magnetic reconnection
6. Polarization current as a driver of this inflow

Acceleration mechanisms

7. Role of $\mathbf{J}_{\rm disp}$ in acceleration by $E_{\parallel}$
   (Song & Lysak, PRL 96, 145002, 2006)

J_{disp} leads to oscillations

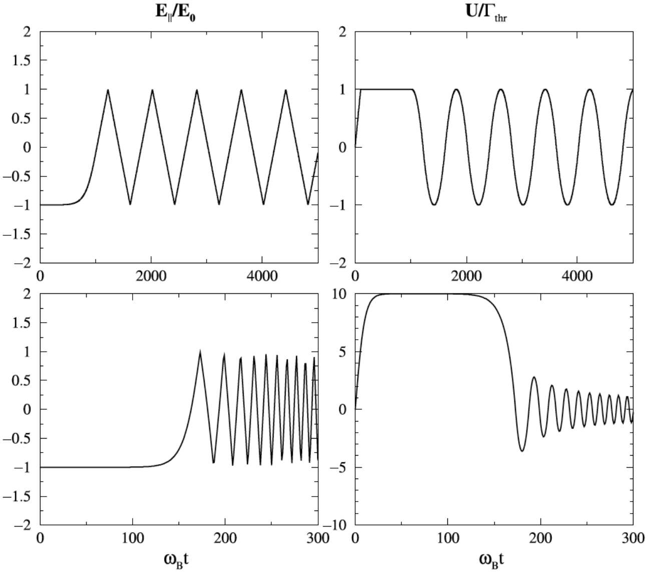


FIG. 1.—Evolution of the $E_{\parallel}$ (*left panels*) and the 4-velocity (*right panels*) in a large-amplitude oscillation with a uniform initial electric field $\tilde{E}_0 = -10^4$ (*top panels*) and $\tilde{E}_0 = -10^6$ (*bottom panels*). The 4-velocity saturates at $U/\Gamma_{th} = 1$, below which pair creation becomes ineffective, and the system oscillates after the charge screening overshoots.

Oscillations saturate through pair creation

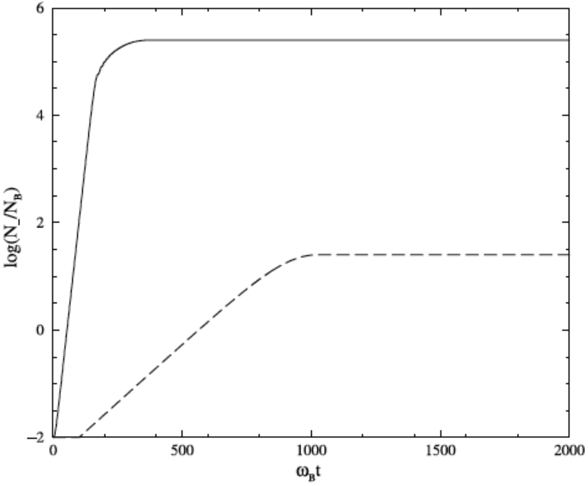


FIG. 2.—Evolution of the pair density, showing an exponential increase until the screening overshoots, after which it remains roughly constant as the system oscillates. The dashed line corresponds to the case with $\tilde{E}_0 = -10^4$ in Fig. 1, and the solid line to $\tilde{E}_0 = -10^6$.

Pulsar magnetospheres cannot be corotating

Plasma charges can screen $E_{\mathrm{ind}||}$, but $\mathbf{E}_{\mathrm{ind}\perp}$ unscreened.

($E_{\mathrm{ind}||}$ subject to violent oscillations
assume screening on average over oscillations)

$\mathbf{E}_{\mathrm{ind}\perp} \neq 0 \Rightarrow$ electric drift $\mathbf{u}_{\mathrm{ind}\perp} = \mathbf{E}_{\mathrm{ind}\perp} \times \mathbf{B}/B^2$.

New result: there exists $\mathbf{u}_{\mathrm{ind}\perp} \neq 0$.

Has nonzero r, θ, ϕ components.

Inconsistent with rotation at any velocity.

Drift velocity increases $\propto \sin\alpha$ with obliquity angle α

Subpulse drifting

Plots of drift velocity in Melrose & Yuen (submitted to ApJ).

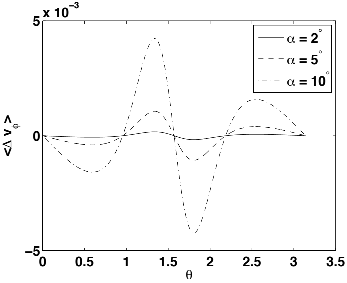


Figure: Inductively induced azimuthal drift velocity

Well studied phenomenon "drifting subpulses"

Simplest explanation: plasma is not corotating!

