Compton Scattering I

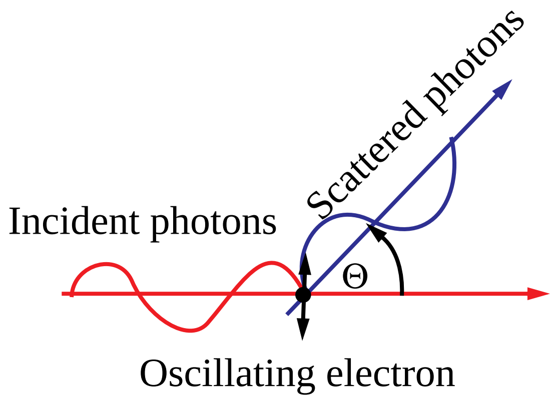
1 Introduction

Compton scattering is the process whereby photons gain or lose energy from collisions with electrons. It is an important source of radiation at high energies, particularly at X-ray to γ-ray energies.

In this chapter, we consider the total energy radiated by relativistic electrons as a result of scattering of soft photons.

2 Scattering from electrons at rest

2.1 Classical approach (Thomson scattering)



When a flux of electromagnetic radiation impinges on an electron, the electron oscillates and radiates electromagnetic radiation (photons) in all directions.

We concentrate on the *number* flux of photons. Let

$$\frac{dN_{\text{inc}}}{dt\,dA} = \text{Incident no of photons per unit time per unit area}$$

$$\frac{dN_{\text{scat}}}{dt\,d\Omega} = \text{No of photons per unit time per steradian scattered by the electron}$$

The differential number of scattered photons is defined in terms of the ***cross-section*** by:

$$\frac{dN_{\text{scat}}}{dt\,d\Omega} = \frac{dN_{\text{inc}}}{dt\,dA}\frac{d\sigma_T}{d\Omega}$$

The ***differential cross section*** for Thomson scattering is:

$$\frac{d\sigma_T}{d\Omega} = \frac{1}{2}r_0^2(1+\cos^2\Theta)$$

$$r_0 = \text{Classical electron radius}$$

$$= \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.818\times10^{-15}\,\text{m}$$

The classical electron radius, r_0, is the “radius” derived by treating the electron as a classical particle and assuming that its rest-mass is equal to its electrostatic potential, i.e.

$$\frac{e^2}{4\pi\varepsilon_0 r_0} = m_e c^2$$

Units

Note the units for the equation describing Thomson scattering

$$\frac{\text{Number per unit time}}{\text{per unit solid angle}} = \frac{\text{Number per unit time}}{\text{per unit area}} \times \frac{\text{Area per unit}}{\text{solid angle}}$$

The ***total cross-section*** for scattering into all solid angles is given by

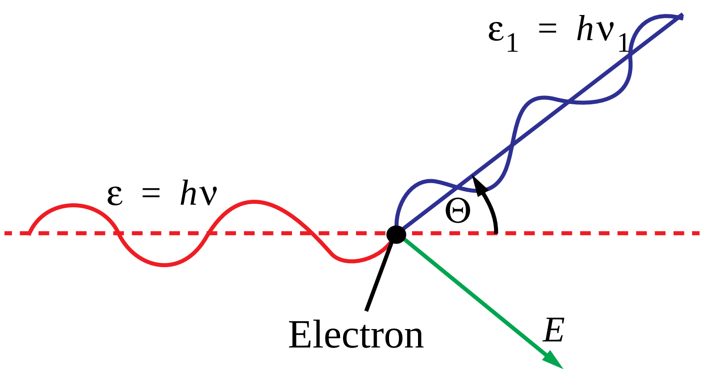
$$\frac{dN_{\text{scat}}}{dt} = \sigma_T \frac{dN_{\text{inc}}}{dtdA}$$

$$\sigma_T = \int_{4\pi} \frac{d\sigma_T}{d\Omega} d\Omega = \frac{2\pi r_0^2}{2} \int_0^{\pi} (1 + \cos^2\Theta) d\Theta = \frac{8\pi}{3} r_0^2$$

$$= 6.65 \times 10^{-29} \ \text{m}^2$$

The quantity σ_T is the Thomson cross-section.

