

Ultracompact binary stars

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feature
article

Double stars with tight, rapid orbits enable astronomers to study issues ranging from binary-star evolution to the internal structure of white dwarfs and neutron stars. In addition, they may emit directly observable gravitational waves.

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Stars most often don't shine alone: Many of the specks of light visible at night actually are binaries comprising two stars orbiting each other. The orbits in general are well described by Newton's law of gravity, just like the planetary orbits in our solar system; the exceptional cases can be explained by Einstein's theory of general relativity. Thus, the closer the stars are to each other, the shorter their orbital period as described by Kepler's third law. At the end of the 1960s, binary stars with periods of less than one hour were discovered, and astrophysicists recognized that the two stars are so close together that an ordinary star like the Sun could not fit in their orbits.^{1,2} The close proximity of the objects suggests that both components are stellar remnants—white dwarfs, helium stars, neutron stars, or even black holes—formed after stars exhaust their nuclear fuel. Box 1 offers a brief tutorial on stellar evolution and describes those remnants.

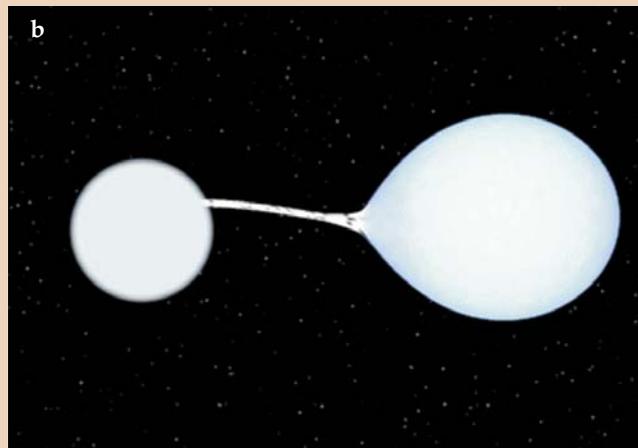
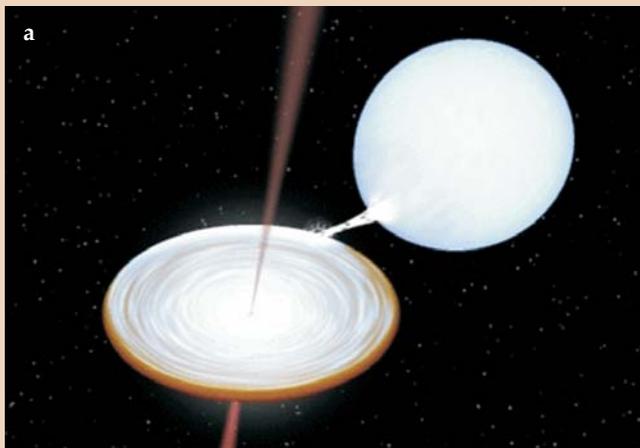
As the binary stars mutually orbit, they emit gravitational waves that remove energy from the binary system. As a result, the two stars get closer to each other. In time, they can get so close that gas from the outer layers of one of the stars falls onto its companion in a process called accretion. For the remainder of this article, I will mostly discuss those so-called interacting ultracompact binaries. By now about 30

such objects are known. They naturally divide in two classes, based on their observed characteristics. In ultracompact x-ray binaries, gas falls onto a neutron star whose potential well is so deep that the gas heats up to millions of kelvin and produces abundant x rays. The second class comprises the AM CVn stars, so named because the first of them was the variable star AM CVn discovered in the constellation Canes Venatici (hunting dogs). In AM CVn systems gas falls onto a white dwarf, which has a much shallower potential well than a neutron star. The gas is heated to only about 100 000 K, and most radiation is emitted at optical and UV wavelengths in the range of 100–600 nm. Most studies of AM CVn systems have been done in that wavelength range, in particular by Joseph Patterson (Columbia University), Jan-Erik Solheim (University of Tromsø, Norway), Brian Warner (University of Cape Town, South Africa), Thomas Marsh (University of Warwick, UK), and their collaborators. Figure 1 shows an artist's impression of both types of ultracompact binary.