Maybe $\mathbf{u}_{\mathrm{ind}\perp}$ explains drifting subpulses.

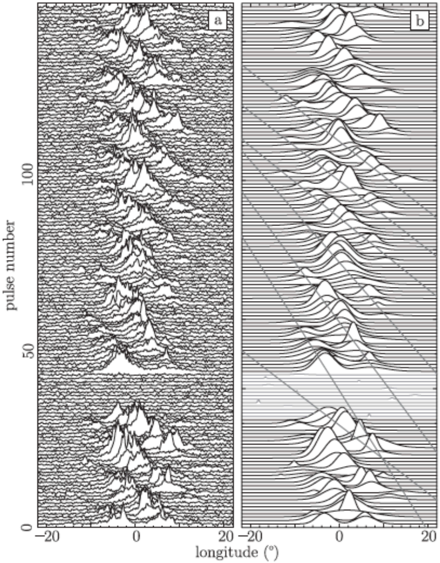
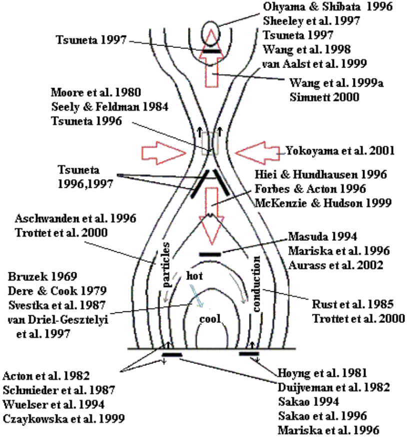


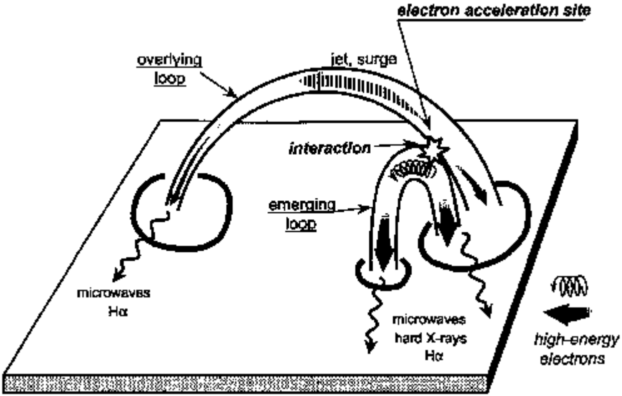
Figure: Drifting subpulses

Solar flares: standard (CSHKP) model



- Helmet streamer above closed loop
- Inflow transports magnetic energy into neutral sheet
- Magnetic reconnection releases energy
- Upflowing energy in CME & Type III electrons
- Downflowing energy produces Hα, hard X-rays, etc.

Emerging-flux model

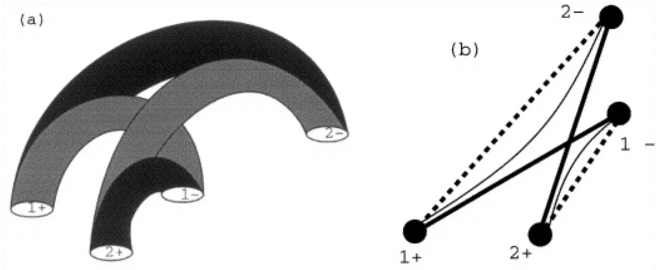


An emerging-flux model (Hanaoka 1999)

Notable differences from CSHKP model

- Driven from below, not from above
- Net current cannot change: I can only be redirected

Quadrupolar model

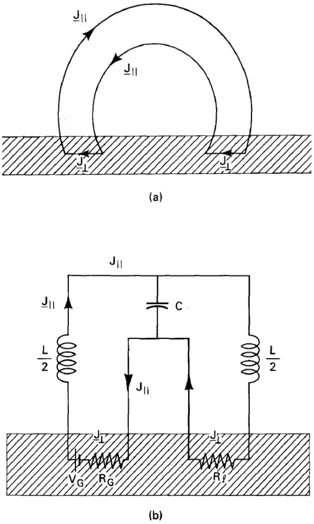


Quadrupolar model Melrose, ApJ **486**, 521 (1997)

- Two initial current-carrying flux tubes
- Reconnection allows transfer of magnetic flux and current
- Two new current-carrying flux tubes form
- Flux and current conserved at each footpoint
- Favorable conditions for energy release Hardy *et al.* PASA 15, 318 (1998)

Model is time-dependent: only initial & final states considered.

