Towards a generic test of the strong field dynamics of general relativity using compact binary coalescence

T.G.F. Li, **W. Del Pozzo**, S. Vitale, C. Van Den Broeck, M. Agathos, J. Veitch, K. Grover, T. Sidery, R. Sturani, A. Vecchio

Outline

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- Method
- Bayesian Inference
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- Implementation
- Results
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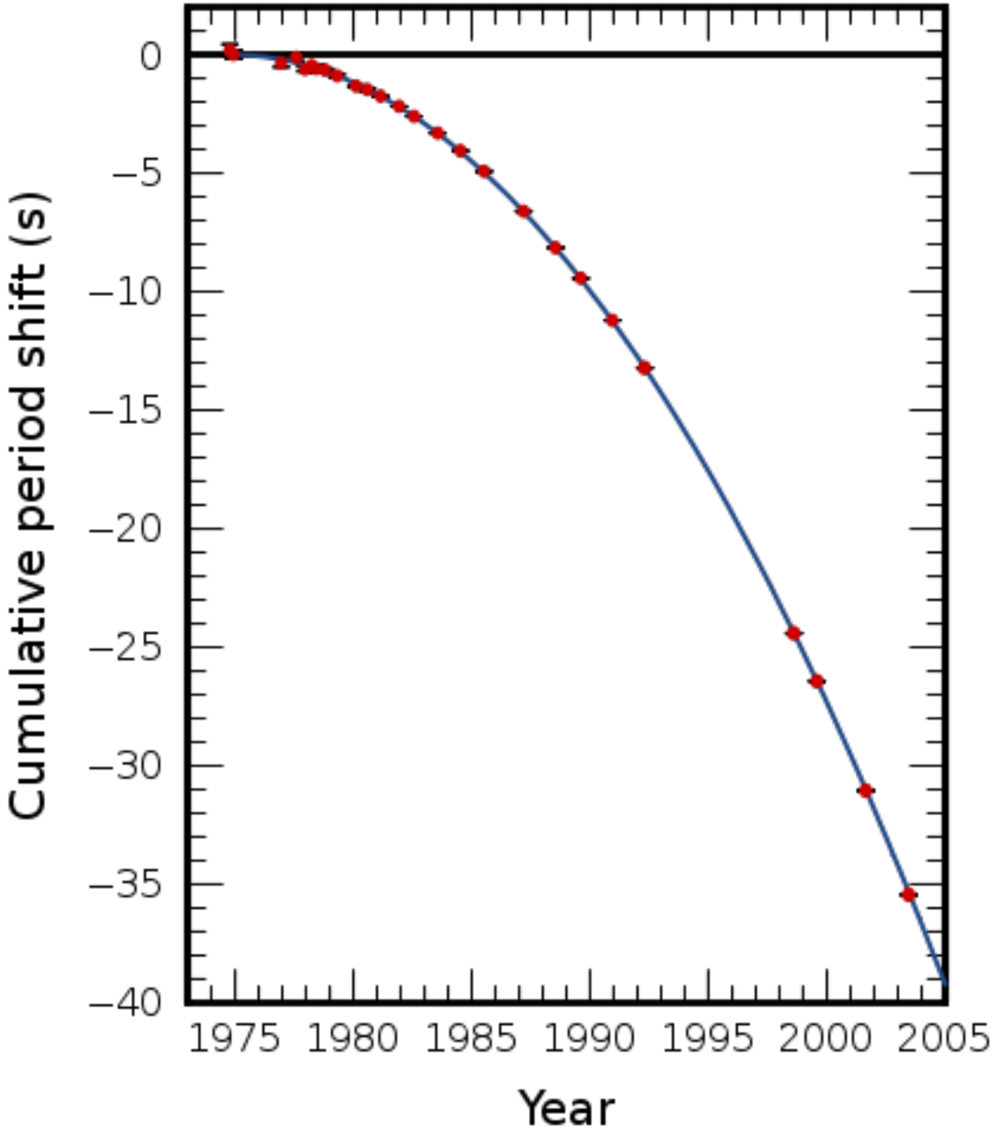
  - GR injections
- Conclusions

Introduction

- General Relativity (GR) has been extensively tested in the weak field, stationary regime (Misner, Thorne & Wheeler 1973)
- First tests of its dynamics came with the Hulse-Taylor pulsar



Introduction

- Binary pulsar tests are still in the weak field regime:

$$GM/(c^2 R) \simeq 4.4 \times 10^{-6}$$
$$v/c \simeq 2 \times 10^{-3}$$

- Compact binary coalescences at the last stable orbit:

$$GM/(c^2 R) = 1/6$$
$$v/c = 1/\sqrt{6}$$

Motivation

- The in-spiral of CBC systems is modeled with great accuracy with the PN formalism (Blanchet 2002)
  - GW carry detailed orbital information
  - At leading amplitude order:
    phase of GW=2 x orbital phase
- Unlike the binary pulsar case (Maggiore 2008), tests of GR with CBC signals are not limited to the 1PN conservative part of the dynamics

Motivation

- CBC provides access to non-linear effects:
  - 1.5PN tail effects, spin-orbit coupling
  - 2PN spin-spin coupling
- Fisher Information Matrix studies suggest that AdvLIGO/Virgo can constrain at least to 1.5PN (Mishra et al. 2010)

Method

- For each GW detection we consider two alternative scenarios:
  1. the signal is described by the predictions of GR
  2. the signal is NOT described by the predictions of GR
- Let's define the corresponding hypotheses:
  1. $\mathcal{H}_{\mathrm{GR}}$
  2. $\mathcal{H}_{\mathrm{modGR}}$

Method

- Let's assume that the GR hypothesis corresponds to this proposition:
  - the GW waveform is described by a non-spinning 3.5PN Taylor series in the frequency domain (what is commonly referred to as TaylorF2):

$$h(f) = \frac{1}{D} \frac{\mathcal{A}(\text{angles}, \mathcal{M}, \eta)}{\sqrt{\dot{F}(\mathcal{M}, \eta; f)}} f^{2/3} e^{i\Psi(\mathcal{M},\eta;f)}$$

$$\Psi(\mathcal{M}, \eta; f) = 2\pi f t_c - \varphi_c - \frac{\pi}{4} + \sum_{i=0}^{7} \left[\psi_i + \psi_i^{(l)} \ln f\right] f^{(i-5)/3}$$

Method

- In GR the phase coefficients of the Taylor expansion are particular functions of the mass parameters, e.g.:

$$\psi_i(\mathcal{M},\eta) = \mathcal{M}^{-5/3} g_i(\eta)(\pi\mathcal{M}\eta^{-3/5})^{(i-5)/3}$$