2.2 Quantum-mechanical particle approach



The above can be derived by approximating photons classically as an electromagnetic wave. It is also useful to treat the scattering from a particle point of view. To do so, we consider the collision between a photon and an electron in the rest frame of the electron, as described in the figure.

The parameters describing the collision are:

$$\varepsilon = h\nu = \text{Initial photon energy}$$
$$\varepsilon_1 = h\nu_1 = \text{Final photon energy}$$
$$m_e c^2 = \text{Initial electron energy}$$
$$E = \text{Final electron energy}$$
$$\Theta = \text{Angle of deflection of photon}$$

Treatment of collision using 4-vectors

The conservation of momentum and energy gives the final photon energy in terms of the initial energy. This can be derived in the following way.

In relativity, the conservation of energy and momentum becomes the conservation of four momentum. As we described in the chapter on relativistic effects, the 4–momentum of a particle with energy $E = \gamma mc^2$ moving in the direction of the unit vector $\boldsymbol{n}$ is described by:

$$\boldsymbol{P} = \left[\gamma mc \;\; \gamma m\boldsymbol{v}\right] = \left[\frac{E}{c} \;\; \frac{E}{c}\frac{\boldsymbol{v}}{c}\right]$$

$$= \frac{E}{c}[1, \beta\boldsymbol{n}]$$

$$\boldsymbol{n} = \text{Unit vector in the direction of motion}$$

Limiting case of photon

In the limit where the mass goes to zero and $\beta \to 1$ but the energy ε remains finite, we have a photon with 4–momentum

$$\boldsymbol{P} = \frac{\varepsilon}{c}[1, \boldsymbol{n}]$$

Let

$$\boldsymbol{P}_{\gamma i} = \text{Initial 4-momentum of the photon} = \frac{\varepsilon}{c}\left[1 \;\; \boldsymbol{n}_{\gamma i}\right]$$

$$\boldsymbol{P}_{\gamma f} = \text{Final 4 momentum of the photon} = \frac{\varepsilon_1}{c}\left[1 \;\; \boldsymbol{n}_{\gamma f}\right]$$

$$\boldsymbol{P}_{ei} = \text{Initial 4-momentum of the electron} = \left[m_e c \;\; \boldsymbol{0}\right]$$

$$P_{ef} = \text{Final 4-momentum of the electron} = \frac{E}{c}\left[1 \;\; \boldsymbol{n}_{ef}\right]$$

Conservation of 4-momentum, tells us that

$$\boldsymbol{P}_{\gamma i} + \boldsymbol{P}_{ei} = \boldsymbol{P}_{\gamma f} + \boldsymbol{P}_{ef}$$

We can rearrange this equation in such a way that the electron momentum drops out. First put

$$\boldsymbol{P}_{ef} = \boldsymbol{P}_{\gamma i} + \boldsymbol{P}_{ei} - \boldsymbol{P}_{\gamma f}$$

then take the 4-dimensional modulus of this equation.

Remember that

1. The modulus of a vector A_μ is given by

$$A^2 = \eta_{\mu\nu}A^\mu A^\nu = -(A^0)^2 + (A^1)^2 + (A^2)^2 + (A^3)^2$$

2. The scalar product of $\boldsymbol{A}$ and $\boldsymbol{B}$ is

$$\boldsymbol{A} \cdot \boldsymbol{B} = \eta_{\mu\nu}A^\mu B^\nu = -A^0B^0 + A^1B^1 + A^2B^2 + A^3B^3$$

