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INNER SIZE OF A DUST TORUS IN THE SEYFERT 1 GALAXY NGC 4151

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ABSTRACT

The most intense monitoring observations yet made were carried out on the Seyfert 1 galaxy NGC 4151 in the optical and near-infrared wave-bands. A lag from the optical light curve to the near-infrared light curve was measured. The lag-time between the V and K light curves at the flux minimum in 2001 was precisely 48^{+2}_{-3} days, as determined by a cross-correlation analysis. The correlation between the optical luminosity of an active galactic nucleus (AGN) and the lag-time between the UV/optical and the near-infrared light curves is presented for NGC 4151 in combination with previous lag-time measurements of NGC 4151 and other AGNs in the literature. This correlation is interpreted as thermal dust reverberation in an AGN, where the near-infrared emission from an AGN is expected to be the thermal re-radiation from hot dust surrounding the central

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engine at a radius where the temperature equals to that of the dust sublimation temperature. We find that the inner radius of the dust torus in NGC 4151 is ~ 0.04 pc corresponding to the measured lag-time, well outside the broad line region (BLR) determined by other reverberation studies of the emission lines.

Subject headings: galaxies: Seyfert — galaxies: active — infrared: galaxies — dust, extinction — galaxies: individual(NGC 4151)

1. INTRODUCTION

One attractive idea of unifying active galactic nuclei (AGNs) of types Seyfert 1 and 2, depending on whether strong, broad emission lines are visible or not, hinges upon the existence of a dust torus that surrounds the broad-line region (BLR) (Antonucci 1993). A direct view of the BLR through the inner hole of the dust torus seen face-on yields a Seyfert 1 spectrum, whereas an obscuration of the BLR by the dust torus seen edge-on results in a Seyfert 2 spectrum. Although this unified model has been supported in many ways already, the strongest justification would come from confirming the existence of the dust torus surrounding the BLR. Since modern technology has not yet reached a point where one might spatially resolve the innermost structure of AGNs, the only method available to uncover it is a lag analysis in light curves, known as reverberation mapping.

The near-infrared continuum ($\lambda \gtrsim 2\ \mu$m) emission of Seyfert galaxies is considered to be dominated by thermal radiation from hot dust surrounding the central engine, based on the decomposition of spectral energy distribution (SED) of AGNs (e.g. Kobayashi et al. 1993). The dust closest to the central engine has higher temperatures, but cannot survive inside some critical radius at which it begins to sublimate. Accordingly, the distribution of dust should be torus-like, with a hole at the critical radius, and the dust at the innermost point of the torus has the maximum temperature equal to the sublimation temperature. This dust produces thermal emission which has a peak luminosity at the corresponding near-infrared wavelength.

Clear evidence that the hot dust is located some distance from the center of the central engine in AGNs is the existence of a lag time between the UV or optical light curve and the near-infrared light curve of an AGN. It is interpreted in terms of light travel time from the central engine, which almost simultaneously emits UV and optical radiation to the surrounding hot dust, which reprocesses the radiation from the central engine to thermal near-infrared radiation. Previous studies reported such lags of $10 - 1000$ days for at most ten AGNs (Clavel, Wamsteker & Glass 1989; Glass 1992; Sitko et al. 1993; Oknyanskij

1993; Nelson 1996; Oknyanskij et al. 1999; Baribaud et al. 1992; Nelson & Malkan 2001; Oknyanskij & Horne 2001), however, the number of targets and the measurement errors were somewhat limited.

To make it possible to carry out much more intense monitoring observations of many AGNs in optical and the near-infrared the MAGNUM (Multicolor Active Galactic Nuclei Monitoring) project installed a 2-meter telescope at the Haleakala Observatories in Hawaii (Kobayashi et al. 1998a; Yoshii 2002; Yoshii, Kobayashi & Minezaki 2003), and started its operation in early 2001.