Circuit model for flares



Spicer (1982)

Generic model for magnetic explosions

Magnetically powered explosions

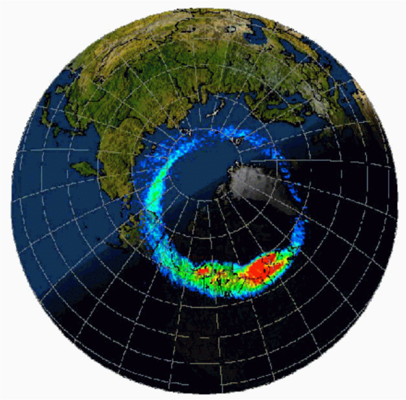
- **magnetospheric substorms**: magnetic energy stored in the Earth's magnetotail is released explosively
- **solar flares**: magnetic energy stored in the solar corona is released explosively
- **magnetar outbursts**: magnetars are neutron stars with magnetic fields that are too strong to be confined to the star, and magnetic energy is released explosively in outbursts

Related electrodynamical problems

- Io-Jupiter interaction: powered by drag of Io through Jovian magnetosphere
- Pulsar electrodynamics: rotation powered systems
- Blandford-Znajek mechanism: accretion powered black holes

Auroral oval during a substorm

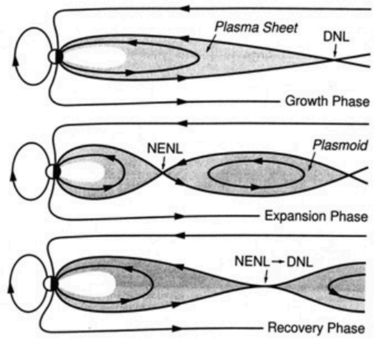
Magnetotail maps onto the auroral oval

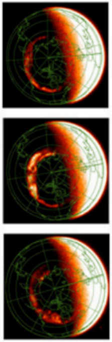


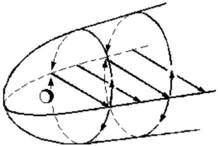
Ψ_B stored in the magnetotail $= \Psi_B$ inside auroral oval

Magnetotail during a substorm

Magnetic flux and current in the magnetotail decrease





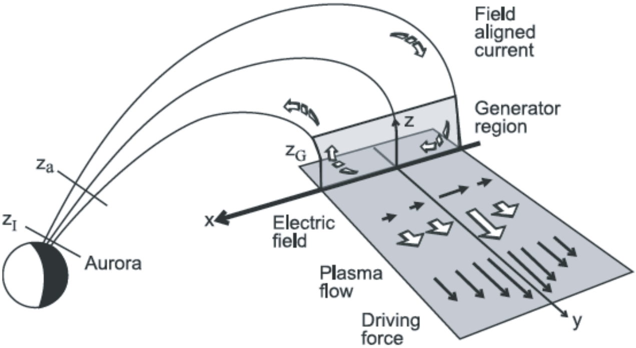


Reconnection (left) decreases stored magnetic energy

Total current across magnetotail (right) decreases

current is partly redirected to close in ionosphere

Current wedge during a substorm



Rönnmark, JGR **107**, 1430 (2002)

Generic features of model

- New current circuit operates during energy release
- Distinct energy-conversion & dissipation-acceleration regions
- Energy transport around circuit is Alfvénic

Global model for solar flares

EMF (first application of $\mathbf{E}_{\rm ind}$)

- Φ set up during flare; due to changing B
- $\Phi = -d\Psi_B/dt$, of order 10^{10} V Swann, Phys. Rev. **43**, 217 (1933)
- EMF is not due to a photospheric dynamo

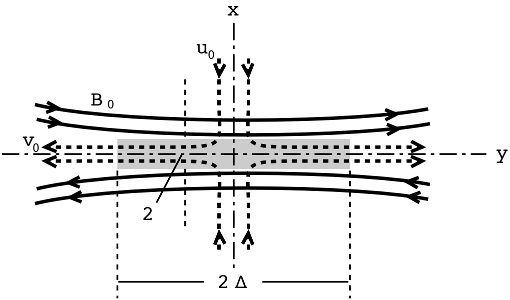
Current

- flare current path = (post-flare path) - (pre-flare path)
- $I \approx 10^{11}$ A: pre-flare currents redirected
- Power $I\Phi = 10^{21}$ W can account for a flare