Formation of ultracompact binaries

The components of ultracompact binaries are stars that have evolved beyond hydrogen-core burning. Thus, to understand how ultracompact binaries arise, one has to follow the evo-

Figure 1. Impressions of ultracompact binary stars. (a) In this ultracompact x-ray binary, a neutron star accretes matter from its companion compact star and forms a bright accretion disk. Part of the accreted material may be expelled as a collimated jet. (b) A white dwarf in an AM CVn star accretes material from its companion. This illustration shows the direct impact phase, in which the accreted material collides directly with the dwarf's surface rather than wrapping around to form a disk. (Created with the BinSim code developed by Robert Hynes.)



Box 1. The life of a star

Stars are gaseous objects that can be understood in terms of relatively simple physical processes: force balance, energy generation, and energy transport. The basic balance in stars is between gravity, which tugs inward, and pressure, which pushes outward. At the beginning of a star's life, the forces are not in balance: A clump of interstellar gas contracts under the sway of gravity. As the cloud becomes smaller, the pressure and temperature in the center increase, and eventually become large enough to trigger the nuclear fusion of hydrogen into helium. That reaction generates a large amount of heat, which further increases the pressure until the contraction is halted. As long as the star burns nuclear fuel, gravity and pressure are balanced. The Sun currently is in this balanced phase.

The nuclear burning, however, cannot go on forever. In time, no hydrogen remains in the core where the nuclear burning takes place. Once the hydrogen is gone, the pressure in the core cannot be sustained; gravity wins the battle of forces, and so the core contracts. Meanwhile, the outer layers of the star expand, typically by an impressive factor of 10 to 100, and the star becomes a red giant.

Although what happens in the stellar core after hydrogen burning is complicated, the quick story is that one of two processes occur.¹⁶ Either the pressure and temperature increase in such a way that the nuclear fusion of heavier elements can generate the pressure necessary to balance gravity, or the pressure and density increase in such a way that the gas becomes degenerate—the atoms come so close together that the electrons lose track of which atom they belong to. In that case, interactions between the electrons produce a pressure that is independent of the temperature. The star has found a way to balance gravity forever with no energy input needed. The formation of a degenerate gas occurs for stars less than about eight times the solar mass M_{\odot} . After the star's outer layers are lost into space due to a stellar wind, all that remains is a compact degenerate core, about the size of Earth but typically 300 000 times more massive, forever cooling and visible as a small, initially hot star called a white dwarf.

In the case of continued nuclear fusion, the core will be burnt into heavier and heavier elements until it is made of iron. But neither fusion nor fission can produce energy from iron, so the iron core has no way of stopping gravity—it collapses. The extreme pressure forces the electrons and protons to combine into a degenerate gas of neutrons that, if the core is not too heavy, generates enough additional pressure to halt the contraction. The resulting neutron star has a radius of about 10 km and a mass of 3×10^{30} kg. If the core is massive enough, even the pressure of the degenerate neutrons cannot stop gravity, and the core collapses into a black hole. Neutron stars are formed from the cores of stars with masses between about 8 and $25 M_{\odot}$; black holes, as far as we know, are created from stars with masses greater than about $25 M_{\odot}$. During the formation of either a neutron star or a black hole, the outer layers of the star are blown into space in a violent explosion called a supernova.

lution of a binary star from the beginning of its life all the way to when two stellar remnants are formed. The formation of stars—and of binaries in particular—is not well understood. I'll skip over that part and begin my discussion of binary evolution with two stars that are in the process of fusing hydrogen. Hydrogen burning lasts for a long time, and the lower the mass of the star, the longer it lasts: Even though massive stars have more hydrogen fuel, they burn it much

more rapidly than do low-mass stars. For a star with a mass of $10 M_{\odot}$, where M_{\odot} is the mass of the Sun, the hydrogen-burning phase lasts several tens of millions of years; for the Sun itself, burning lasts for about 10 billion years.