3. The magnitude of the 4-momentum of a particle is given by:

$$P^2 = -m^2c^2$$

4. Magnitude of 4-momentum of a photon is:

$$P^2 = 0$$

Now take the modulus of the equation for $\boldsymbol{P}_{ef}$:

$$|\boldsymbol{P}_{ef}|^2 = |\boldsymbol{P}_{\gamma i} + \boldsymbol{P}_{ei} - \boldsymbol{P}_{\gamma f}|^2$$

$$\Rightarrow -m_e^2c^2 = |\boldsymbol{P}_{\gamma i}|^2 + |\boldsymbol{P}_{ei}|^2 + |\boldsymbol{P}_{\gamma f}|^2$$
$$+ 2\boldsymbol{P}_{\gamma i}\cdot\boldsymbol{P}_{ei} - 2\boldsymbol{P}_{\gamma i}\cdot\boldsymbol{P}_{\gamma f} - 2(\boldsymbol{P}_{ei}\cdot\boldsymbol{P}_{\gamma f})$$

$$-m_e^2c^2 = 0 - m_e^2c^2 + 0 + 2\left(-\frac{\varepsilon}{c}m_e c\right)$$
$$-2\left(-\frac{\varepsilon\varepsilon_1}{c^2} + \frac{\varepsilon\varepsilon_1}{c^2}\boldsymbol{n}_i\cdot\boldsymbol{n}_f\right) - 2\left(-m_e c\frac{\varepsilon_1}{c}\right)$$

This simplifies to:

$$\varepsilon m_e - \left(\frac{\varepsilon\varepsilon_1}{c^2} - \frac{\varepsilon\varepsilon_1}{c^2}\cos\Theta\right) - m_e\varepsilon_1 = 0$$

$$\varepsilon_1[m_e c^2 + \varepsilon(1-\cos\Theta)] = \varepsilon$$

$$\varepsilon_1 = \frac{\varepsilon}{1 + \dfrac{\varepsilon}{m_e c^2}(1-\cos\Theta)}$$

One can see immediately from this equation that:

$$\varepsilon \ll m_e c^2 \Rightarrow \varepsilon_1 \approx \varepsilon$$

2.3 Energy and wavelength change

The wavelength of a photon is given by:

$$\varepsilon = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\varepsilon}$$

The above equation for the energy can be expressed as:

$$\lambda_1 - \lambda = \lambda_c(1-\cos\theta)$$

$$\lambda_c = \frac{h}{m_e c} = \text{Compton wavelength of electron} \approx 0.0246\ \text{A}$$

The wavelength change after scattering is of order the Compton wavelength. For long wavelengths, $\lambda \gg \lambda_c$, the change in wavelength is small compared to the initial wavelength. Equivalently, when $\varepsilon \ll m_e c^2$ energy is conserved ($\varepsilon_1 = \varepsilon$) to a good approximation.

2.4 The Klein-Nishina cross-section

When $\varepsilon \sim m_e c^2$ as well as the relativistic effects implied by conservation of energy and momentum, quantum mechanical effects also change the electron cross-section from the classical value. The differential cross-section is given by the *Klein-Nishina* formula:

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{r_0^2}{2}\frac{\varepsilon_1^2}{\varepsilon^2}\left(\frac{\varepsilon_1}{\varepsilon} + \frac{\varepsilon}{\varepsilon_1} - \sin^2\Theta\right)$$

As

$$\varepsilon \to \varepsilon_1$$

$$\frac{d\sigma_{KN}}{d\Omega} \to \frac{r_0^2}{2}(2 - \sin^2\Theta) = \frac{1}{2}r_0^2(1 + \cos^2\Theta) = \frac{d\sigma_T}{d\Omega}$$

Integrating the above expression over solid angle gives the following expression for the total Klein-Nishina cross-section:

$$\sigma_{KN} = \frac{3}{4}\left[\frac{1+x}{x^3}\left\{\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right\}\right]$$

$$+ \frac{1}{2x}\ln(1+2x) - \frac{(1+3x)}{(1+2x)^2}$$

$$x = \frac{h\nu}{m_e c^2}$$

Limits

Nonrelativistic regime ($x \ll 1$):

$$\sigma = \sigma_T\left(1 - 2x + \frac{26x^2}{5}\right)$$

Extreme relativistic regime ($x \gg 1$):

$$\sigma = \frac{3}{8}\sigma_T x^{-1}\left(\ln 2x + \frac{1}{2}\right)$$

$$\to 0 \quad \text{as} \quad x \to \infty$$

That is, electrons are less efficient scatterers of high energy photons.

