Extreme Mass Ratio Binaries

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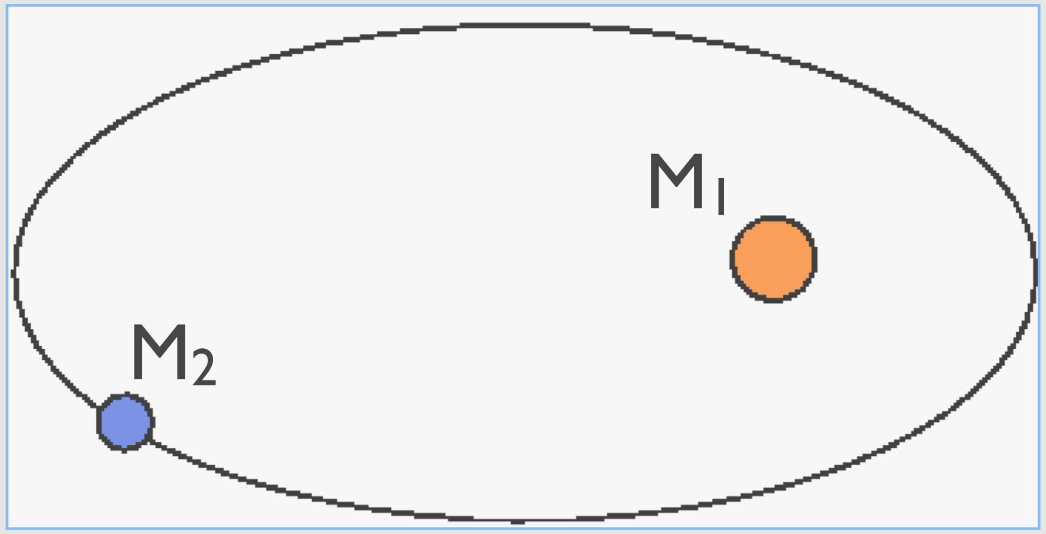
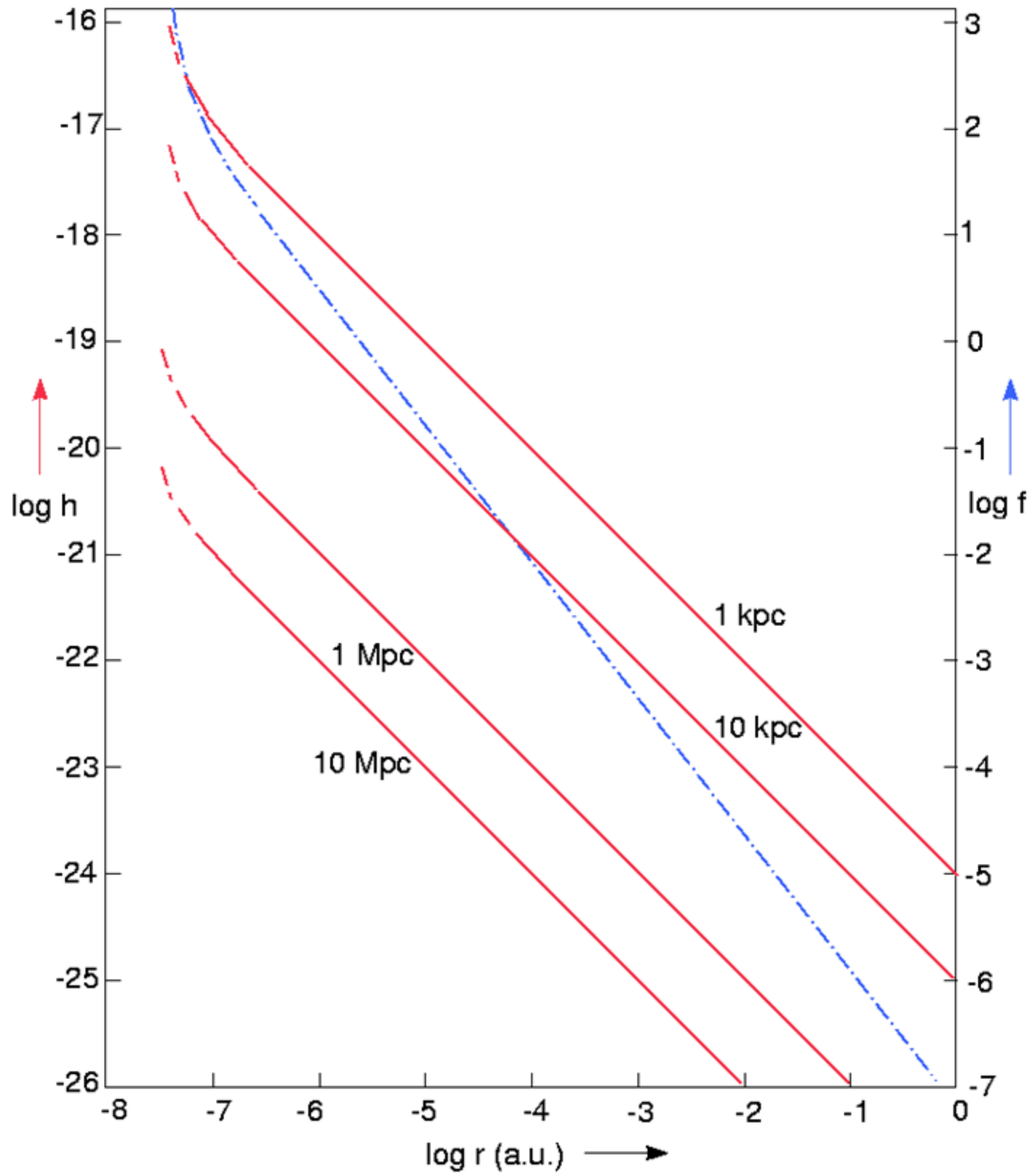
NIKHEF

Dutch Gravitational Wave meeting
19-01-2012

Newtonian binaries

$$W_{rad} = 0.43 \times 10^{14} \frac{\left(\frac{m_1}{M_\odot}\right)^2 \left(\frac{m_2}{M_\odot}\right)^2 \left(\frac{m_1+m_2}{2M_\odot}\right)}{\left(\frac{r}{1\text{A.U.}}\right)^5} \quad \text{J s}^{-1}$$