As the more massive star in a binary evolves into a red giant, as described in box 1, its outer layers invariably expand. If the less massive companion is close enough, it can interact with those outer layers, and that interaction can alter the evolution of both giant and companion. The giant might completely lose its outer layers and become a bare core—a shortcut on the path to becoming a white dwarf or a helium core in which further nuclear burning takes place. The outer layers might end up accreted onto the companion, or they might be lost from the binary system. In either case, the redistribution of mass and the possible loss of angular momentum change the separation between the stars.

In the 1970s, Bohdan Paczyński and Jeremiah Ostriker (both now at Princeton University) independently realized that some observed binaries had such short periods that they must have lost an enormous amount of angular momentum in the course of their evolution. Moreover, that evolution would have included a phase in which one of the stars was an extremely large giant. Paczyński proposed an explanation for how the angular momentum was lost. Somehow, he argued, the companion of the giant found itself inside the giant's outer layers, or envelope. During that common-envelope phase, friction reduced the velocity of the companion, which implies a decrease in the orbital separation and a transfer of angular momentum from the orbit into the envelope of the giant. Along with angular momentum, orbital energy was deposited in the envelope, whose matter was then unbound from the giant's core (see figure 2).

Ultracompact binaries have such short orbital periods that they must have gone through at least one common-envelope phase. Indeed, both stars in ultracompact binaries are remnants; the initially lower-mass star has also evolved and lost its outer layers. So ultracompact binaries have gone through two phases of interaction, likely two common-envelope phases.

The picture I have painted of the common-envelope phase is simplistic, and modeling the phase in detail has proven to be difficult, despite good efforts by Ronald Taam (Northwestern University) and others. Astrophysicists' knowledge about the formation of ultracompact binaries is severely limited by their lack of understanding of the common-envelope phase. That ignorance is expressed by a so-called efficiency parameter, which determines how easily the envelope is unbound. In my opinion, it has become ever more clear that the current understanding is very incomplete, and that many binaries expected to enter a common-envelope phase either avoid it altogether or somehow avoid losing most of their angular momentum and stay rather widely separated. But that opinion is controversial.

Why they are so interesting

The close proximity of the two stars in an ultracompact binary and the high frequency of their orbit are intrinsically fascinating to many of us in astrophysics. Intrinsic interest aside, ultracompact binaries have received attention for a number of other reasons. To name a few: AM CVn stars give insights about binary evolution, ultracompact x-ray binaries give astrophysicists a peek deep inside stars, and both types of binary are sources of gravitational waves. Those considerations have led to a surge of scientific activity over the past 10 years.³ Still, the community remains small enough that I think I know personally everyone working on ultracompact binaries. Many of those