- Define $\mathcal{H}_i$ as the hypothesis:
  - the coefficient $\psi_i(\mathcal{M},\eta)$ is the exact function predicted by GR
- In practice, we take the GR hypothesis to be:

$$\mathcal{H}_{GR} = \mathcal{H}_0 \text{ and } \mathcal{H}_1 \text{ and } \mathcal{H}_2 \text{ and } \ldots \equiv \bigwedge_{i=0}^{7} \mathcal{H}_i$$

Method

- In these terms, $\mathcal{H}_{\mathrm{modGR}}$ corresponds to:
  - one or more of the coefficients is not the function predicted by GR: $\psi_i^{\mathrm{modGR}} \neq \psi_i(\mathcal{M}, \eta)$

- In logical terms:

$$\mathcal{H}_{\mathrm{modGR}} = (\mathrm{not}\ \mathcal{H}_0)\ \mathrm{or}\ (\mathrm{not}\ \mathcal{H}_1)\ \mathrm{or}\ (\mathrm{not}\ \mathcal{H}_2)\ \mathrm{or}\ \ldots \equiv \bigvee_{i=0}^{7} \neg \mathcal{H}_i$$

- From the definition of the two hypotheses:

$$\mathcal{H}_{\mathrm{modGR}} = \mathrm{not}\ \mathcal{H}_{\mathrm{GR}}$$

Method

- Our purpose is to develop a pipeline able to discriminate between the two aforementioned hypotheses

- The natural framework in which carry out such program is *Bayesian Inference*

Bayesian Inference

- To "rank" two competing hypotheses we use the odds ratio:

$$O_{X,Y} = \frac{P(X|D,I)}{P(Y|D,I)} = \frac{P(X|I)}{P(Y|I)}\frac{P(D|X,I)}{P(D|Y,I)}$$

$$= \frac{P(X|I)}{P(Y|I)} B_{X,Y}$$

Bayes' factor

prior odds

- and:

$$P(D|X,I) \equiv \mathcal{L}(X) = \int d\vec{\lambda} P(\vec{\lambda}|I) P(D|\vec{\lambda}, X, I)$$

marginalised likelihood

Bayesian Inference

- The odds ratio can be generalised to consider:
  - arbitrary number of disjoint hypotheses
  - arbitrary number of sources
  - both

Logically disjoint hypotheses

- The $\mathcal{H}_{\mathrm{modGR}}$ is defined as the composition of non-disjoint hypotheses $\mathcal{H}_i$
- If we define $H_{i_1 i_2 \ldots i_k}$ as:
  - the phasing coefficient $\psi_{i_1}, \ldots, \psi_{i_k}$ do not have the functional form predicted by GR, but *all* the remaining $\psi_j, \; j \notin \{i_1, i_2, \ldots, i_k\}$ do,

  they are disjoint, and we can write:

$$\mathcal{H}_{\mathrm{modGR}} = \bigvee_{i_1 < i_2 < \ldots < i_k} H_{i_1 i_2 \ldots i_k}$$

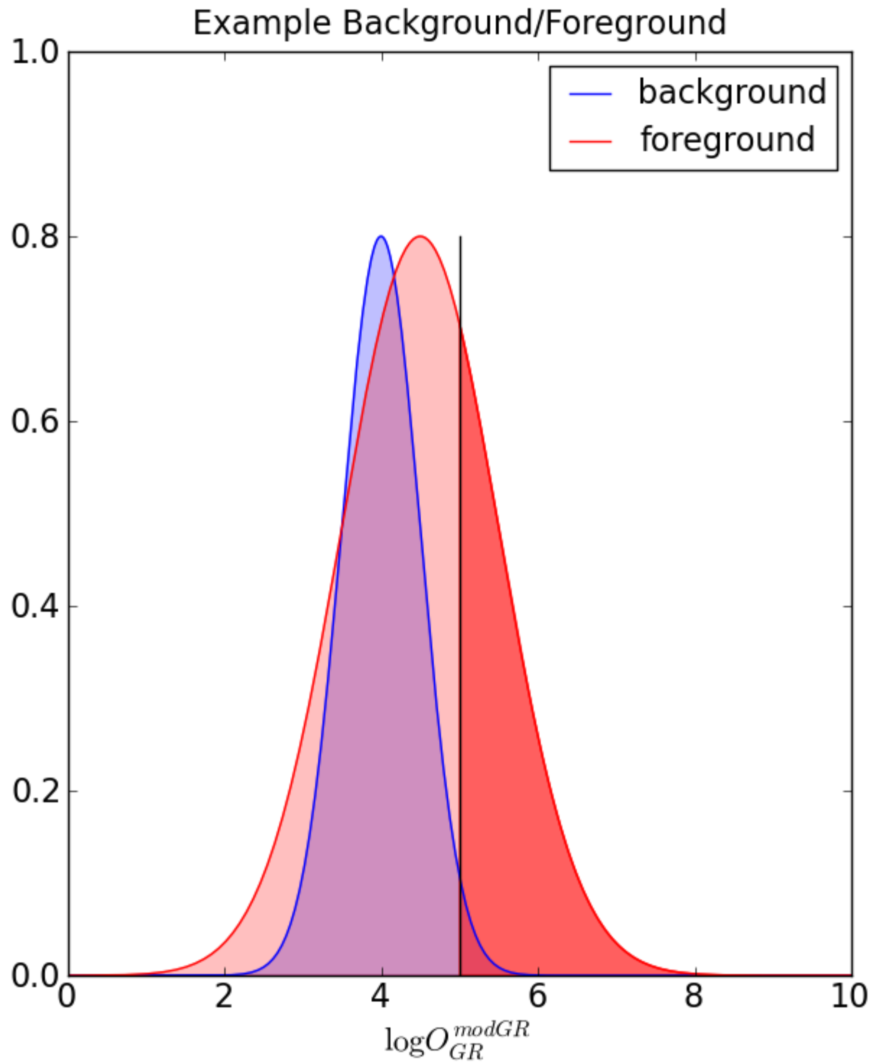
Implementation

- The hypothesis $H_{i_1 i_2 \dots i_k}$ is tested by a waveform where we allow to vary the parameters $\{\vec{\theta}, \psi_{i_1}, \psi_{i_2}, \dots, \psi_{i_k}\}$, keeping all remaining phase coefficients to their GR values

- In practice $\psi_i = \psi_i^{\mathrm{GR}}(\mathcal{M}, \eta)\ [1 + \delta\chi_i]$ with $\delta\chi_i \in [-0.25, 0.25]$
- The evidence is computed using a nested sampling algorithm

Implementation

- Since noise can mimic a deviation from GR, the bare odds ratio may not sufficient for a claim
  - We use GR injections to compute a background distribution of odds ratio
  - When analysing non-GR injections we compare with the background to assess the "efficiency"