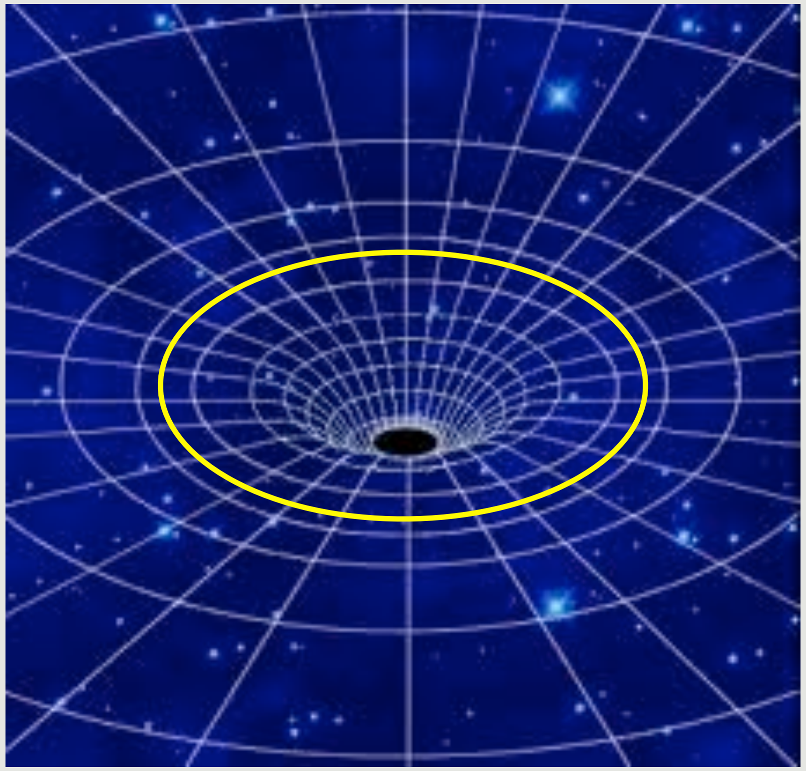


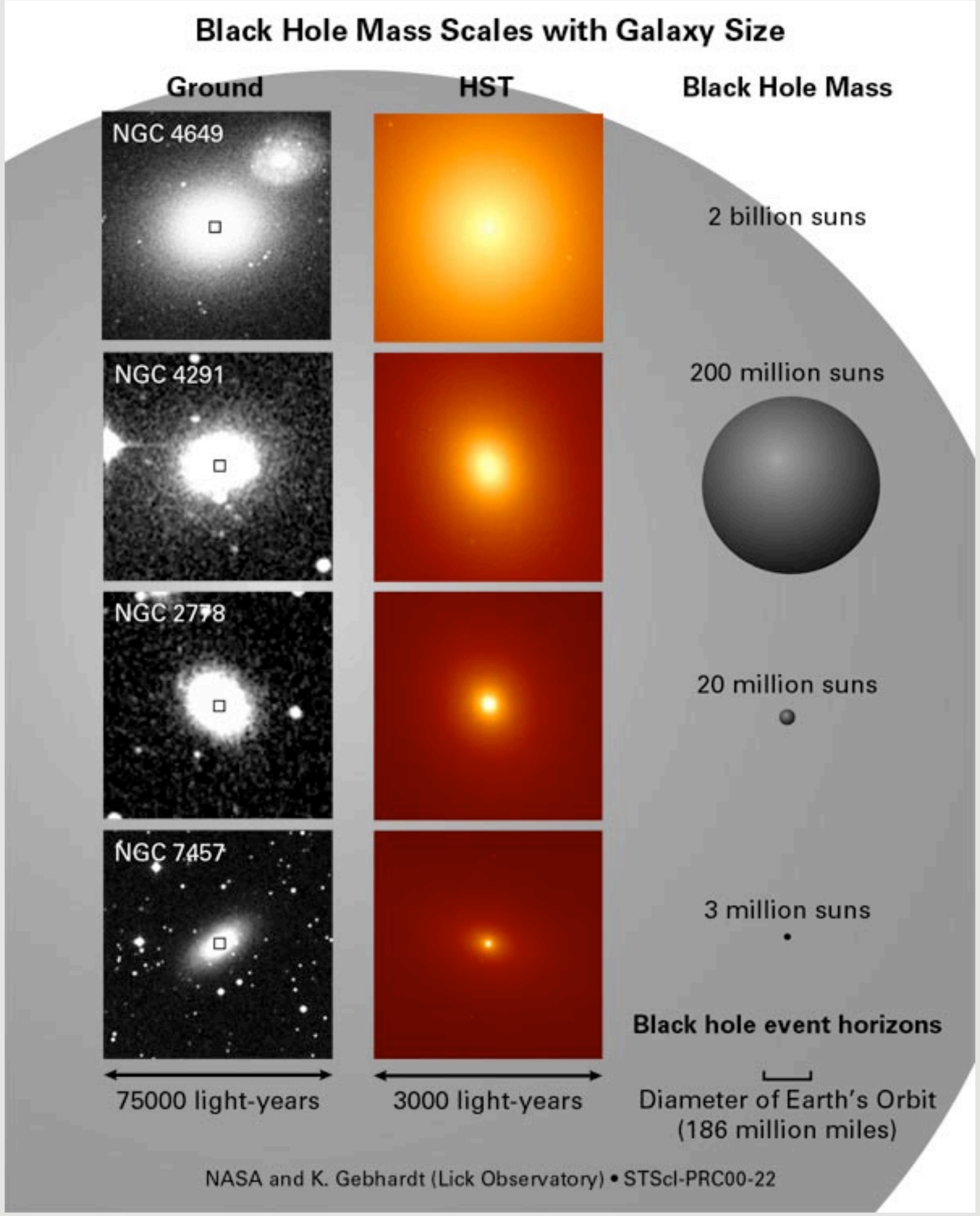


$$f = 0.6 \times 10^{-7} \frac{\sqrt{(M_1+M_2)/M_\odot}}{(r/[1\,\text{a.u.}])^{3/2}} \text{ Hz}$$

M >> m: extreme mass ratio binary

Schwarzschild geometry





Gravitational waves

Fluctuating space-time geometry:

$$\boxed{g_{\mu\nu} = \bar{g}_{\mu\nu} + 2\kappa h_{\mu\nu}} \qquad \kappa^2 = 8\pi G$$

geometry of background

Einstein equations:

$$\bar{D}^\lambda \tilde{h}_{\lambda\mu} = 0 \longrightarrow \bar{D}^2\,\tilde{h}_{\mu\nu} - 2\bar{R}_{\mu\kappa\nu}{}^{\lambda}\,\tilde{h}^{\kappa}_{\lambda} = -\kappa\, T_{\mu\nu}$$