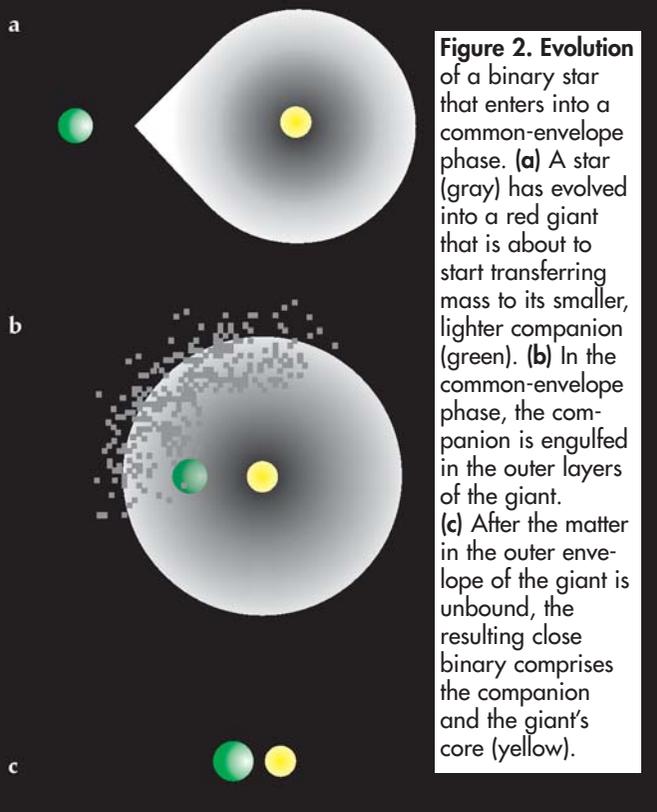


Figure 2. Evolution of a binary star that enters into a common-envelope phase. (a) A star (gray) has evolved into a red giant that is about to start transferring mass to its smaller, lighter companion (green). (b) In the common-envelope phase, the companion is engulfed in the outer layers of the giant. (c) After the matter in the outer envelope of the giant is unbound, the resulting close binary comprises the companion and the giant's core (yellow).

One of the scenarios for AM CVn formation starts with a pair of white dwarfs that are formed in a close binary after two common-envelope phases. The binary emits gravitational-wave radiation and thus loses energy and angular momentum. The orbital period gets ever shorter as the binary continues to emit gravitational waves. If the binary initially had a period of about 10 hours or less, then within the 10 billion years of the Milky Way's existence, the binary's period would decrease to just a few minutes, and one of the white dwarfs would start to lose mass to its companion.¹⁴ That mass transfer leads either to a complete merger of the binary into one object or to the formation of an AM CVn system that subsequently evolves back to having a longer orbital period.

The two other formation scenarios involve initially non-degenerate helium cores as the mass donors.⁵ Because those cores are larger than white dwarfs, they cannot get as close to their partners as white dwarfs can. Thus, AM CVn stars formed according to these other scenarios always have periods longer than 10 minutes. One way, then, to establish the likelihood of the double-white-dwarf scenario is to find the relative numbers of sources with periods greater and less than 10 minutes. Two surveys capable of finding AM CVn stars with periods less than 30 minutes are under way or will start soon: the Rapid Time Survey (RATS), led by Gavin Ramsay (University College London) and Pasi Hakala (University of Helsinki, Finland), and OmegaWhite, led by Paul Groot (Radboud University Nijmegen, the Netherlands).⁶

Another way to distinguish the scenarios is to measure the mass of the mass-losing star at a fixed period.^{5,7} A relatively large mass would indicate that the system has evolved from a non-degenerate helium donor. Recent work suggests just such an evolution for the binary star AM CVn.⁸

A window into a star's interior

Of the few known ultracompact x-ray binaries, many reside in globular clusters—dense assemblies, typically of a million stars or so, that formed around the same time as the Milky Way. In those dense clusters, stars have a high probability of colliding, or better, interacting with other stars. Those interactions probably change how close binaries form.^{9,10} Stellar interactions are extremely unlikely for nonclustered stars in the Milky Way simply because the density is low. For non-clustered systems, the formation of ultracompact x-ray binaries is probably similar to the formation of AM CVn stars.¹¹

Through work by Deepto Chakrabarty and his group at MIT, and through independent studies by a group of which I was a part, it has become clear that, in contrast to the mass-

colleagues have contributed to the science I will discuss here, but in this brief survey I cannot credit all of them in detail.

AM CVn stars are the largest group of ultracompact binaries. Astrophysicists have identified 17 AM CVn systems and can now start to investigate the statistics of that population. One reason for such investigations is to probe the three distinctly different scenarios that have been proposed for the formation of AM CVn stars. But determining the number and statistics of AM CVn systems will also teach something about binary evolution in general and about the common-envelope phase in particular, because the number of AM CVn systems in our galaxy depends heavily on the efficiency of the common-envelope phase.