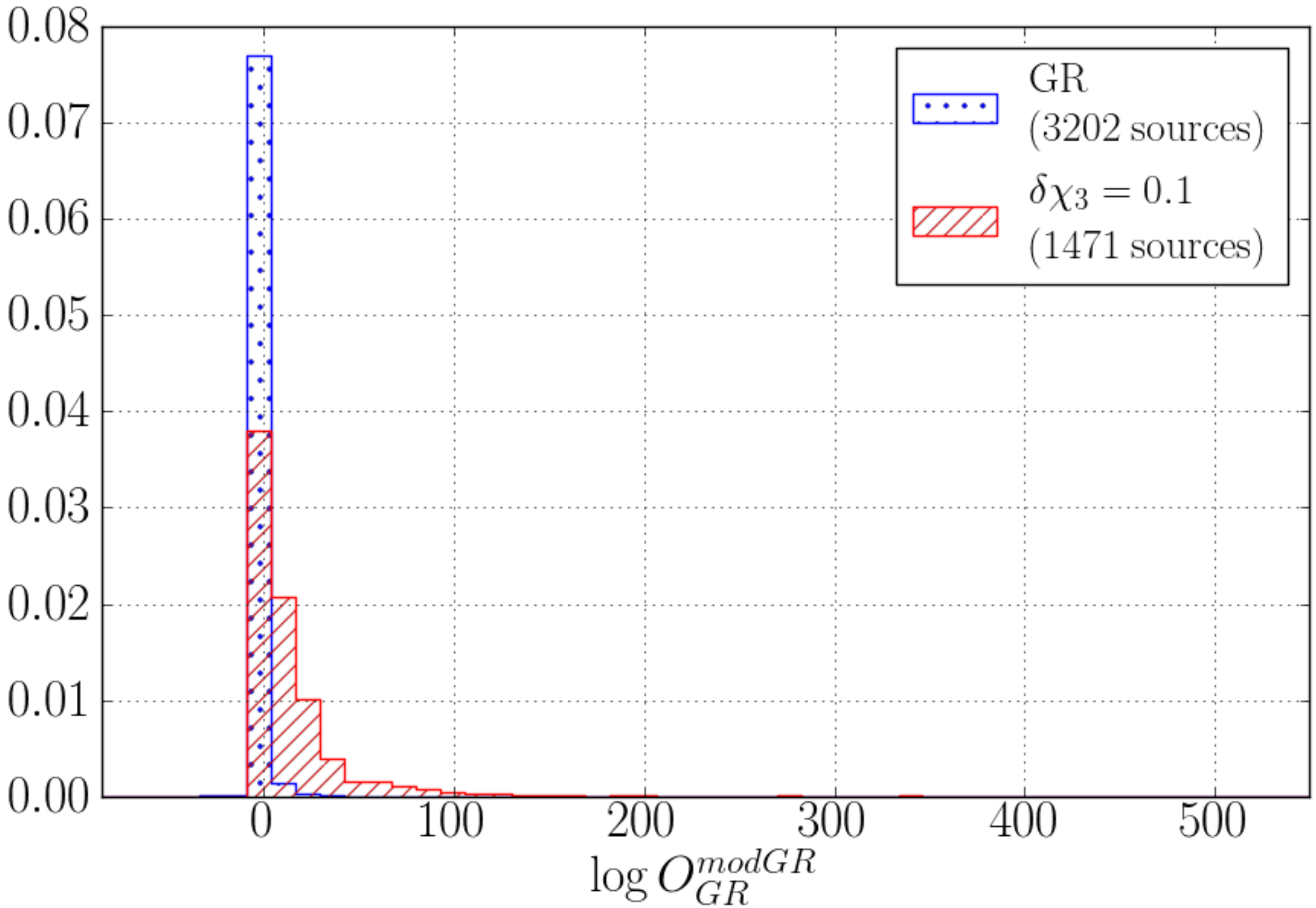


Results

- We performed experiments injecting a variety of modGR signals in simulated data from a network of 2 Advanced LIGO and Advanced Virgo
- BNS sources:
  - uniform in sky position and orientation
  - distances $\in$ [100,400] Mpc
  - masses $\in$ [1,2] Msun
- Tested three coefficients: ψ_1 , ψ_2 , ψ_3