Gravitational waves

distribution of mass and momentum

2 transverse polarization states

$$h_+ - ih_\times \;=\; \frac{1}{r}\sum_{lm}\left(\Psi^{lm}_{ZM}(r,t) - 2i\int_{-\infty}^{t}\Psi^{lm}_{RW}(r,t')dt'\right)$$

$$\times \;\; (angular \;\; functions).$$

Zerilli-Moncrief and Regge-Wheeler:

$$\left(\partial_t^2 - \partial_{r^*}^2 + \left(1 - \frac{2GM}{r}\right)V_A(r)\right)\psi_A = S_A$$

Power:

$$P = \frac{1}{32\pi}\sum_{lm}\frac{(l+2)!}{(l-2)!}\left(|\dot{\Psi}^{lm}_{ZM}|^2 + 4|\Psi^{lm}_{RW}|^2\right)$$

Loss of angular momentum:

$$\frac{dL}{dt} = \frac{i}{128\pi}\sum_{lm} m\frac{(l+2)!}{(l-2)!}\left(\dot{\Psi}^{lm}_{ZM}\Psi^{*lm}_{ZM} + 4\Psi^{*lm}_{RW}\int_{-\infty}^{t}\Psi^{*}_{RW}(t')dt'\right) + c.c$$

Sources: orbiting mass

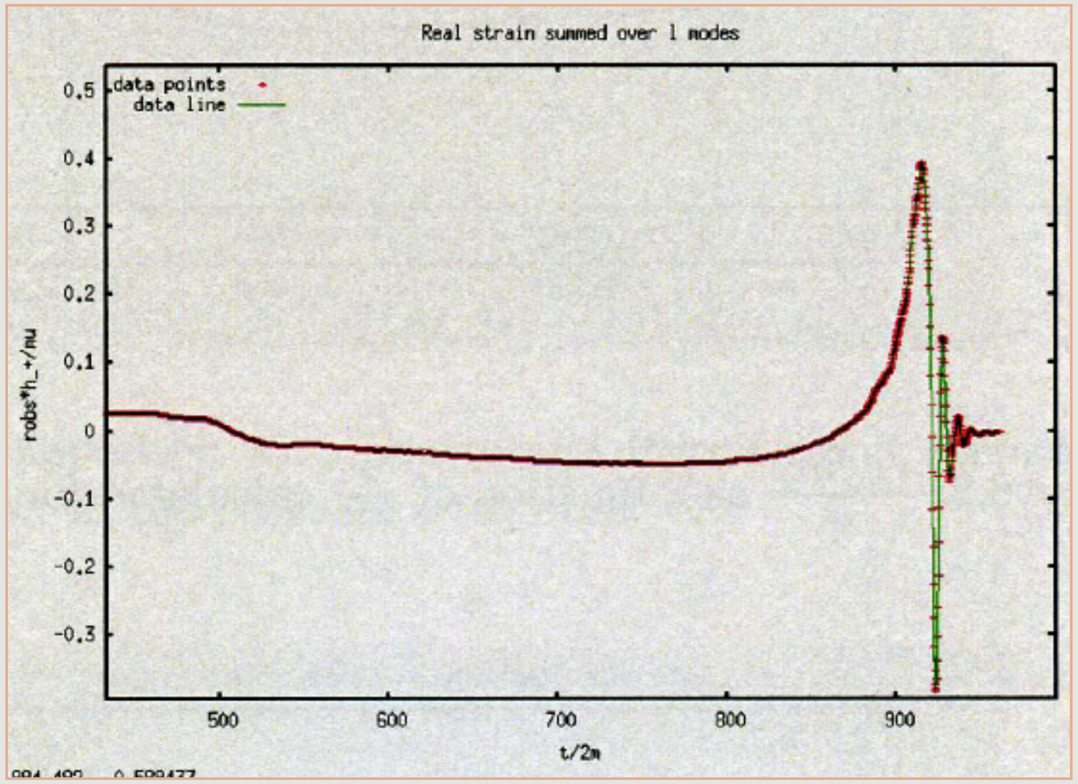
Geodesics (free fall)

$$\frac{D^2 x^\mu}{D\tau^2} \equiv \frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\lambda\nu}{}^{\mu} \frac{dx^\lambda}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

A. Exact solutions:

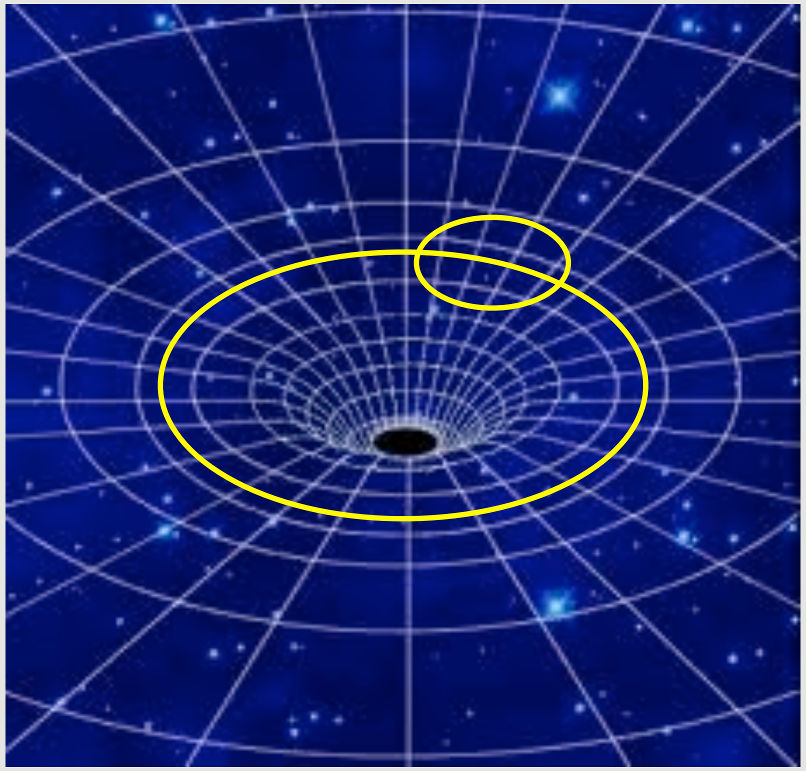
- straight plunge

neutron star falling into galactic black hole:
$f \sim 10^{-3}$ Hz
$h \sim 10^{-17}$



m/M = 0.5 x 10^{-5}

- circular orbits $t = \frac{\tau}{\sqrt{1 - \frac{3M}{R}}}, \qquad \varphi = \sqrt{\frac{M}{R^3}} \frac{\tau}{\sqrt{1 - \frac{3M}{R}}}$