3 Scattering from electrons in motion

The above applies to an electron at rest. For most applications, the electrons are moving, sometimes with relativistic velocities so that we need to consider the details of electron scattering in this case. We do so by extending the results for scattering by a stationary electron to moving electrons using a change of frame defined by the Lorentz transformation.

3.1 Rest frame and electron frame

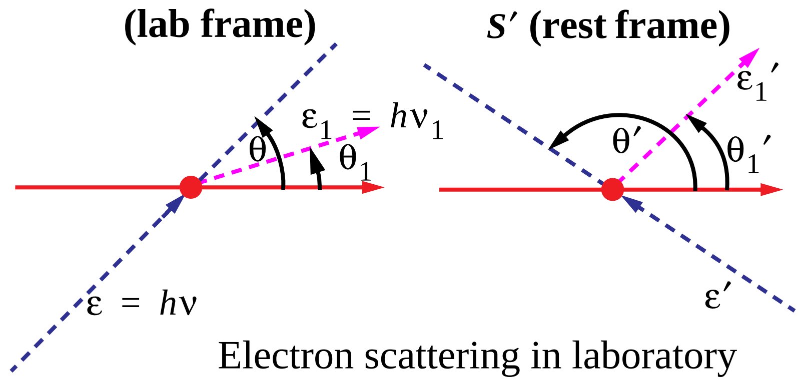
Here we use a device that it used a lot in High Energy Astrophysics, we determine the energy transfer in an arbitrary frame by Lorentz transforming to and from a frame in which the electron is at rest. In this application “something” is the electron.

Assume that in the rest frame (S')

$$h\nu' \ll m_e c^2$$

so that the energy change in the rest frame can be neglected. The photon-electron collision in the two frames is as depicted in the following diagram:

Notation:



Electron scattering in laboratory frame and rest frame of electron

Note that all angles are measured clockwise from the positive -axis defined by the electron velocity.

$$\varepsilon = \text{Initial photon energy in lab frame}$$

$$\theta = \begin{array}{c}\text{Angle between initial photon direction} \\ \text{and electron velocity in lab frame}\end{array}$$

$$\varepsilon' = \text{Initial photon energy in electron frame}$$

$$\theta' = \begin{array}{c}\text{Angle between initial photon direction} \\ \text{and electron velocity in rest frame}\end{array}$$

$$\varepsilon_1 = \text{Scattered photon energy in lab frame}$$

$$\theta_1 = \text{Scattered photon angle in lab frame}$$

$$\varepsilon_1' = \text{Scattered photon energy in rest frame}$$

$$\theta_1' = \text{Scattered photon angle in rest frame}$$

3.2 Transformation between frames

The frame S' is the frame in which the electron is at rest. The various angles refer to the angle between the direction of the photon (pre- or post-collision) and the x-axis. In the lab frame S the electron has velocity v and Lorentz factor

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = (1-\beta^2)^{-1/2}$$

$$\beta = \frac{v}{c}$$

Recall the transformation between energies between two relatively moving frames. For massive particles:

$$E' = \gamma E\left(1 - \beta\frac{v_p}{c}\cos\theta\right)$$

where E is the energy of the particle and v_p is its velocity.

For photons (where we denote energy by ε):

$$\varepsilon' = \gamma\varepsilon(1-\beta\cos\theta) = \gamma\varepsilon(1-\beta\mu)$$

where $\mu = \cos\theta$

The Lorentz factor $\gamma = (1-\beta^2)^{-1/2}$.