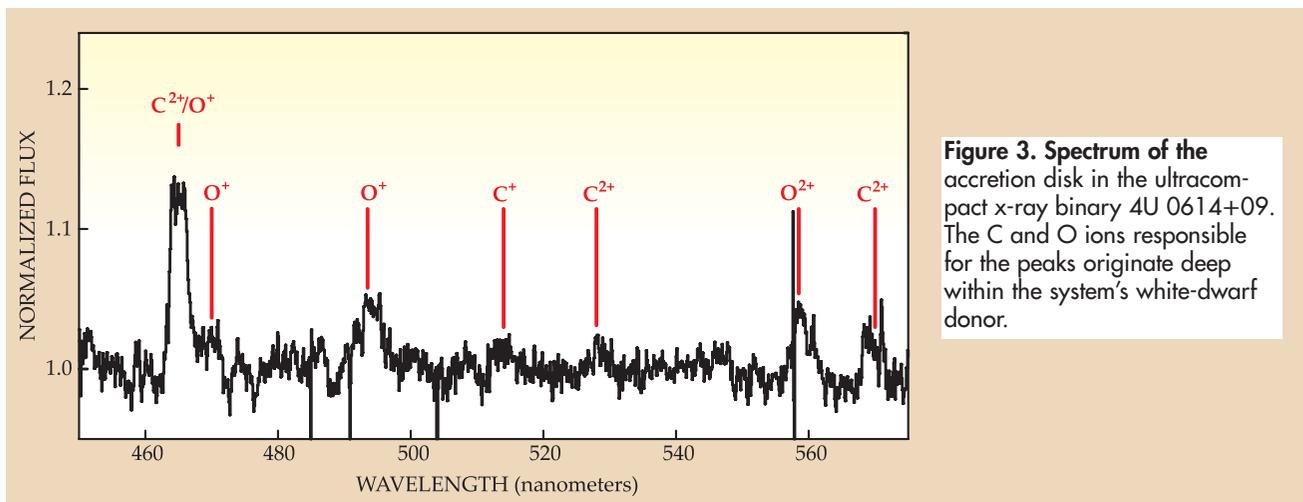


Figure 3. Spectrum of the accretion disk in the ultracompact x-ray binary 4U 0614+09. The C and O ions responsible for the peaks originate deep within the system's white-dwarf donor.

Box 2. Gravitational waves and their detection

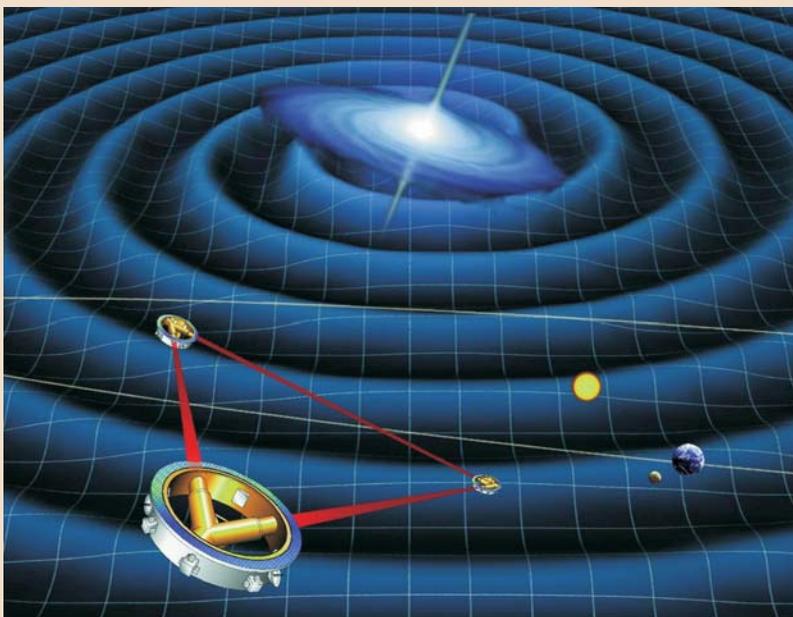
Einstein's theory of general relativity predicts that accelerated masses, like accelerated charges, emit radiation. However, the radiation produced by masses is not electromagnetic; rather, it takes the form of gravitational waves in spacetime. That is, when a gravitational wave passes, spacetime periodically expands and contracts. The undulation of spacetime is difficult to measure, because measuring rulers expand and contract as well. Indeed, gravitational waves have never been detected directly. Physicists, though, are convinced that gravitational waves exist because they have been detected indirectly via their effects on the orbital motion of binary pulsars.