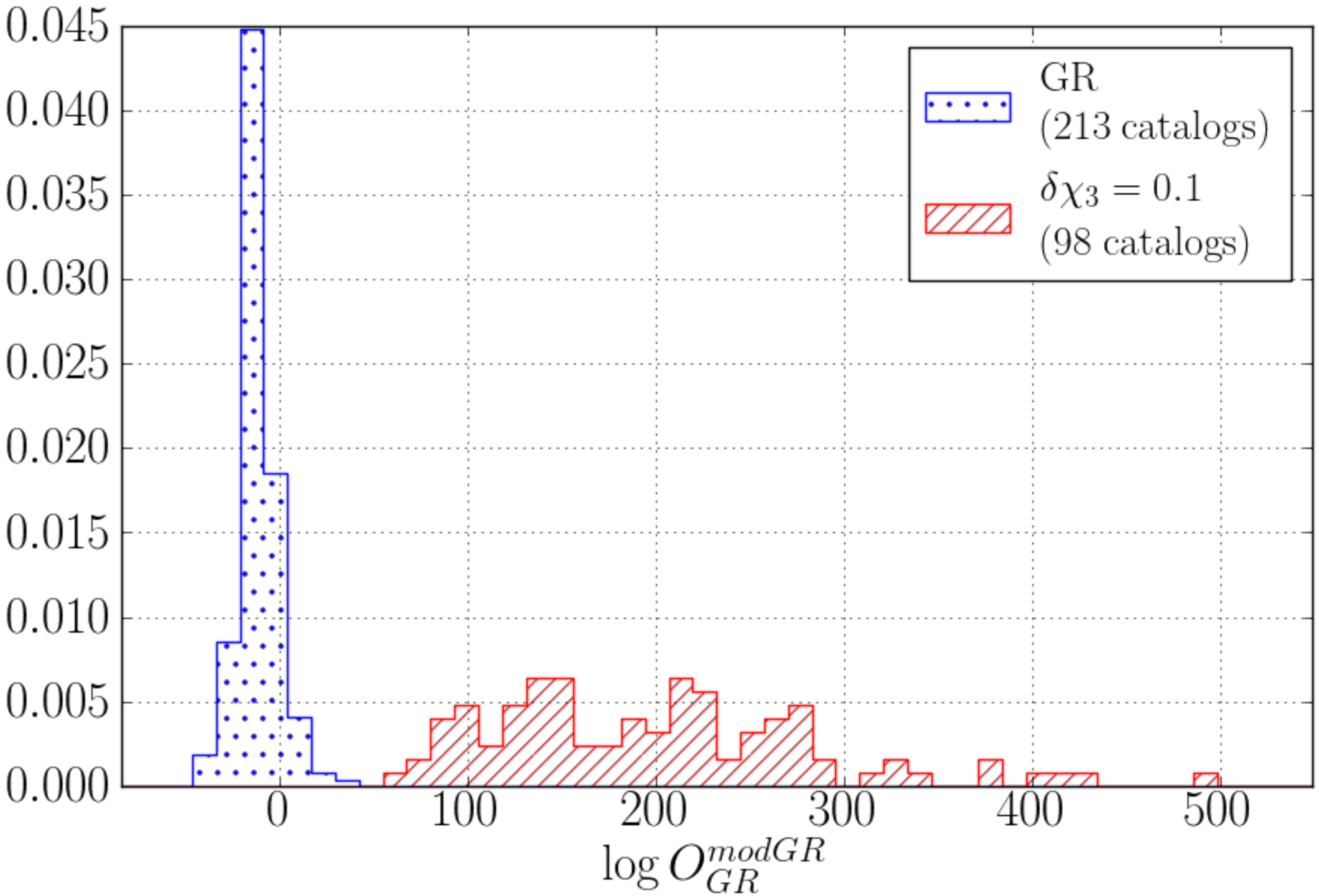
Results – modGR injections

- $\delta\chi_3 = 0.1$ injections (1.5PN), single source



Results – modGR injections

- $\delta\chi_3 = 0.1$ injections (1.5PN), 15 sources catalog



Results – modGR injections

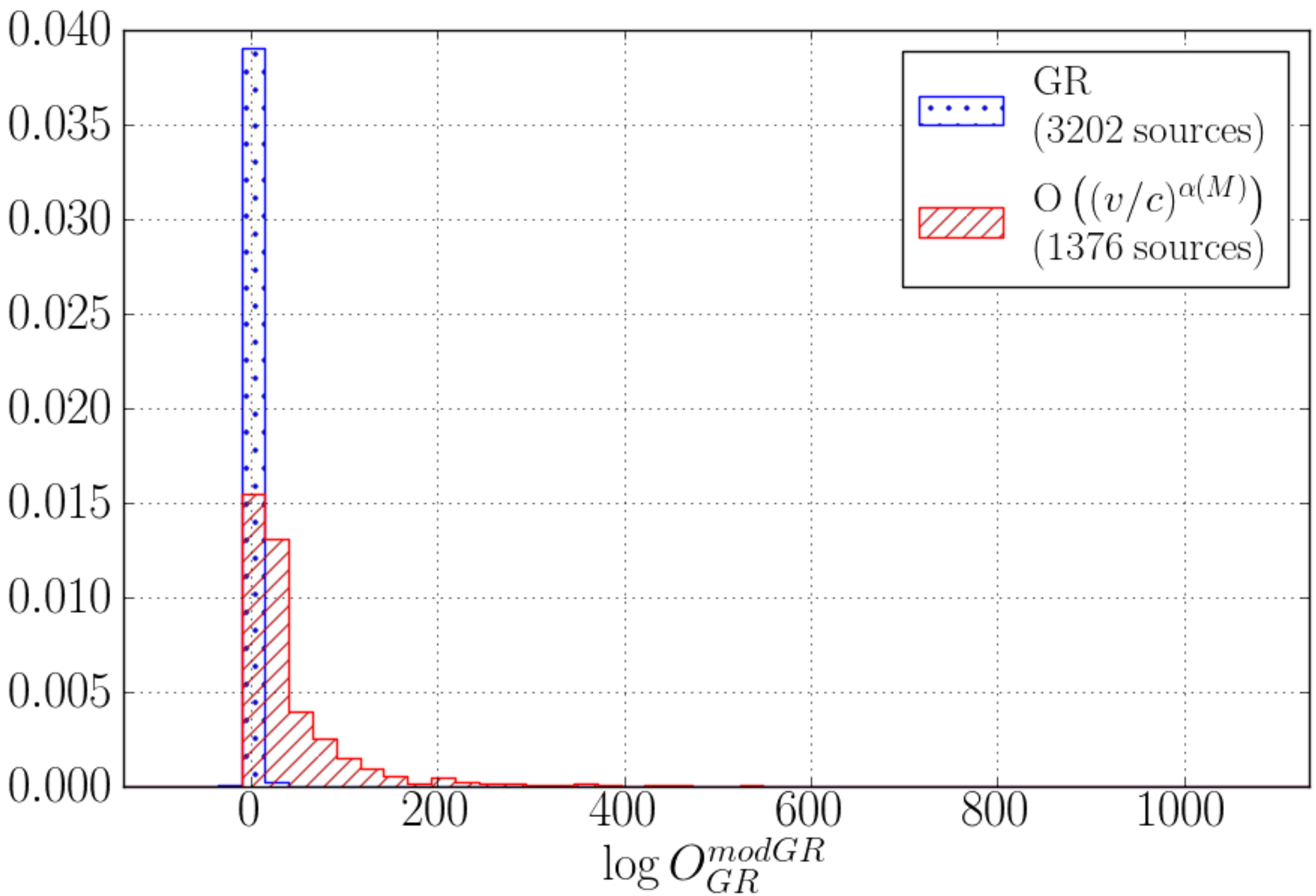
- The pipeline is able to detect deviations in one of the PN coefficients
- It cannot pinpoint the nature of the deviation
- Can it detect "arbitrary" (non-PN) deviations?
- We performed an experiment injecting signals with non-PN phase deviations:

$$\Psi^{\mathrm{GR}}(\mathcal{M},\eta;f) \to \Psi^{\mathrm{GR}}(\mathcal{M},\eta;f) + \frac{3}{128\eta}(\pi M f)^{-2+M/(3M_{\odot})}$$