In this *Letter*, as an early result of the MAGNUM project, we present the $V(0.55\ \mu\text{m})$ and $K(2.2\ \mu\text{m})$ light curves of a bright Seyfert 1 galaxy NGC 4151 for the period including the epoch of flux minimum in 2001, and report a measurement of lag-time between them based on a cross-correlation analysis. Due to the high frequency and high signal-to-noise ratio of our monitoring observations, the measured lag is the most precise ever made, not only for NGC 4151 but also for any other AGNs. Then it was compared with the previously measured lag-times of other AGNs in literature, and interpreted by thermal dust reverberation in NGC 4151. The lag-time between the V and K light curves corresponds to the light travel-time between the central engine and the inner radius of the dust torus, and with this result, we argue in favor of the unified model that NGC 4151 has a dust torus whose innermost radius is well outside the BLR.

2. OBSERVATION

Monitoring observations of NGC 4151 started January, 2001 in the V band and the K band, using the multicolor imaging photometer (MIP) mounted on the MAGNUM telescope (Kobayashi et al. 1998a,b). The field of view of the MIP is 1.5×1.5 arcmin^2, and it enables simultaneous imaging in optical and near-infrared by splitting the incident beam into two different detectors, a SITe CCD (1024×1024 pixels, 0.277 arcsec/pixel) and an SBRC InSb array (256×256 pixels, 0.346 arcsec/pixel).

We obtained photometric data for NGC 4151 between January and December 2001, consisting of 44 nights in the V band and 49 nights in the K band. With the exception of the three months during solar conjunction, from mid-August to the beginning of November, the average interval of the observations was less than a week. This monitoring has provided the most frequent simultaneous optical and near-infrared sampling of NGC 4151 to date.

The telescope was pointed at NGC 4151 and two reference stars with $(\Delta\alpha, \Delta\delta) = (16.1', 5.7')$ and $(-11.3', -9.3')$ alternately, and then it was dithered. In this manner, high signal-to-noise ratios were achieved with total on-source integration time of a few minutes for each observation. The typical FWHMs of the point spread function (PSF) during the observation were 1.5 arcsec in the V band and 1.1 arcsec in the K band, respectively.

The images were reduced using IRAF [9] . We followed standard procedures for image reduction, with small corrections for the non-linear response of the detectors. For the infrared image reduction, a sky image assembled from dithered images pointed at the reference stars was subtracted prior to flat-fielding. Then, flat-field corrections were applied using dome flats for the optical images, or an illumination-corrected dome flats for the near-infrared images.

The nuclear flux of NGC 4151 was measured relative to the reference stars, then the fluxes of these stars were calibrated. Aperture photometry within $\phi = 8.3$ arcsec was applied to all images, then the fluxes of the NGC 4151 nucleus and the reference stars were compared for each dithering set. Finally, the nuclear flux relative to the reference stars was estimated from the average of the relative fluxes for all of the dithering sets. The photometric errors for data taken each night were usually better than $\sigma = 0.01$ mag. The flux ratio between the reference stars was also monitored; it was confirmed that they were not variable stars.

The fluxes of the reference stars were calibrated with respect to photometric standard stars taken from Landolt (1992) and Hunt et al. (1998), and a small correction for the filter color term in the V band was applied. The flux calibration errors were less than 0.01 mag in V and about 0.01 mag in K. The flux of the host galaxy within the aperture was estimated by fitting the galaxy profile using the GALFIT (Peng et al. 2002). The flux of the host galaxy in the V band was slightly dependent on the seeing FWHM, and was corrected for. The flux of the host galaxy was subtracted from the aperture flux; the errors arising from this subtraction were about 0.1 mag. We note that the subtraction provides only a flux offset, which does not affect the cross-correlation function analysis below.

3. RESULTS

Figure 1 shows the V and K light curves of NGC 4151 nucleus in 2001. The V flux minimum was observed at epoch MJD $\approx$ 51980; the K flux minimum was clearly delayed behind the V flux minimum by about 50 days. While the V light curve exhibited some

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[9]IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

variability on short time-scales comparable to the typical monitoring interval of several days, the K light curve changed smoothly for the whole period of the monitoring observations. This indicates that the K band emitting region is extended more than a scale of several light-days.