The reverse and forward transformations are:

$$\varepsilon' = \gamma\varepsilon(1-\beta\cos\theta) = \gamma\varepsilon(1-\beta\mu)$$

$$\varepsilon_1 = \gamma\varepsilon_1'(1+\beta\cos\theta_1) = \gamma\varepsilon_1'(1+\beta\mu_1)$$

Hence, except for values of θ near 0 ($\mu \approx 1$), the photon picks up a factor of γ when we transform to the rest frame and except for values of θ_1 near π ($\mu_1 \approx -1$), we pick up a further factor of γ when we transform the energy of the scattered photon back to the lab frame.

Energy gain from Thomson scattering

Assuming that Thomson scattering applies in the rest frame, (i.e. $\varepsilon_1' = \varepsilon'$) then the ratios of energies in going from lab frame to rest frame and then back to lab frame are of order

$$1:\gamma:\gamma^2$$

Hence, in being scattered by an electron, a photon increases in energy by a factor of order γ^2. Obviously for relativistic electrons, this can be substantial.

Condition for Thomson scattering in the rest frame

Since, the energy of the photon in the rest frame is of order $\gamma\varepsilon$, then the condition for Thomson scattering to apply in the rest frame is:

$$\gamma\varepsilon \ll m_e c^2 \Rightarrow \gamma h\nu \ll m_e c^2$$

$$h\nu \ll \frac{m_e c^2}{\gamma}$$

Example

Consider scattering of radio emitting photons by electrons with a Lorentz factor of order 10^4. First, assuming that the Thomson limit applies in the rest frame, the typical photon frequency produced is

$$\gamma^2\nu \sim 10^8 \times 10^9 \text{ Hz} \sim 10^{17} \text{ Hz}$$

i.e. X-ray frequencies.

Is the condition for the Thomson limit satisfied? We require the initial soft photon energy to satisfy:

$$\nu \ll \frac{m_e c^2}{h\gamma} = \frac{9.11\times10^{-31} \times (3\times10^8)^2}{6.6\times10^{-34} \times 10^4} = 10^{16}$$

and this is easily satisfied for radio photons.

4 Emitted power resulting from inverse Compton scattering

The scattering of photons by energetic electrons, frequently results in a transfer of energy from the electrons to the photons. When this is the case, the process is known as inverse Compton scattering.

4.1 Single electron power

Isotropic distribution of photons

Photons impinging on a population of electrons have a distribution of directions. For simplicity and with physical applications in mind, we consider a distribution of photons which is isotropic in the lab frame. Let

$$
\begin{aligned}
f(\mathbf{p})d^3p &= \text{No density of photons within range } d^3p \\
n(\varepsilon)d\varepsilon &= \text{No density of photons within } d\varepsilon
\end{aligned}
$$

In the lab frame, where the photons are assumed isotropic:

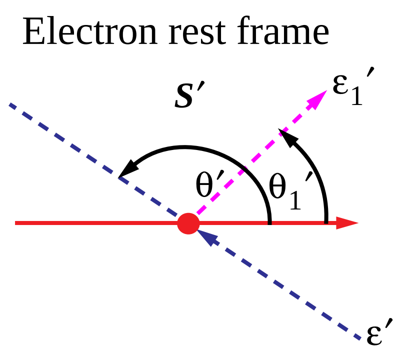
$$4\pi p^2 f(p)dp = n(\varepsilon)d\varepsilon$$

The calculation of the single electron power follows the following scheme.

