Numerical Methods in Babylonian Astronomy

Teije de Jong

Astronomical Institute ‘’Anton Pannekoek”
University of Amsterdam

Babylonian theory of planetary motion (4th century BC)

- Based on the periodicity observed in the dates of heliacal rising/ setting (called first appearance or disappearance by the Babylonian astronomers) and of the stationary points of the planets
- Taking into account in an experimental way the variation in angular velocity of the Sun, Moon and planets (due to the eccentricity of the planetary orbit)
- Numerically described by so-called “step” functions in System A and “zig-zag” functions (approximations to trigonometric functions) in System B
- Example: Computation of Eastern stationary points of Jupiter for 199 – 139 BC recorded on tablet AO 6476 (= ACT No. 600*)

  *ACT = O. Neugebauer, Astronomical Cuneiform Texts, 3 Vols., London (1955)

AO 6476 – hand-written copy

AO 6476 – translation

Year	*Time interval*	*Date*	*Position*
113 U	48; 5,10	I 28;41,40	8; 6 (10)
114	48; 5,10	II 16;46,50	14; 6 (11)
115 A	48; 5,10	IV 4;52	20; 6 (12)
116	48; 5,10	IV 22;57,10	26; 6 (1)
117	48; 5,10	VI 11; 2,20	2; 6 (3)
118 A	45;54,10	VII 26;56,30	5;55 (4)
119	42; 5,10	VIII 9; 1,40	5;55 (5)
120	42; 5,10	IX 21; 6,50	5;55 (6)
121 A	42; 5,10	XI 3;12	5;55 (7)

Jupiter – System A theory

- Parameters:
  - Angular velocity:
    $\Delta\lambda$ = 30º per "time step" for λ = 85º - 240º, and
    $\Delta\lambda$ = 36º per "time step" for λ = 240º - 85º
  - Transition from "slow" to "fast" arc by linear interpolation so that accuracy of the scheme is retained (no accumulating errors)
  - "Time step" Δt ("tithis") = $\Delta\lambda$ (º) + 12;05,10
    - 1 tithi = 1/30 of lunar month → greatly simplifies computation and is always accurate within ½ day
- The beauty of this system lies in its:
  - Numerical simplicity
  - Constant "accuracy" over hundreds of years
- How were these parameters determined from observation?

The periods of Jupiter

- In the procedure text BM 34221 [+ 5 other fragments] (= ACT No. 812) we find the following periods of Jupiter:
  12 years + 5°; 71 years - 6°; 83 years - 1°; ...; 427 years [exact]
- Synodic phases of Jupiter (first appearances etc.) recur at the same position in the ecliptic (± the indicated difference in longitude) after one such period
- These periods are remarkably accurate; they can be reproduced by using modern parameters (1 sidereal year = 365.256363 days and 1 synodic period of Jupiter = 398.884 days) and using the fact that the sun moves about 1° per day [check this by computation!]
- The "exact" long period of 427 years can be constructed by taking linear combinations of the the 12-year, 71-year and 83-year periods [check for which linear combinations the cumulative differences cancel to zero and try to understand the Babylonian choice]