In order to obtain a more quantitative estimate of the lag-time between the V and K light curves, a cross-correlation analysis was applied. A cross-correlation function (CCF) of irregularly sampled photometric data was computed based on the linear interpolation method (Gaskell & Peterson 1987; White & Peterson 1994), as modified below. The first method is a "bidirectionally interpolated" CCF. Each flux data point of one band, $f_1(t_i)$, at an epoch t_i, is paired with the linearly interpolated flux of the other band, $f_2(t_i + \tau)$, with a lag-time, τ, then the CCF(τ) is computed from all data pairs $(f_V(t_i), f_K(t_i + \tau))$ and $(f_V(t_j - \tau), f_K(t_j))$, not the average of two CCFs each from $(f_V(t_i), f_K(t_i + \tau))$ and $(f_V(t_j - \tau), f_K(t_j))$ as is often done. The second method is to obtain an "equally sampled" CCF. Both the V and K light curves are linearly interpolated to produce the flux data points every day, then the CCF(τ) is computed between those equally sampled light curves. The third method is the MCCF (Oknyanskij 1993, the MCCF parameter MAX was 10 days).

A Monte-Carlo simulation was carried out to estimate the lag-time error using the CCF analysis. Many CCFs were calculated from artificial light curves, constructed by interpolating the observed light curves. In the artificial light curves, we incorporated not only the photometric errors at the observed data points, but also the short time-scale flux variation between the observed data points which serves to increase the scatter of the light curves at the interpolated data points. The flux variation was incorporated so that the structure function of the artificial light curves reproduced that of the observed light curve. We note that the error estimation here incorporated only the random errors caused by the photometric error and the flux variation, and there may be some possible systematic error. The details of the CCF computation and the simulation are described in Suganuma et al. (2003).

Figure 2 shows the CCFs between the V and K light curves of NGC 4151. Only the photometric data points (37 points in V and 42 points in K) before the solar conjunction were used. All CCFs have a peak at the same lag-time of $\Delta t = 48$ days. The average and the standard deviation of the lag-time at the CCF peak calculated from the simulation were 47.9 and 1.1 days for the "bidirectionally interpolated" CCF, 47.5 and 2.4 days for the "equally sampled" CCF, and 47.6 and 0.9 days for the MCCF. To be conservative, hereafter we use $\Delta t = 48^{+2}_{-3}$ days as representative of the measured lag-time and the standard deviation error. The estimated lag-time of $\Delta t = 48$ days indicate that the K band emitting region was located at distance of 0.04 pc away from the central engine.

4. DISCUSSION

Figure 3 shows the lag-time of NGC 4151, together with those of other quasars and Seyfert 1 galaxies in the literature, plotted against their absolute V magnitudes. The lag-times between the UV or optical light curve and the K light curve were taken from the original references (Clavel, Wamsteker & Glass 1989; Glass 1992; Sitko et al. 1993; Nelson 1996; Oknyanskij et al. 1999). The lag-time data are restricted to those that were estimated by CCF analyses applied to the light curves. The absolute V magnitudes were estimated from their apparent magnitudes and redshifts in a range of $cz = 1000 - 50000$ km/s, assuming cosmological parameters of $(h_0, \Omega_0, \lambda_0) = (0.7, 0.3, 0.7)$ (Bennett et al. 2003). The flux of the host galaxy within the aperture was subtracted by using the image decomposition technique, or the flux variation gradient (Bahcall et al. 1997; Kotilainen, Ward, & Williger 1993; Winkler et al. 1992; Winkler 1997; Glass 1992; Nelson 1996, ; this work). Galactic extinction was corrected for (Schlegel, Finkbeiner, & Davis 1998), but the intrinsic extinction in AGNs was ignored here for simplicity, because they were all Seyfert type 1 like AGNs. A K-correction was applied assuming a power-law spectrum of $f_\nu \propto \nu^{-0.44}$ (Vanden Berk et al. 2001). The observed lag-time was multiplied by a factor of $(1+z)^{-1}$ to correct for the cosmological time dilation [10].