Isotropic distribution of photons in lab frame → Anisotropic distribution of photons in rest frame of electron

↓

Compute power in rest frame, using conservation of energy in rest frame → Transform back to lab frame and calculate power in that frame



We consider the geometry at the left. Consider photons incident at an angle θ' to the x-axis which are scattered into a range of angles indicated by $\theta_1{}'$. Consider the incident flux on the electron due to photons within a region $d^3\mathbf{p}'$ of momentum space, where

$$d^3\mathbf{p}' = p'^2 dp' \sin\theta' d\phi'$$

The ***number density*** of photons in this region of momentum space is:

$$\delta n = f(\boldsymbol{p}')d^3\boldsymbol{p}'$$

Therefore:

$$\text{Incident photon flux per unit area per unit time} = \delta n \times c = cf(\boldsymbol{p}')d^3\boldsymbol{p}'$$

$$\text{No of photons scattered per unit time} = \sigma_T \times cf(\boldsymbol{p}')d^3\boldsymbol{p}'$$

In the rest frame the energy of the scattered photons remains the same. Hence, the energy per unit time, i.e. the power, of the scattered radiation contributed by a single electron, is:

$$\delta P' = \sigma_T \times \varepsilon' \times cf'(\boldsymbol{p}')d^3\boldsymbol{p}'$$

Integrating over all momenta in the *rest* frame:

$$P' = c\sigma_T \int \varepsilon' f'(\boldsymbol{p}')d^3\boldsymbol{p}'$$

Transformation back to lab frame

We know that the distribution function is invariant under Lorentz transformations:

$$f'(\boldsymbol{p}') = f(\boldsymbol{p})$$

We also need to determine the transformation law for ε and $d^3\boldsymbol{p}$ as a result of the Lorentz transformations between S and S'. Determining the transformation of $d^3\boldsymbol{p}$ involves determining the Jacobean of the transformation from S' to S.Using the transformations for photons derived from the 4-momentum, we have for the spatial momentum components:

$$p_x' = \gamma\left(p_x - \frac{\varepsilon}{c}\beta\right) = \gamma(p_x - p\beta)$$

$$p_y' = p_y \qquad p_z' = p_z$$

$$\frac{\partial p_x'}{\partial p_x} = \gamma\left(1 - \frac{\partial p}{\partial p_x}\beta\right) = \gamma\left(1 - \frac{p_x}{p}\beta\right)$$

$$= \gamma(1 - \beta\cos\theta) = \gamma(1 - \beta\mu)$$

$$\frac{\partial p_x'}{\partial p_y} = -\gamma\beta\frac{p_y}{p} \qquad \frac{\partial p_x'}{\partial p_z} = -\gamma\beta\frac{p_z}{p}$$

$$\frac{\partial p_y'}{\partial p_y} = \frac{\partial p_z'}{\partial p_z} = 1$$

For the energy

$$\varepsilon' = \gamma(\varepsilon - \beta c p_x) = \gamma\varepsilon\left(1 - \beta\frac{p_x}{p}\right)$$
$$= \gamma\varepsilon(1 - \beta\cos\theta) = \gamma\varepsilon(1 - \beta\mu)$$

Jacobean of the transformation between lab momentum space and rest frame momentum space

$$J = \begin{vmatrix} \gamma(1-\beta\mu) & -\gamma\beta\frac{p_y}{p} & -\gamma\beta\frac{p_z}{p} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \gamma(1-\beta\mu)$$

Hence,

$$d^3\boldsymbol{p}' = \gamma(1 - \beta\cos\theta)d^3\boldsymbol{p}$$

$$\varepsilon' f(\boldsymbol{p}')d^3 p' = \gamma^2(1-\beta\cos\theta)^2\varepsilon f(p)d^3\boldsymbol{p}$$

The power radiated in the rest frame is:

$$P' = c\sigma_T\int\varepsilon f(\boldsymbol{p}')d^3\boldsymbol{p}' = c\sigma_T\int\gamma^2(1-\beta\cos\theta)^2\varepsilon f(p)p^2 dp d\Omega$$
$$= \frac{c\sigma_T\gamma^2}{4\pi}\int(1-\beta\cos\theta)^2\varepsilon n(\varepsilon)d\varepsilon\sin\theta d\theta d\phi$$

The integral is over variables in the lab frame where the photon distribution is isotropic.