- Effective PN order varies with mass, between 0.5PN and 1.5PN

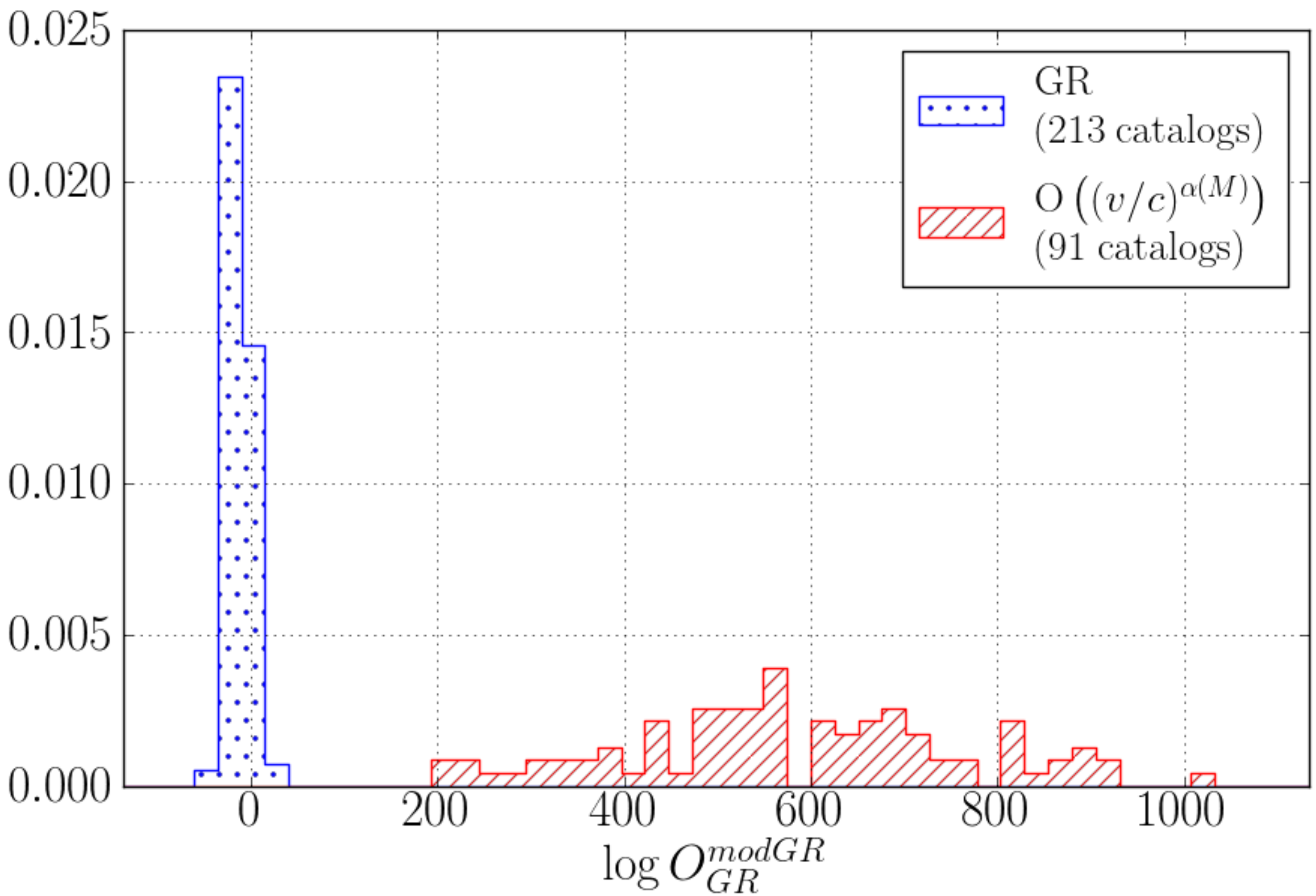
Results – modGR injections

- non-PN injections



Results – modGR injections

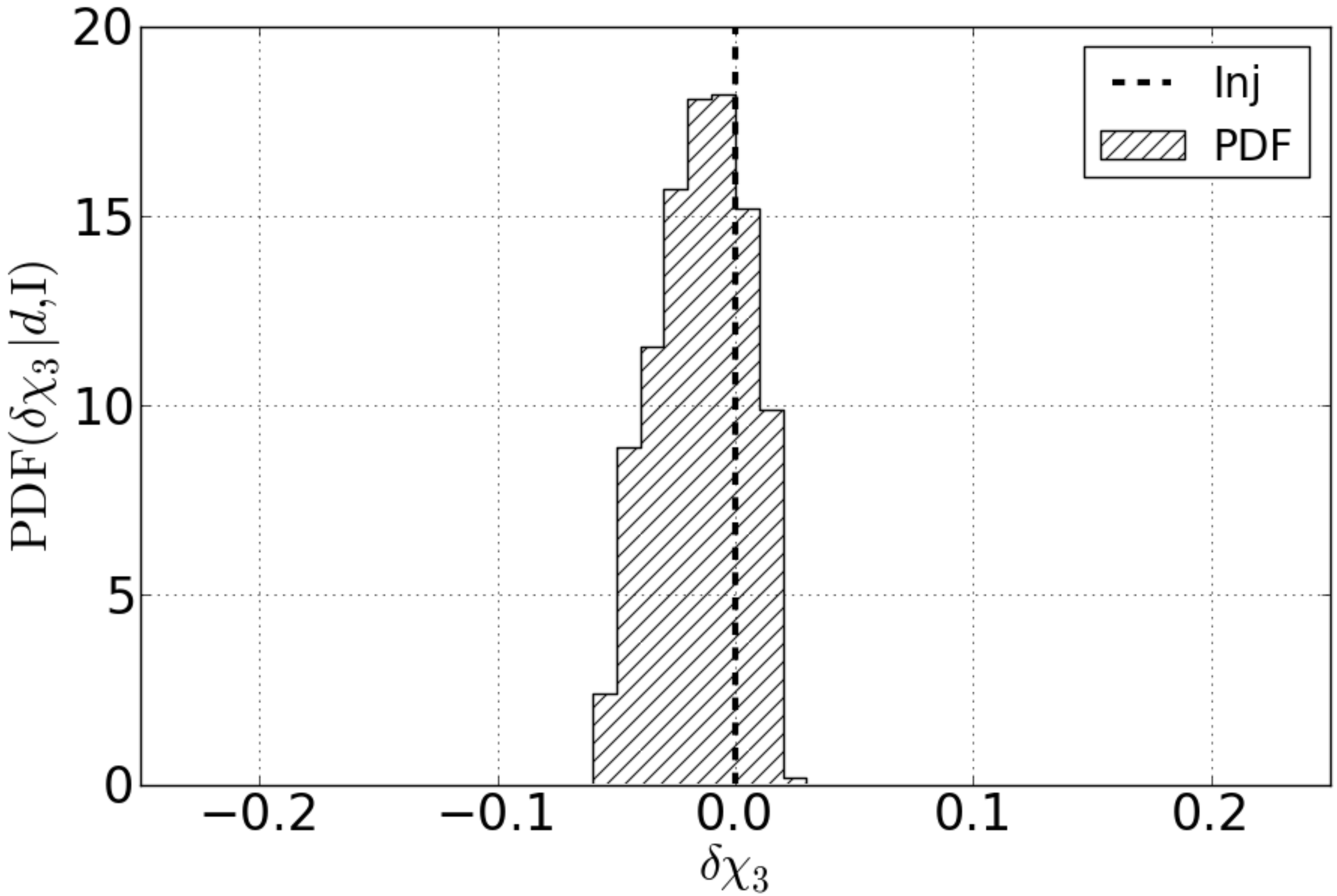
- non-PN injections



Results – GR injections

- When no deviation from GR is found, signals can constrain individual coefficients
- Use of posterior probability distributions for the shifts in the phasing coefficients $\delta\chi_i$
- Typical accuracy ~2% at 1-sigma level at SNR ~20