It is clearly seen in Figure 3 that the lag-time is correlated with the absolute V magnitude of the source, spanning over a range of 100 times in luminosity and 10 times in lag-time, in the sense that a luminous AGN has a larger lag-time. A similar correlation has been previously noted by Oknyanskij & Horne (2001), using the UV luminosity instead of V. The inclined line ($\log_{10} \Delta t = -2.15 - M_V/5.0$) shown in Figure 3 is a fit to the data, with the expected slope of $\Delta t \propto L^{0.5}$ from the reverberation of thermal emission of the hot dust surrounding the central engine. The inner radius of the dust torus is determined by the highest sublimation temperature of the dust, around $T \approx 1800$ K for graphite grains (Salpeter 1977) or $T \approx 1500$ K for silicate grains (Huffman 1977). The dust at the innermost limit of the torus produces strong thermal emission in the near-infrared. Since the sublimation radius is proportional to the square root of luminosity of the central engine (Barvainis 1987), the lag-time between the UV/optical and the K light curves, which would corresponds to a light

[10] Assuming that the near-infrared emission at wavelength λ is dominated by the thermal emission from heated dust of temperature T which peaks at λ, the location of such dust at radius r for given central luminosity scales as $r \propto T^{-2.8}$ (Barvainis 1987), leading to $\Delta t \propto \lambda^{2.8}$. Then, the lag-time correction for the cosmological wavelength dilation is expected to be a factor of $(1+z)^{2.8}$, opposite to that for the cosmological time dilation (Sitko et al. 1993; Oknyanskij & Horne 2001). In the case of GQ Comae, having the largest redshift in the sample shown in Figure 3, this correction is equal to $(1 + 0.165)^{2.8} \approx 1.5$, placing the point (c) even closer to the inclined line.

travel-time between the central engine and the inner radius of the dust torus, should be proportional to the square root of the luminosity of the central engine. Several studies have applied dust reverberation models to Fairall 9 (Barvainis 1992) and Mrk 744 (Nelson 1996), and estimated maximum temperatures for the circumnuclear dust to be comparable to the expected sublimation temperature.

The good correlations that exist between the reported lag-times and the absolute V magnitude along the expected slope from thermal dust reverberation, including our new result for NGC 4151, indicate that the K band emission of NGC 4151 is likely to be dominated by the thermal radiation of hot dust at its innermost radius; the radius of the dust torus would be ~ 0.04 pc in order to correspond to the observed lag-time of $\Delta t = 48$ days.

There are measurements of shorter lag-times for NGC 4151 in the past, such as 18 ± 6 days in $1969 - 1981$ and 35 ± 8 days in $1993 - 1998$ by Oknyanskij et al. (1999). Given a two-sigma level of their measurement error, then it may be possible to interpret that the recovering time-scale of the inner dust was so long, more than a few years, that the inner radius of the dust torus was not reduced after the most luminous state in $1993 - 1998$. However, the past data suffered from relatively large lag-time errors. In order to make it clear, it must be important to continue intense monitoring observation of NGC 4151 to measure the lag-times at different epochs accurately, and in addition, monitoring the near-infrared color is also important because the dust temperature at the inner torus should change with flux variation when the inner radius of the dust torus is constant.

Next, the measured lag-time for the dust torus is compared with similar estimates for the BLR, based on broad emission-line reverberation measurements. Clavel, Wamsteker & Glass (1989) observed not only the lag-times of near-infrared emission but also those of far-UV broad emission-lines, and concluded that the BLR lies inside the dust shell in the Seyfert 1 galaxy Fairall 9. Although we did not observe broad emission-lines at the same time, the measured lag-time of 48^{+2}_{-3} days for K band emission was quite precise, and should correspond to the inner radius of the dust torus. Therefore, it is worthwhile to compare it with previous measurements of the lag-times of broad emission-lines. It is noted that the previous reports differ somewhat between the lines observed and the observations, such as $\Delta t = 4 \pm 3$ days for CIV and MgII (Clavel et al. 1990), $\Delta t = 9 \pm 2$ days for Hβ and Hα (Maoz et al. 1991), $\Delta t = 0 - 3$ days for Hβ and $\Delta t = 0 - 2$ days for Hα (Kaspi et al. 1996), $\Delta t = 1.8 - 4.4$ days for CIV (Ulrich & Horne 1996), and $\Delta t = 4 \pm 2$ days for Hβ and Hα (Oknyanskij & van Groningen 1997).