We obtain for the power:

$$P' = c\sigma_T\gamma^2\left(1 + \frac{\beta^2}{3}\right)\int_0^\infty \varepsilon n(\varepsilon)d\varepsilon$$

The quantity

$$U_{\rm ph} = \int_0^\infty \varepsilon n(\varepsilon)d\varepsilon$$

is the photon energy density in the lab frame.

Emitted power in the lab frame

How does the power emitted in the rest frame relate to the power emitted in the lab frame? We can write these powers as

$$P' = \frac{dE_1'}{dt'} \qquad P = \frac{dE_1}{dt}$$

where dE_1 and dE_1' are the energies in bundles of radiation emitted in time intervals dt and dt' respectively.

From the Lorentz transformation between the electron rest frame and the lab frame:

$$dt = \gamma\left(dt' + \frac{\beta dx'}{c}\right) = \gamma dt'$$

since in the rest frame of the electron, $dx' = 0$.

Consider a bunch of photons emitted with energy dE_1' and x-momentum $dp_x' = (dE_1'/c)\cos\theta_1$ at the angle θ_1'. The Lorentz transformed energy of this bunch of photons in the lab frame is

$$dE_1 = \gamma(dE_1' + \beta c\,dp_x') = \gamma(dE_1')(1 + \beta\cos\theta_1')$$

However, scatterings with $\cos\theta_1' > 0$ and $\cos\theta_1' < 0$ are equally likely, because of the symmetry of the Thomson cross-section, so that the averaged contribution to dE_1 is

$$\langle dE_1 \rangle = \gamma dE_1'$$

Hence the power in the lab frame is:

$$\frac{\langle dE_1 \rangle}{dt} = \frac{\gamma dE_1'}{\gamma dt'} = \frac{dE_1'}{dt'}$$

That is,

$$P = P'$$

Hence the scattered power in the lab frame is

$$P = c\sigma_T\gamma^2\left(1 + \frac{\beta^2}{3}\right)U_{\mathrm{ph}}$$

4.2 Number of scatterings per unit time

In the electron rest frame:

$$\frac{dN'}{dt'} = \text{Number of scatterings per unit time}$$

$$= c\sigma_T \int f(\boldsymbol{p}')d^3\boldsymbol{p}'$$

Again we transform this equation to the lab frame:

$$\begin{aligned}\frac{dN'}{dt'} &= c\sigma_T\int\gamma(1 - \beta\cos\theta)f(p)p^2dp\sin\theta d\theta d\phi \\ &= \frac{2\pi\gamma c\sigma_T}{4\pi}\int(1 - \beta\cos\theta)n(\varepsilon)d\varepsilon\sin\theta d\theta \\ &= \gamma c\sigma_T N_{\mathrm{ph}}\end{aligned}$$

where the ***photon number density*** is

$$N_{\mathrm{ph}} = \int n(\varepsilon)d\varepsilon$$

In transforming dN'/dt' to the lab frame, we note that dN' is a number and so is Lorentz invariant, so that

$$\boxed{\frac{dN}{dt} = \gamma^{-1}\frac{dN'}{dt'} = c\sigma_T N_{\rm ph}}$$

4.3 Nett energy radiated

We write the above equation as

$$\frac{dN}{dt} = c\sigma_T N_{\rm ph} = c\sigma_T \int n(\varepsilon)\,d\varepsilon$$

Hence, the number of scatterings per unit time per unit photon energy by a single electron is $c\sigma_T n(\varepsilon)$. Hence the energy removed from photons within $d\varepsilon$ is $c\sigma_T \varepsilon n(\varepsilon) d\varepsilon$. Hence, the energy removed from the photon field is given by:

$$\frac{dE_1}{dt} = -c\sigma_T \int \varepsilon n(\varepsilon)\,d\varepsilon = -c\sigma_T U_{\rm ph}$$