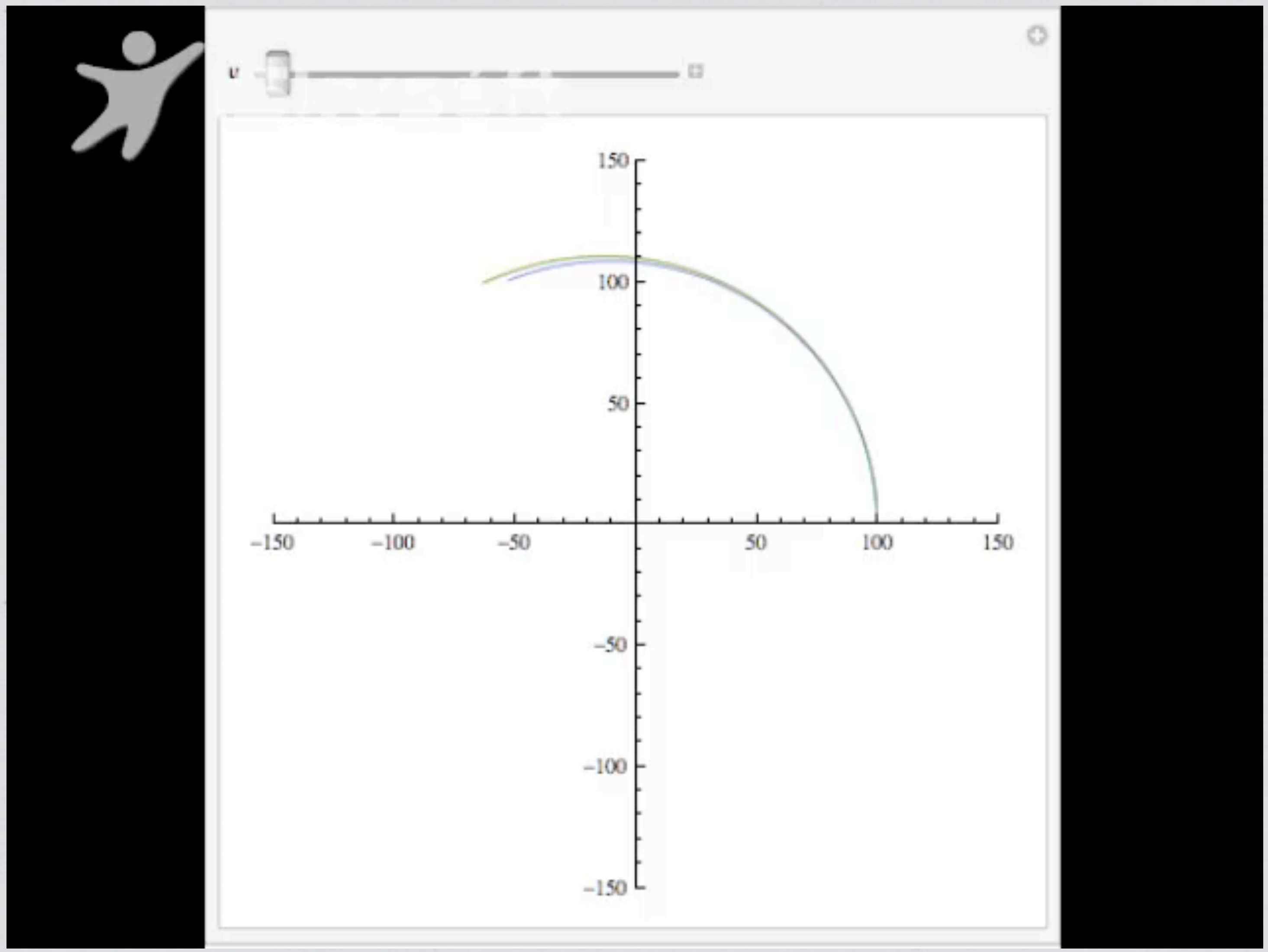


$$T^2 = \frac{4\pi^2}{M} R^3$$

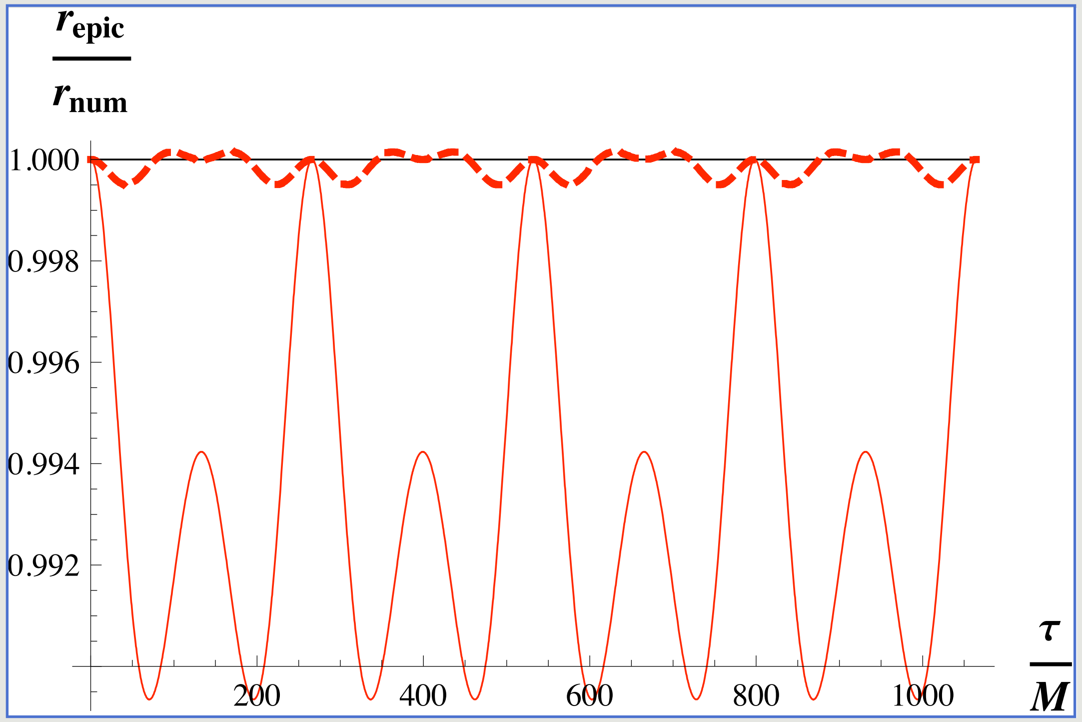
B. Non-circular orbits:
expand in eccentricity