It has been claimed that the spatial distribution of the BLR clouds should be more extended than that estimated from the lag-times of the observed emission lines (Maoz et al. 1991; Ulrich & Horne 1996). Even if this is the case, the inner radius of the dust torus in NGC 4151 is found to be well outside the BLR, therefore, the BLR in NGC 4151 can be observed directly without strong obscuration unless the dust torus is edge-on. Also the near-infrared emission dominated by thermal radiation from hot dust at the innermost region is observed without strong obscuration due to outer cold dust. These results are consistent with the unified model of Seyfert types of AGNs.

5. CONCLUSION

An optical and near-infrared monitoring observation of the Seyfert 1 galaxy NGC 4151 was carried out, and a lag-time between the V and K light curves at the flux minimum in 2001 was estimated at $\Delta t = 48^{+2}_{-3}$ days by a cross-correlation analysis. We present that the lag-time between the UV or optical and the K light curves is correlated with the absolute V magnitude of AGN, and conclude that the K band emission of NGC 4151 is dominated by the thermal radiation of innermost hot dust whose temperature is comparable to dust sublimation, and that the inner radius of the dust torus of NGC 4151 is ~ 0.04 pc, which is well outside the BLR.

The ongoing intense multicolor monitoring observations of NGC 4151 and many other AGNs in the MAGNUM project will not only study the detailed geometry of the dust tori and further examine the unified model, but also constrain the real-time change in size of the dust tori and the time-scales of dust formation and destruction under extreme circumstances in AGNs from measurements of Δt at different epochs along the light curves and the near-infrared color with flux variation.

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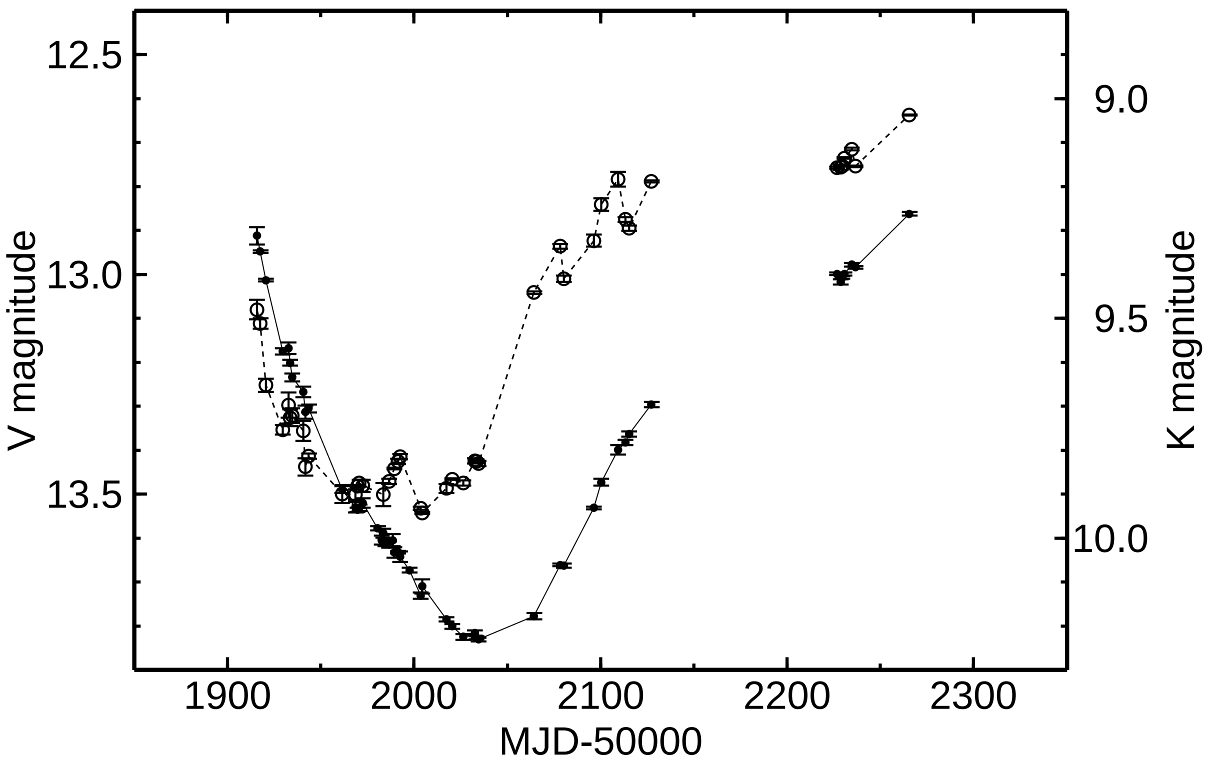