Therefore, the nett energy radiated is:

$$\frac{dE_{\rm rad}}{dt} = P_{\rm compt} = c\sigma_T U_{\rm ph}\left[\gamma^2\left(1+\frac{1}{3}\beta^2\right)-1\right]$$

$$\boxed{= \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_{\rm ph}}$$

where we have used

$$\gamma^2\beta^2 = \gamma^2 - 1$$

4.4 Comparison of synchrotron and inverse Compton power

We already know that the *synchrotron* power of an electron is given by

$$\boxed{P_{\rm synch} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_B}$$

$$U_B = \frac{B^2}{2\mu_0}$$

Hence,

$$\frac{P_{\rm compton}}{P_{\rm synch}} = \frac{U_{\rm ph}}{U_{\rm B}}$$

From this expression, we can see that the inverse Compton power can be comparable to the synchrotron power, when the photon energy density is comparable to the magnetic energy density. This is often the case at the bases of jets, so that these regions are often strong X-ray emitters.

5 Inverse Compton emission from the microwave background

A regime in which inverse Compton emission is important is in the extended regions of radio galaxies, where the energy density of the microwave background radiation may be comparable to the magnetic energy density.

Consider the energy density of blackbody radiation:

$$\varepsilon_{BB} = aT^4 \qquad a = \frac{8\pi^5 k^4}{15 h^3 c^3} = 7.57\times10^{-16}\,\mathrm{J\ m^{-3}\ K^{-4}}$$

This is equal to the energy density of a magnetic field when

$$\frac{B^2}{2\mu_0} = aT^4 \Rightarrow B = \sqrt{2\mu_0 a}\,T^2 = 4.4\times10^{-11}\,T^2$$

For the microwave background, $T = 2.7$ K.

Therefore, when

$$B < B_{\mathrm{crit}} = 3.2\times10^{-10}\,T = 3.2\,\mu\mathrm{G}$$

the radiation from inverse Compton emission is more important than synchrotron radiation as an energy loss mechanism for the electrons.

6 Inverse Compton emission from a thermal plasma

The above expressions are derived without any restriction on the energies of the electrons. If we consider a thermal distribution, then,

$$\gamma^2 \approx 1 \qquad \langle\beta^2\rangle = \frac{\langle v^2\rangle}{c^2} = \frac{3kT}{mc^2}$$

Hence, the total Compton emission from a single electron is given by:

$$P_{\mathrm{Compton}} = \left(\frac{4kT}{mc^2}\right) c\sigma_T U_{\mathrm{ph}}$$

The volume emissivity from the plasma with electron number density n_e is:

$$j_{\mathrm{Compton}} = \left(\frac{4kT}{mc^2}\right) c\sigma_T n_e U_{\mathrm{ph}}$$

Compton scattering of "soft" photons by hot thermal electrons in the coronae of accretion disks is thought to be responsible for the X-ray emission from AGN and for the X-ray emissivity of galactic black hole candidates.

7 Mean energy of scattered photons

By dividing the radiated power by the number of scattered photons per unit time, we can calculate the mean energy per scattered photon.

$$\begin{aligned}\langle \varepsilon_1 \rangle &= \frac{\frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_{\mathrm{ph}}}{c\sigma_T N_{\mathrm{ph}}} \\ &= \frac{4}{3}\gamma^2\beta^2 \frac{U_{\mathrm{ph}}}{N_{\mathrm{ph}}} = \frac{4}{3}\gamma^2\beta^2 \langle \varepsilon \rangle\end{aligned}$$

Inverse Compton emission from relativistic electrons

Thus for photons scattering off relativistic electrons, the mean amplification of energy per scattering is $4\gamma^2/3$, supporting the order of magnitude estimate of γ^2 for this parameter.

Clearly, for relativistic electrons, the energy gain is substantial, underlining the importance of inverse Compton emission as an astrophysical process.