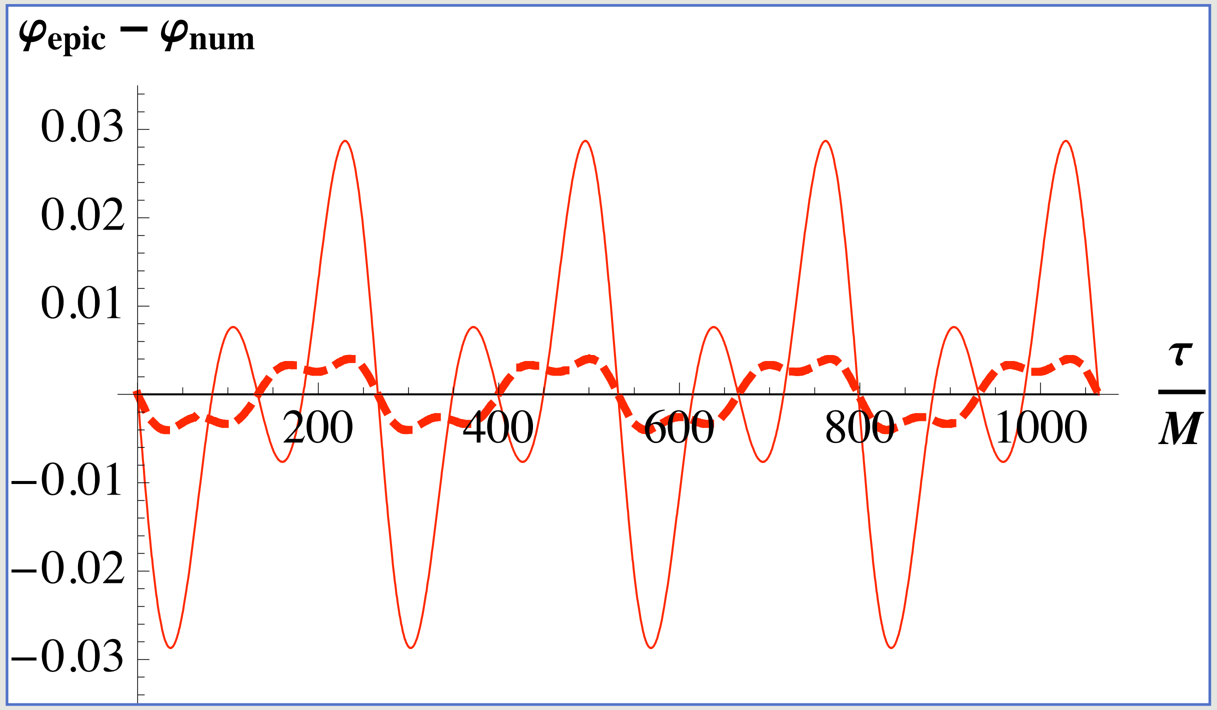
$$r = \frac{a}{1 + e \cos y}$$

$$\left(\frac{dy}{d\varphi}\right)^2 = 1 - \frac{2M}{a}(3 + e \cos y)$$


v
150
100
50
−150
−100
−50
50
100
150
−50
−100
−150