Fig. 1.— The V (open circles connected with dashed lines) and the K (closed circles connected with solid lines) light curves of NGC 4151 nucleus in 2001. The flux from the host galaxy is subtracted. The flux minimum of the K light curve is clearly delayed behind that of the V light curve. The monitoring observation was interrupted due to the solar conjunction at MJD = 52130 $\sim$ 52220.

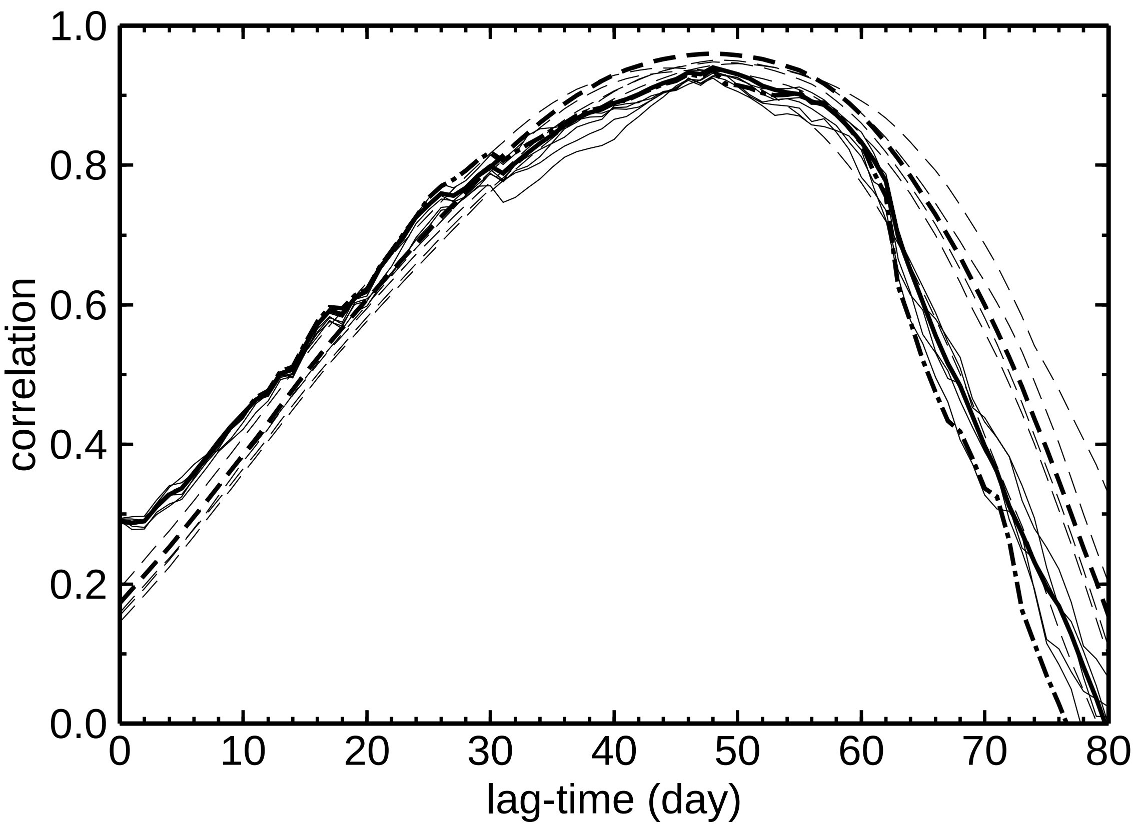

Fig. 2.— The cross-correlation functions (CCFs) between the V and K light curves of NGC 4151 at the flux minimum in 2001. The thick solid line is the "bidirectionally interpolated" CCF, the thick dashed line is the "equally sampled" CCF, and the thick dash-dotted line is the MCCF; all of them have a peak at a lag-time of $\Delta t = 48$ days. The thin solid and dashed lines are the examples of CCFs calculated from the simulated light curves to estimate the lag-time errors.