Results – GR injections

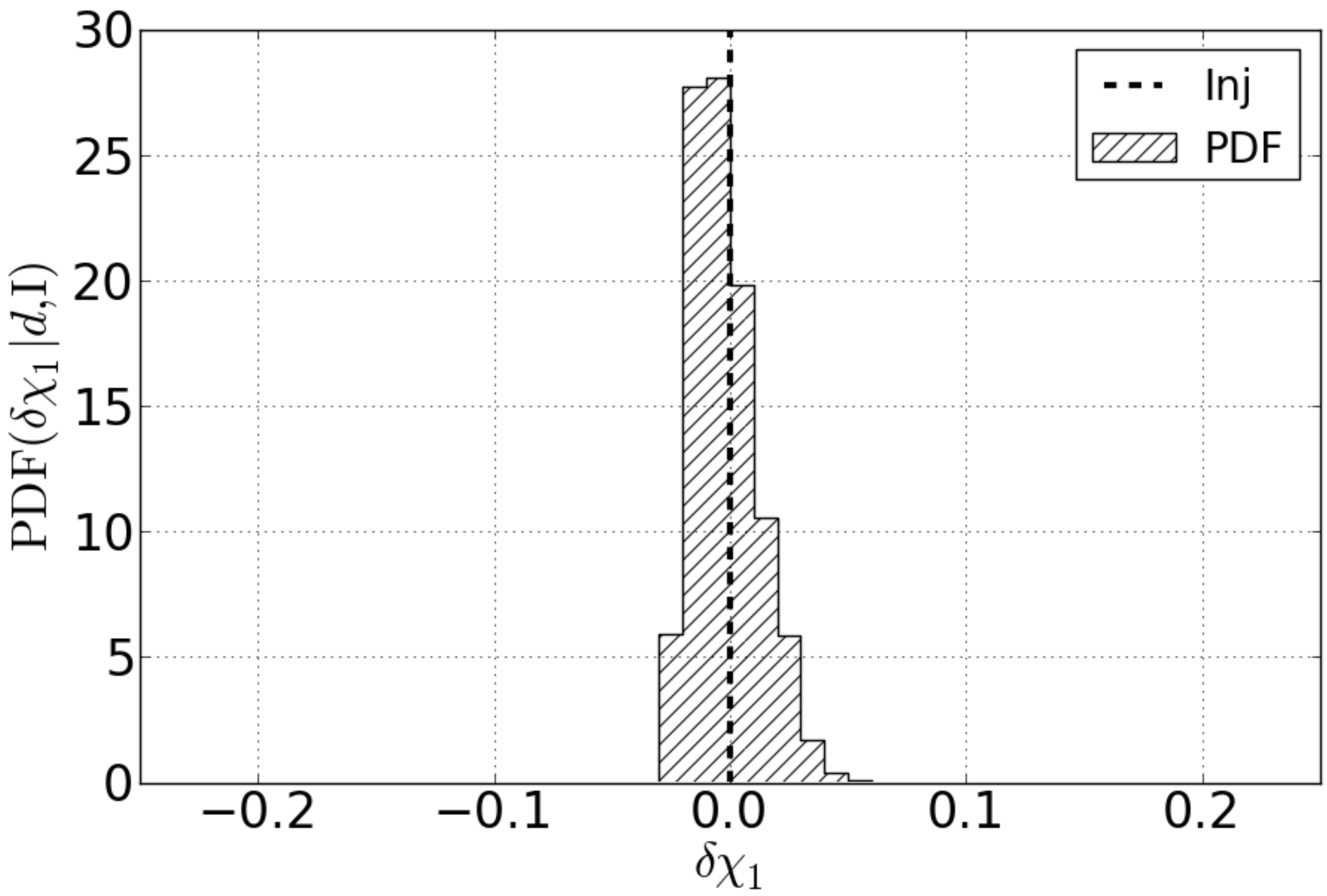


Conclusions

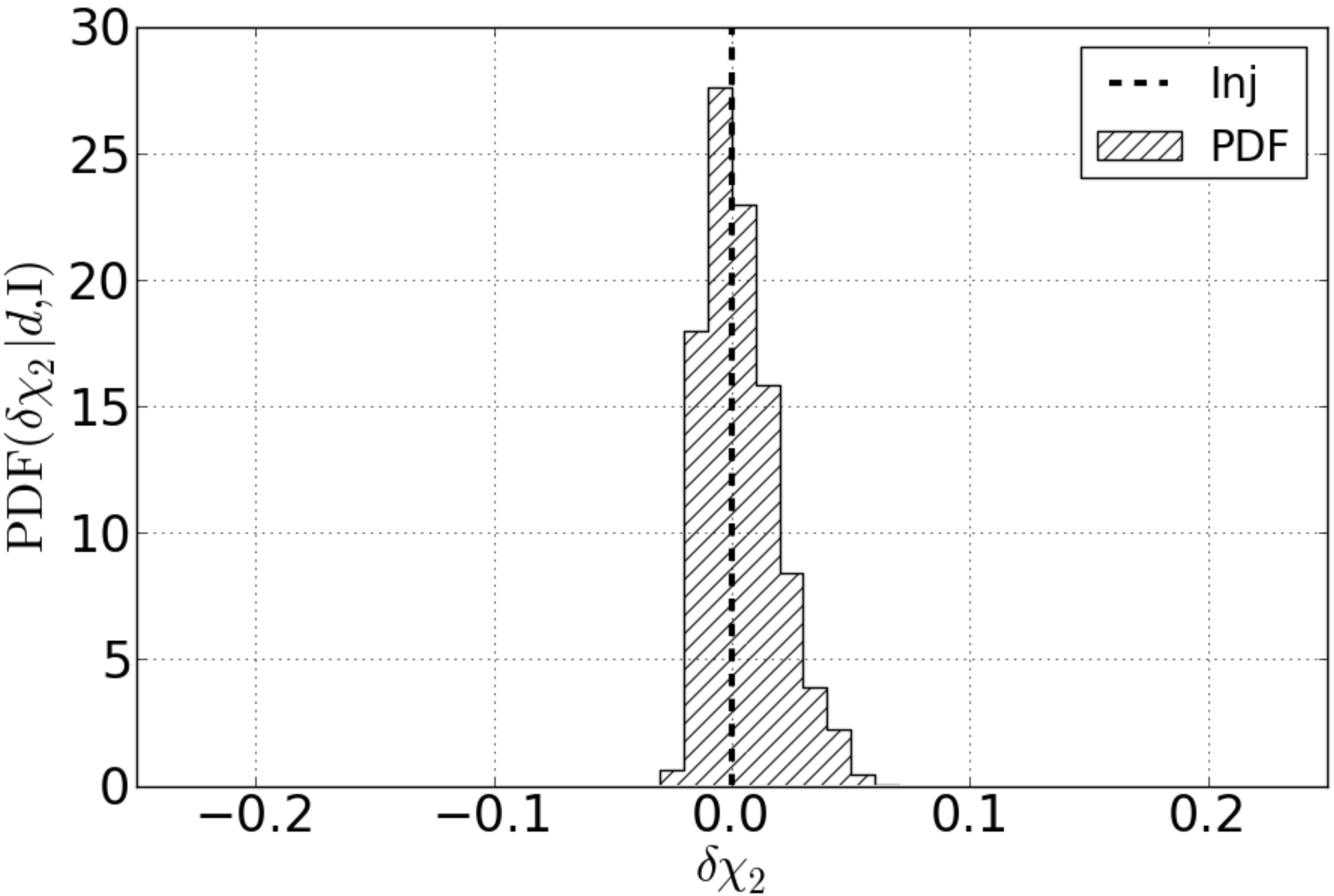
- GW probe the strong field dynamics of GR
- Developed a pipeline for the detection of deviations from GR:
  - detect "arbitrary" deviations from GR
  - constrain GR well beyond binary pulsar
- Our method does not pinpoint the nature of the deviation
  - complement with other approaches (e.g., ppE)
- Need to use more realistic waveforms:
  - spins
  - higher harmonics
  - ...
- For more details:
  - Li et al, **arXiv:1110.0530, arXiv:1111.5274**