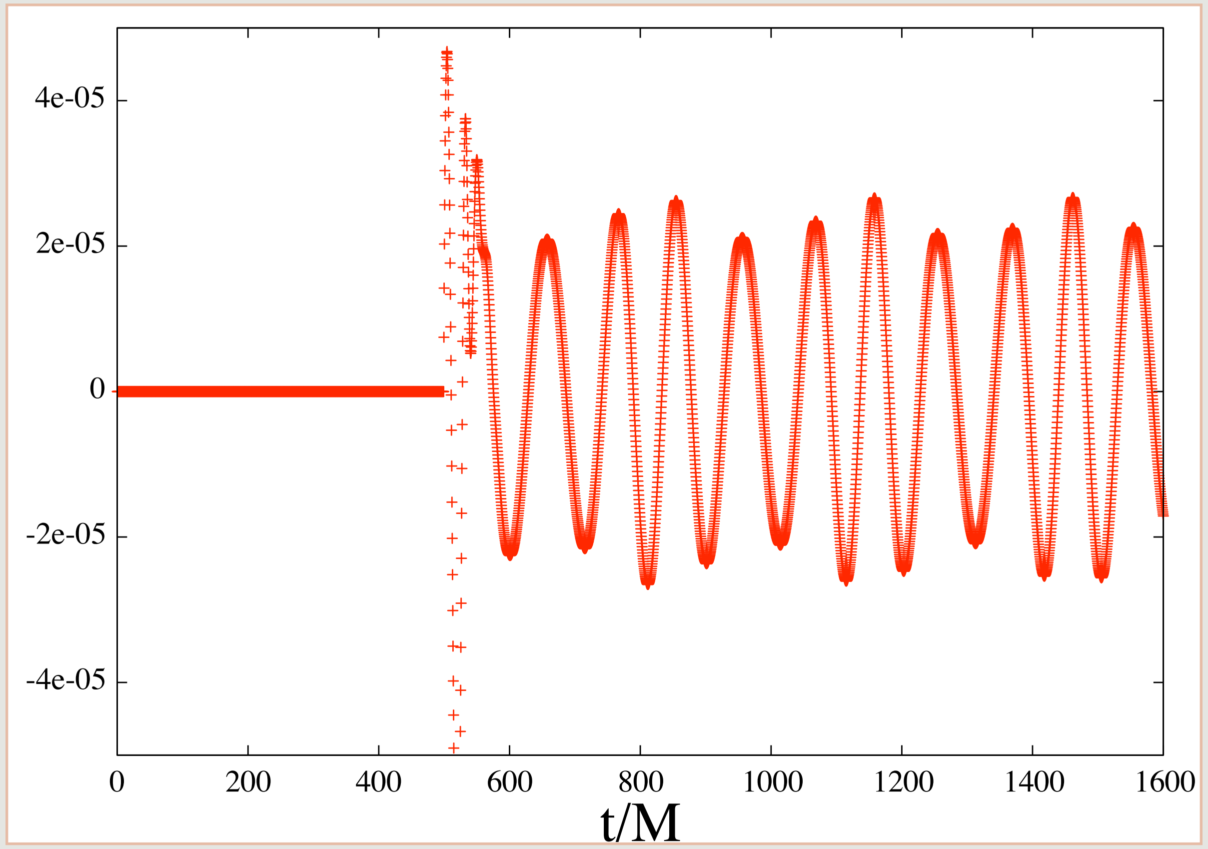
Accuracy of geodesic approximation



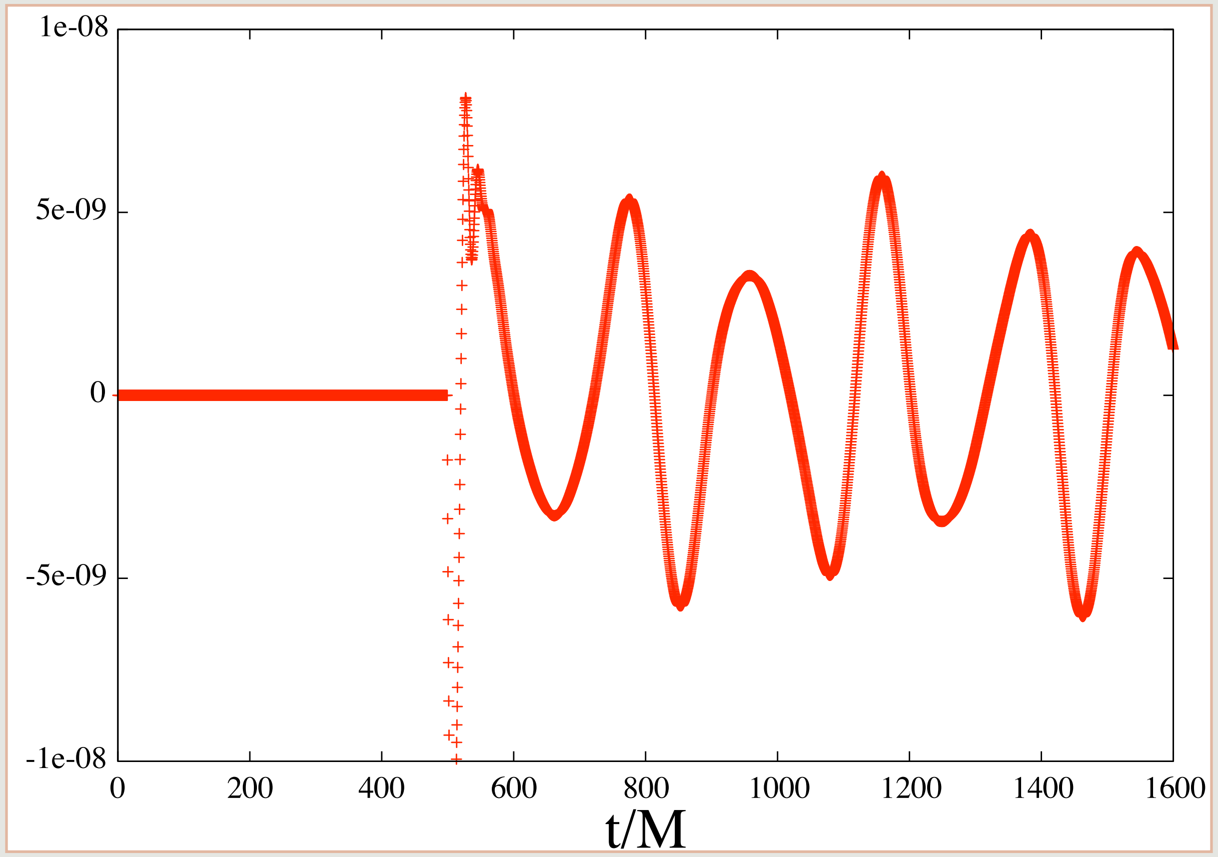


Wave forms

a = 10M, e = 0.1, m/M = 10^{-5}

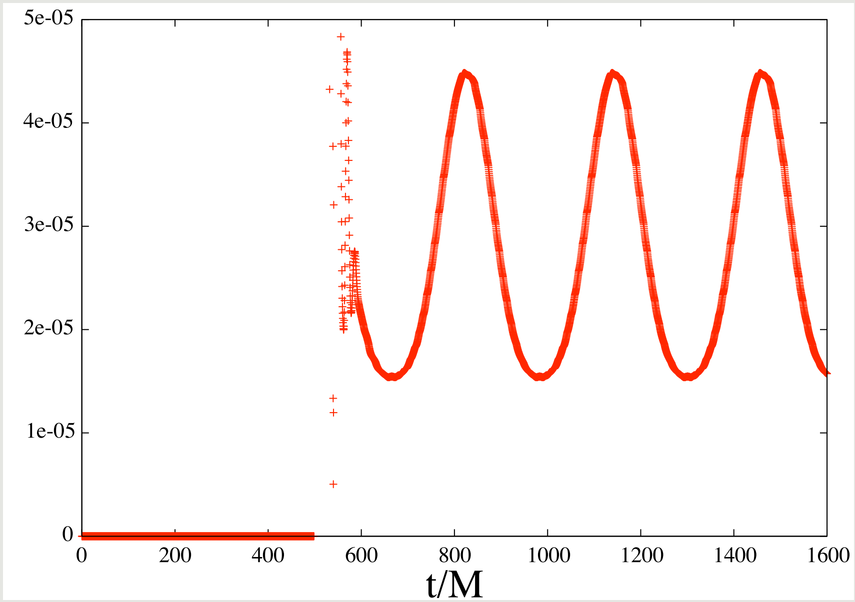