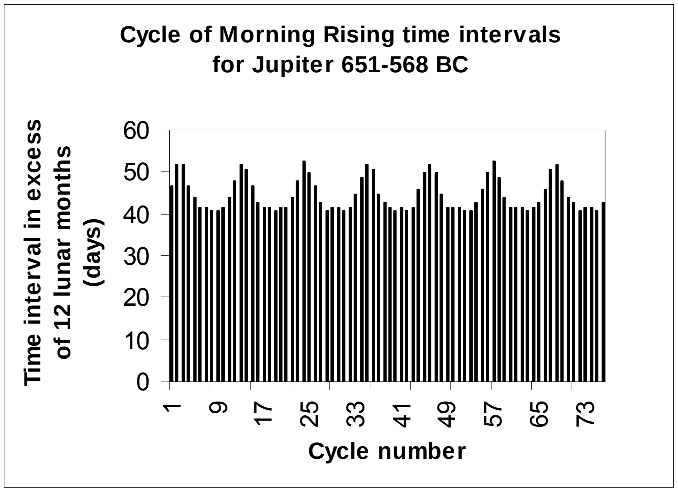
System A theory of Jupiter

- The 427-year period was used by the Babylonians to construct their system A theory of Jupiter
- In 427 years Jupiter completes 36 revolutions and 427 – 36 = 391 synodic periods
- The next step in the process was to introduce an ideal ecliptic coordinate system of 360° (analogous to the ideal year of 12 month of 30 tithis each?)
- We then have 36 x 360° = 391 x $\Delta\lambda \rightarrow \Delta\lambda$ = 33;08,45°, the increase in longitude between two consecutive synodic phases of the same kind (e.g. from first appearance to first appearance or from stationary point to stationary point, etc.)
- Similarly we have 391 x (360 + Δt) = 427 years, where t is measured in tithis (theoretical days) and where (360 + Δt) is the time interval between successive synodic phenomena of the same kind and Δt is the time in excess of one year of 12 months of 30 tithis each

System A theory for Jupiter - 2

- Now inserting 1 year = 12;22,08 months (derived from the Saros) = 12;22,08 x 30 tithis we find Δt = 45;13,53 tithis
- Using the value of $\Delta\lambda$ derived earlier we find $\Delta t = \Delta\lambda$ + 12;05,08, often approximated to 12;05,10, a number that we have encountered already in the numerical procedure in AO 6476
- Although it seems strange to us to mix quantities in degrees and in days (tithis) in one equation it is numerically justified by the fact that we are dealing with synodic phenomena (for which the distance between the sun and Jupiter is the same) and that the sun moves on average 1° per day
- The next step in the development of the theory is to take into account that the interval in time and in longitude between two successive synodic phases of the same kind is not constant (see the next two figures)

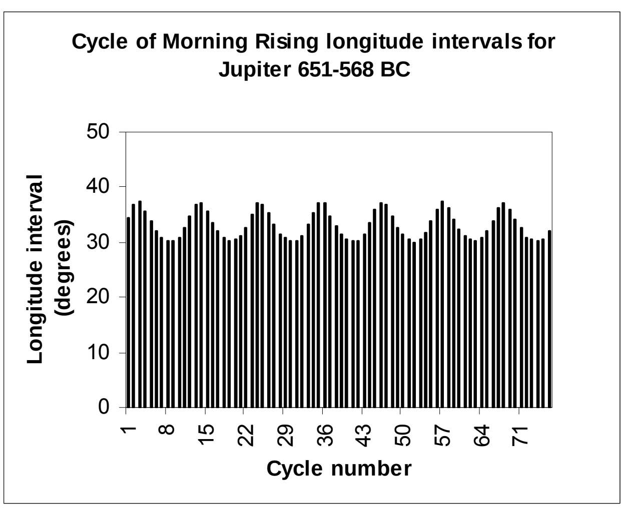
Observational data



Average time interval between successive first appearances of Jupiter equals:

12 lunar months + 45 ± 4 days

Observational data - 2



Average longitude interval between positions of Jupiter at successive first appearances:

33º ± 3º

System A theory of Jupiter - 3

- To take account of the variable longitude and time intervals the Babylonian astronomers adopted a step function with two angular velocities:

  30º per timestep1 of 42;05,10 (= 30 + 12;05,10) tithis and 36º per timestep2 of 48;05,10 (= 36 + 12;05,10) tithis
- Strict periodicity over 427 years is ensured by requiring that the number of synodic periods (time steps) in one revolution of Jupiter be unaffected by the division of the orbit in a "fast" arc (L) and a "slow" arc (360º – L), where L can be found from the condition

  L/36º + (360º – L)/30º = 391(synodic periods)/36 (revolutions)
- We then find L = 205º and (360º – L) = 155º
- The Babylonian astronomers chose the transition from slow motion to fast motion at 25º Gem (85º) and from fast to slow at 0º Sag (240º)
- This division implies that the perihelion of Jupiter is located halfway the "fast" arc at 12;30º Pisces, in reasonable agreement with the real value of 20º Pisces

System A theory for Jupiter - 4

- When in one time step the boundary between the slow and the fast arc (240º) or the fast and the slow arc (85º) was passed a new intermediate value of the longitude increment was found by interpolation
- Since 30 and 36 are both multiples of 6 the calculation for Jupiter is quite simple (in the sexagesimal system)
- For some of the other planets the calculation can be more numerically involved but for all planets the slow and fast arc were chosen such that simple ratios resulted
- To compute all planetary positions at one synodic phase (e.g. its first appearance) over a full period of 427 years only one accurate initial position and date (taken from the diaries) was needed