Bonus Slides

Results – GR injections



Results – GR injections



An example

- Consider the case in which we want to test only 2 coefficients:

$$\mathcal{H}_{\mathrm{modGR}} = H_1 \vee H_2 \vee H_{12}$$

- the odds ratio is:

$$
\begin{aligned}
{}^{(2)}O^{\mathrm{modGR}}_{\mathrm{GR}} &\equiv \frac{P(H_1 \vee H_2 \vee H_{12}|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})} \\
&= \frac{P(H_1|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})} + \frac{P(H_2|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})} + \frac{P(H_{12}|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})} \\
&= \frac{P(H_1|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})}\, B^1_{\mathrm{GR}} + \frac{P(H_2|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})}\, B^2_{\mathrm{GR}} + \frac{P(H_{12}|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})}\, B^{12}_{\mathrm{GR}}
\end{aligned}
$$

An example

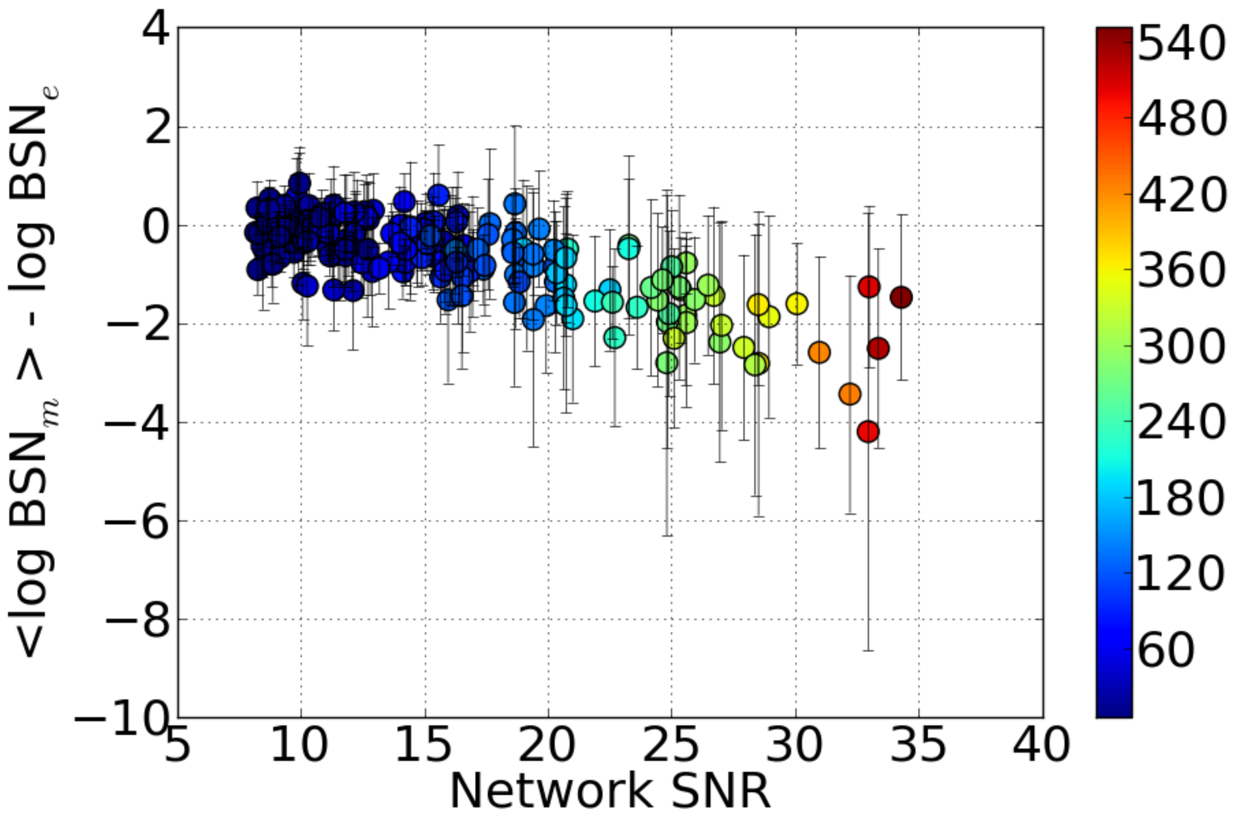
- where

$$B_{\mathrm{GR}}^{1} = \frac{P(d|H_1,\mathrm{I})}{P(d|\mathcal{H}_{\mathrm{GR}},\mathrm{I})}, \; B_{\mathrm{GR}}^{2} = \frac{P(d|H_2,\mathrm{I})}{P(d|\mathcal{H}_{\mathrm{GR}},\mathrm{I})}, \; B_{\mathrm{GR}}^{12} = \frac{P(d|H_{12},\mathrm{I})}{P(d|\mathcal{H}_{\mathrm{GR}},\mathrm{I})}$$

  are the Bayes' factors for each component hypothesis
- To test 2 coefficients we need 3 quantities
- This generalises to an arbitrary number of coefficients:
  - for N_T coefficients, we need $2^{N_T} - 1$ tests.