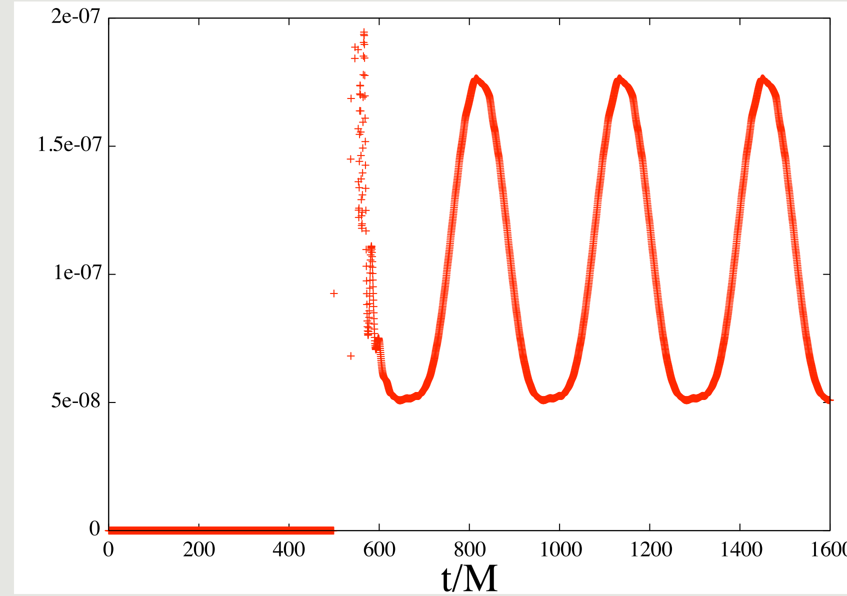
Re ZM



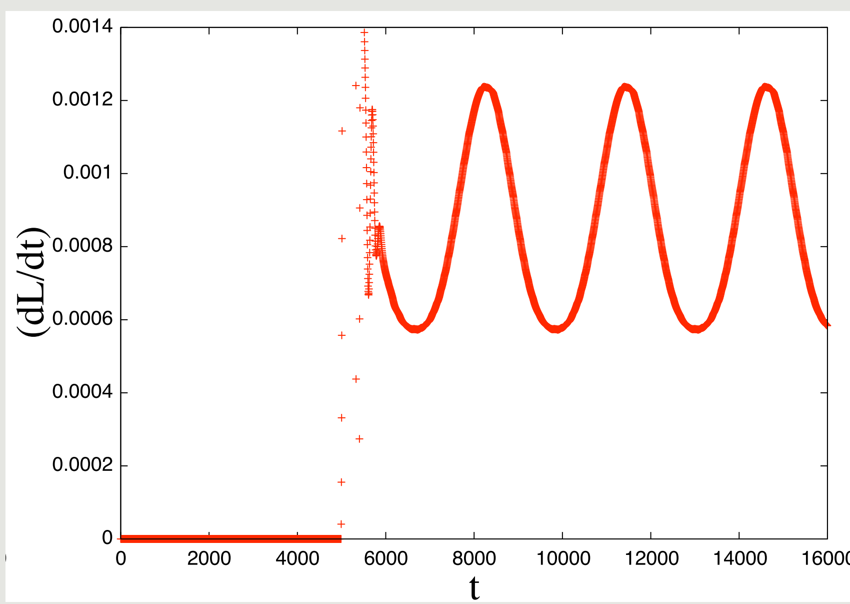
Re RW

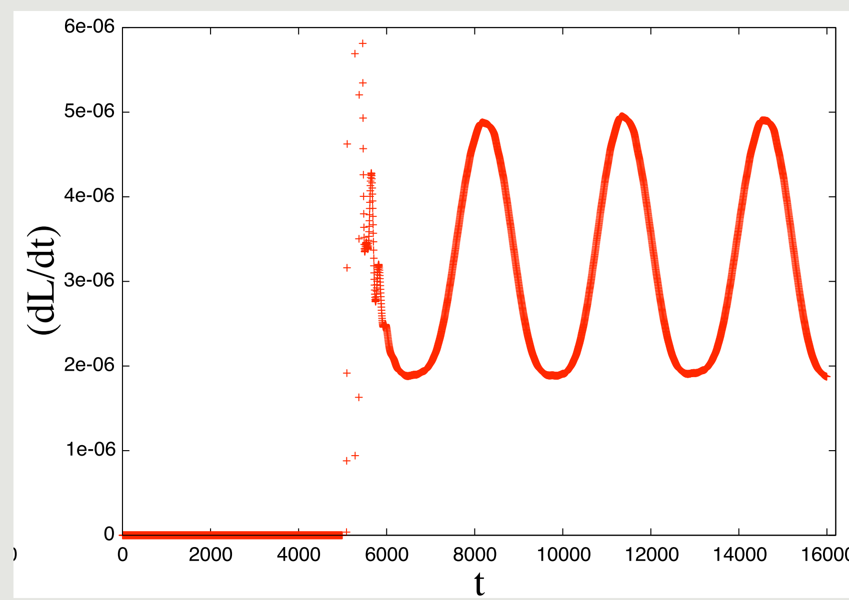
Power at r = 500 M





Loss of angular momentum

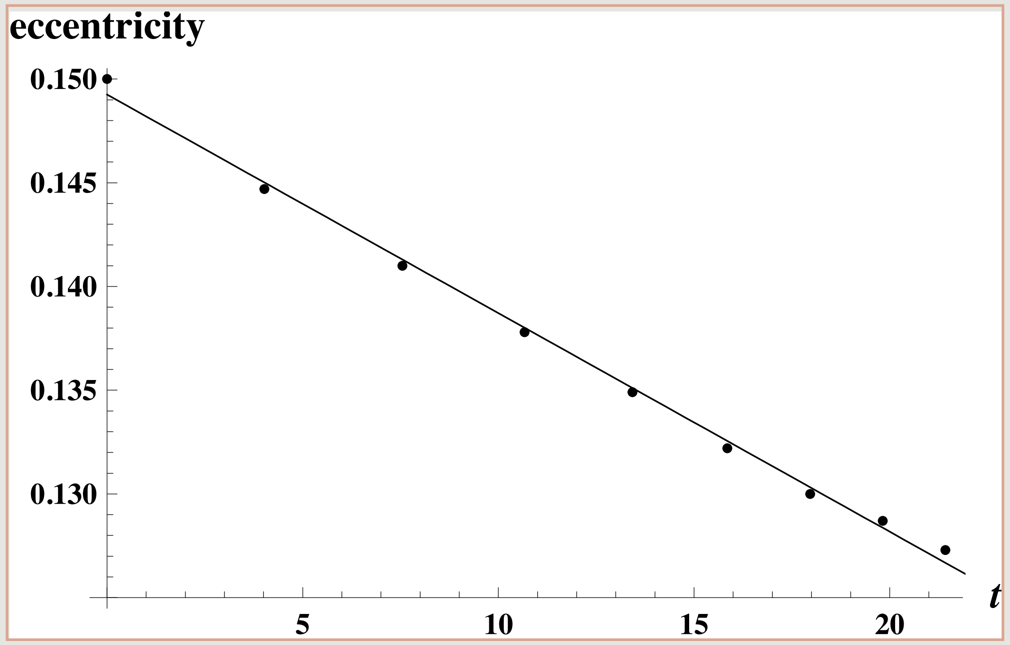