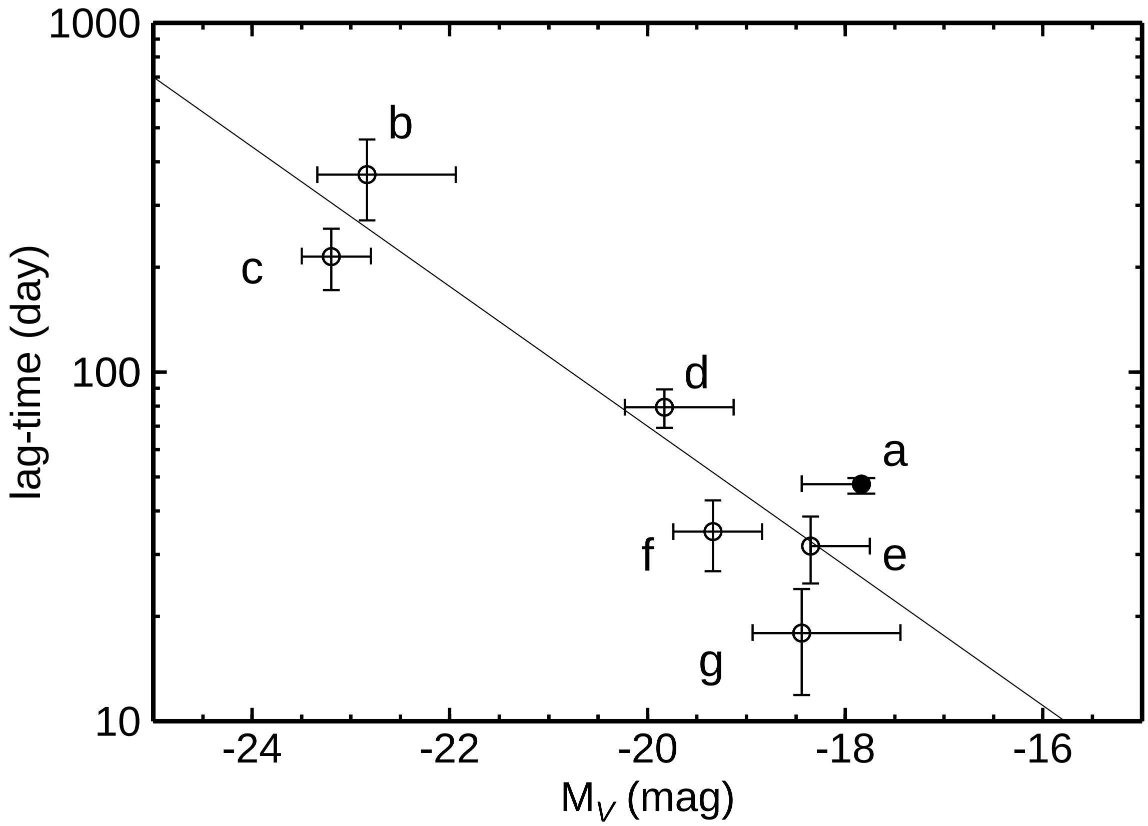

Fig. 3.— The lag-times between the UV or optical and the K light curves of AGNs against the absolute V magnitudes. The closed circle (a) is this work (new data for NGC 4151); the open circles are taken from literature: (b) Fairall 9 (Clavel, Wamsteker & Glass 1989), (c) GQ Comae (Sitko et al. 1993), (d) NGC 3783 (Glass 1992), (e) Mrk 744 (Nelson 1996), (f) and (g) NGC 4151 (Oknyanskij et al. 1999). The inclined line was fitted assuming the expected slope from thermal dust reverberation, such that the lag-time is proportional to the square root of the luminosity. It should be noted that the "error bar" in magnitude represents the observed range of variation during the observations.