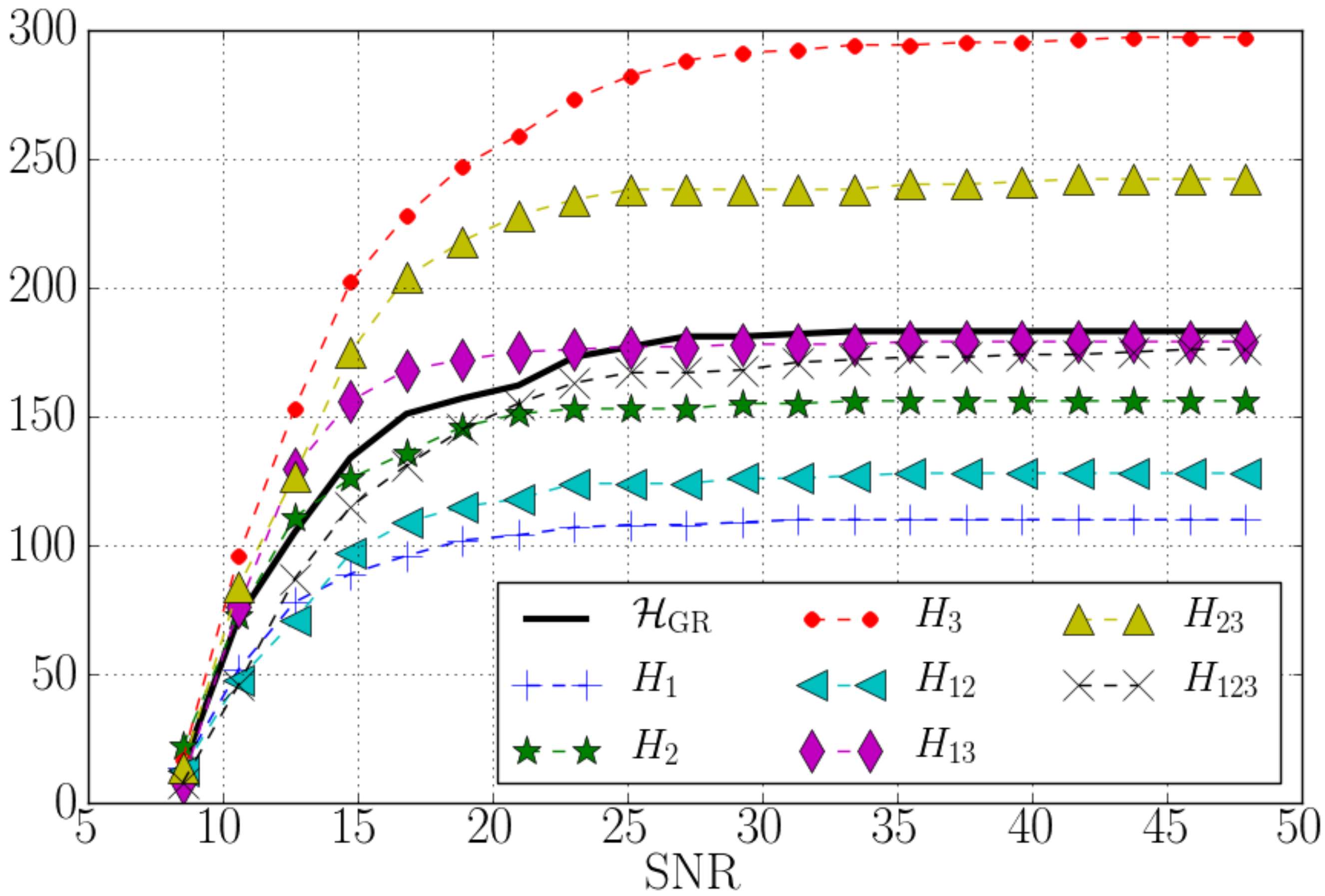
Effect of calibration errors

- Calibration errors do not affect the Bayes' factor significantly (Vitale et al, 2011)



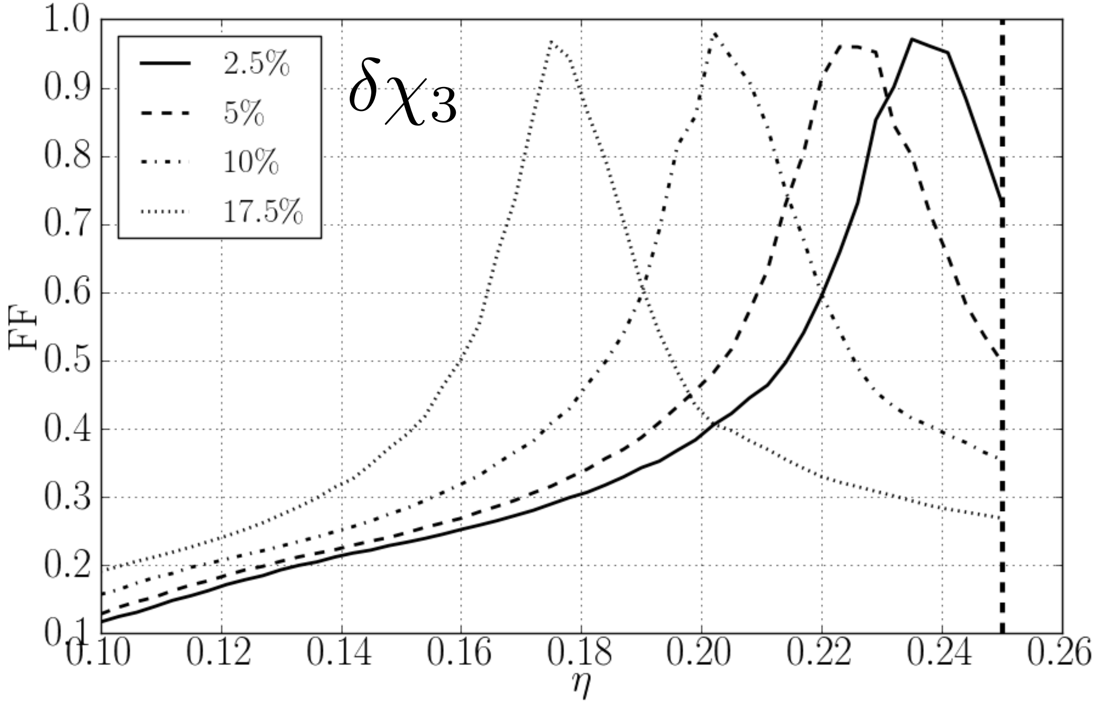
Detection with GR templates

- GR templates can still detect modGR signals



Detection with GR templates

- But a significant bias is introduced



- See also Del Pozzo et al (2011), Cornish et al (2011)