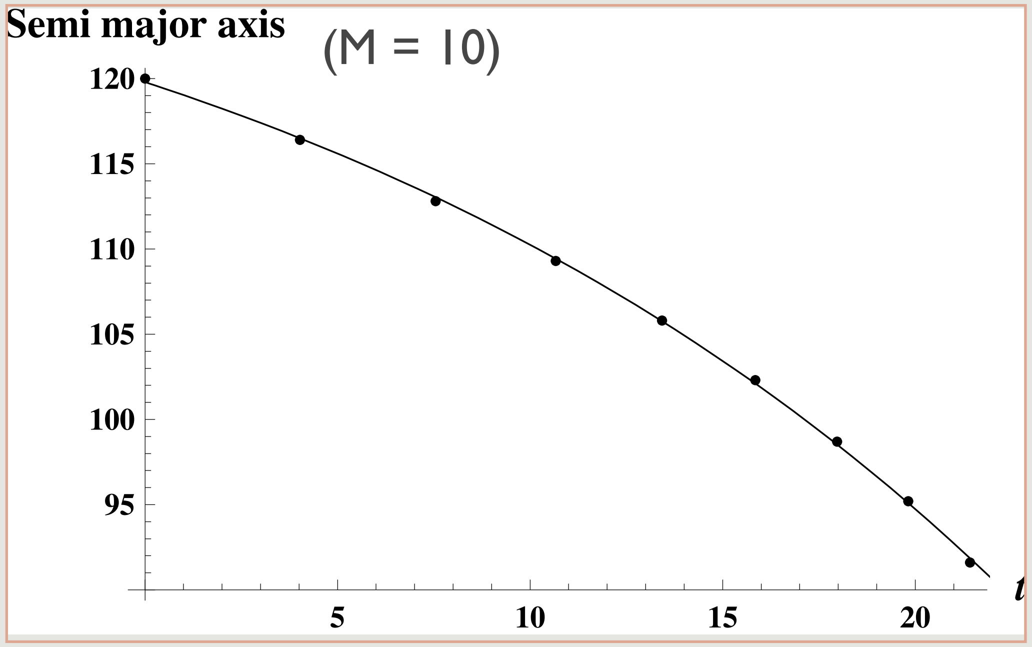


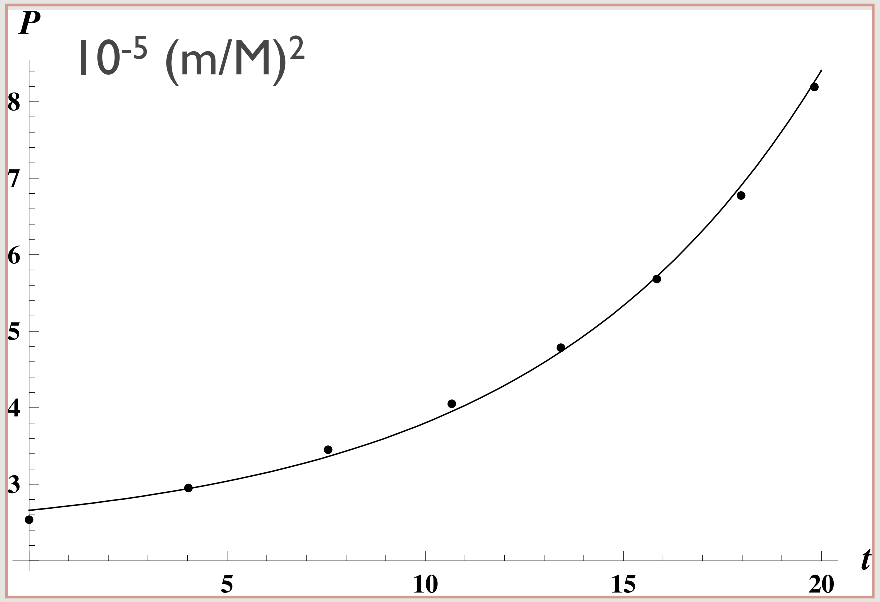
ZM

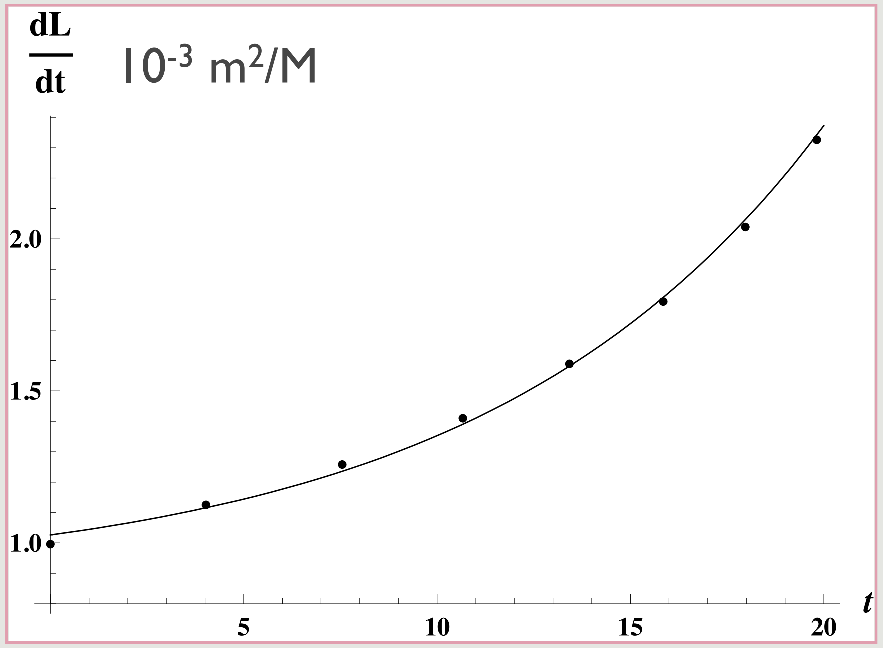
RW

Orbital evolution









initial values: $a = 12M$, $e = 0.15$