Plasma inflow in reconnection

Inductively induced inflow (second application of $\mathbf{E}_{\text{ind}}$)

- Reconnection model with $\partial \mathbf{B}/\partial t \neq 0$
- $=> \mathbf{E}_{\text{ind}} => \mathbf{u}_{\text{ind}} = \mathbf{E}_{\text{ind}} \times \mathbf{B}/B^2$
- $\partial B_z/\partial t = \partial E_y/\partial x => E_y => u_x = E_y/B_x$
- $\mathbf{u}_{\text{ind}}$ transports magnetic flux into current sheet



Sweet-Parker reconnection model

Driver for reconnection

Polarization current (third application of $\mathbf{E}_{\text{ind}}$)

- Displacement current $\varepsilon_0 \partial \mathbf{E}/\partial t \neq 0$
- => polarization drift (from "orbit theory")
- => polarization current $\mathbf{J}_{\text{pol}} = (c^2/v_A^2)\varepsilon_0 \partial \mathbf{E}/\partial t$
- $d\mathbf{u}_{\text{ind}}/dt$ due to force $\mathbf{J}_{\text{pol}} \times \mathbf{B}$

Inflow driven by Maxwell stress through $\partial \mathbf{B}/\partial t$

Acceleration by parallel electric field

Simplest acceleration mechanism for energetic particles:

electric field, $E_\parallel$, parallel to magnetic field.

Compelling evidence for this mechanism for auroral electrons,

accelerated 1–3 km above the Earth's surface.

Strong evidence for $E_\parallel$-acceleration of electrons in solar flares.

Challenges for solar flares

- "Number problem" $\dot{N} \approx 10^6\, I/e$, inferred from HXR data
- Energy of accelerated electrons $\approx 10^{-6} e\Phi$
- Acceleration region in the chromosphere? Brown *et al.*, A&A **508** 993 (2009)

Role of $\mathbf{J}_{\rm disp}$ in acceleration by $E_\parallel$

We still do not understand acceleration by $E_\parallel$

Cross-field $\mathbf{E}_{\rm ind}$ induces $\mathbf{u}_{\rm ind} = \mathbf{E}_{\rm ind} \times \mathbf{B}/B^2$

$E_\parallel$ created where $\mathbf{u}_{\rm ind}$ changes (with t) along $\mathbf{B}$

Quote from Song & Lysak, PRL **96**, 145002 (2006)
in the context of auroral acceleration:

"… appeal to generalized Ohm's law for $E_\parallel$ generation has misled and hindered research on reconnection and auroral acceleration … processes causing and supporting $E_\parallel$ … either neglected or not yet discovered (Fälthammar 1990)"

"$E_\parallel$ acceleration is associated with … the parallel displacement current"

Fiducial numbers

Magnetospheric substorm

$I = 10^6\,\text{A}, \quad \Phi = 10^5\,\text{V}, \quad T = 10^4\,\text{s}.$

Solar flare

Energy $= 10^{23}\,\text{J}, \quad$ Power $= 10^{21}\,\text{W}$

$I = 10^{11}\,\text{A}, \quad \Phi = 10^{10}\,\text{V}, \quad T = 10^2\,\text{s}.$

Other numbers

$B = 10^{-2}\,\text{T}, \quad \ell = 10^7\,\text{m}, \quad \mu_0 = 4\pi \times 10^{-7}$

$I = B\ell/\mu_0 = 10^{11}\,\text{A}, \quad \Phi = B\ell^2/T = 10^{10}\,\text{V},$

$L = \mu_0\ell = 10\,\text{H}, \quad B^2\ell^3/\mu_0 = LI^2 = 10^{23}\,\text{J}.$

Magnetar outburst

$I = 3 \times 10^{15}\,\text{A}, \quad \Phi = 10^{18}\,\text{V}, \quad T = 0.1\,\text{s}.$

Conclusions

Using only half of Maxwell's equations is not justified in general.

Time-dependent $\mathbf{B} => \mathbf{E}_{\rm ind} \neq 0$

Implications of neglect of $\mathbf{E}_{\rm ind}$ & $\mathbf{J}_{\rm disp}$:

1. Electrostatic models for pulsars are violently unstable
2. Pulsar plasma cannot be corotating
   $=>$ natural explanation for drifting subpulses?
3. EMF in magnetic explosions due to time-dependent Ψ_B
4. Inflow into reconnection driven by Maxwell stress due to $\partial\mathbf{B}/\partial t$
5. Acceleration by $E_\parallel$ involves $\mathbf{J}_{\rm disp}$
   relevant to auroral electrons, solar-flare electrons.