A number of experiments designed to directly detect gravitational waves are under development.¹⁷ These include LIGO (Laser Interferometer Gravitational-Wave Observatory), VIRGO, GEO600, and *LISA* (Laser Interferometer Space Antenna). The relevant experiment for ultracompact binaries is *LISA*.

LISA, run jointly by the European Space Agency and NASA, is a constellation of three spacecraft flying in a triangular formation that has sides about 5 million kilometers long.¹⁸ Laser signals will be exchanged between the satellites, and any passing gravitational wave will cause a very small phase offset that can be detected by the interferometer. The artist's impression above, courtesy of NASA's Jet Propulsion Laboratory at Caltech, shows the satellite, with Earth and the Sun establishing the scale. The gravitational waves depicted come from the merger of two super-massive black holes.

In contrast to ground-based detectors, *LISA* will be sensitive at frequencies below 1 Hz and will have its best sensitivity between 1 and 10 mHz. That optimal sensitivity range corresponds to the minutes-long orbital periods of ultracompact binaries.

In addition to observing gravitational waves from ultracompact binaries in our galaxy, *LISA* will be able to detect certain extreme events throughout the universe: the merger of super-massive black holes and the spiraling of compact stars into super-massive black holes. Presently *LISA* is in the formulation phase and is planned for launch around 2015; unfortunately the fiscal year 2007 NASA budget is worrisome for fundamental science in general and *LISA* in particular.



losing stars in AM CVn systems, which are all helium rich, the mass-losing stars in ultracompact x-ray binaries are sometimes helium rich, but sometimes they are composed of carbon and oxygen. When one sees signatures of C and O, as in the spectrum of figure 3, one is looking at the cores of stars that have evolved through helium burning. Indeed, because much of the core material has already transferred to the accreting neutron star, spectroscopic observations of C and O provide a window into the deep interior of stellar cores. In particular, they allow astrophysicists to study the process of helium burning because the C/O ratio depends sensitively on the details of the burning.

Ultracompact x-ray binaries are interesting for a number of other reasons. In particular they allow astrophysicists to study neutron stars outside the hydrogen-rich x-ray-binary environment in which those stars are usually found and to probe fundamental physics associated with neutron-star structure.¹² Observations of x-ray pulsations from many of those ultracompact systems have enabled astrophysicists to determine the rotation of the neutron star; in particular Craig Markwardt (University of Maryland, College Park, and NASA's Goddard Space Flight Center) has led a productive research team that has measured such rotations using the *Rossi X-ray Timing Explorer* (*RXTE*). As had been expected on theoretical grounds, the neutron stars are spinning rapidly—several hundred times per second—due to the angular momentum gained from infalling matter. The measurements give credence to the long-standing hypothesis that rapidly

spinning neutron stars observed as millisecond radio pulsars are descendants of accreting neutron stars in binary systems. However, scientists have yet to establish the exact role of ultracompact binaries in the formation of those pulsars.

I expect more ultracompact x-ray binaries will be found through continued monitoring of the sky with *RXTE* and other satellites that will search for transient x-ray sources. In addition, Peter Jonker (Netherlands Institute for Space Research) is currently developing dedicated x-ray and optical surveys to search for ultracompact x-ray binaries.

Ultracompact binaries will be an important source of gravitational waves for the *Laser Interferometer Space Antenna*, a gravitational-wave detector scheduled for launch around 2015 (see box 2). *LISA* will be able to detect so many ultracompact binaries, in particular the double-white-dwarf precursors of AM CVn systems, that binaries will form a noise background in the detector. Still, the space antenna will be able to distinguish individual ultracompact binaries; theoretical work shown in figure 4 indicates that some ten thousand new binaries may be detected. Those include a number of the known AM CVn stars that are guaranteed to be detected and can thus be used to test the instrument.

LISA observations will provide astrophysicists with details of the orbital periods and mass distributions of ultracompact binaries and thus will enable critical tests of models. Potentially, *LISA* will completely change how scientists view the formation and evolution of ultracompact binaries. In addition, by detecting thousands of objects spread

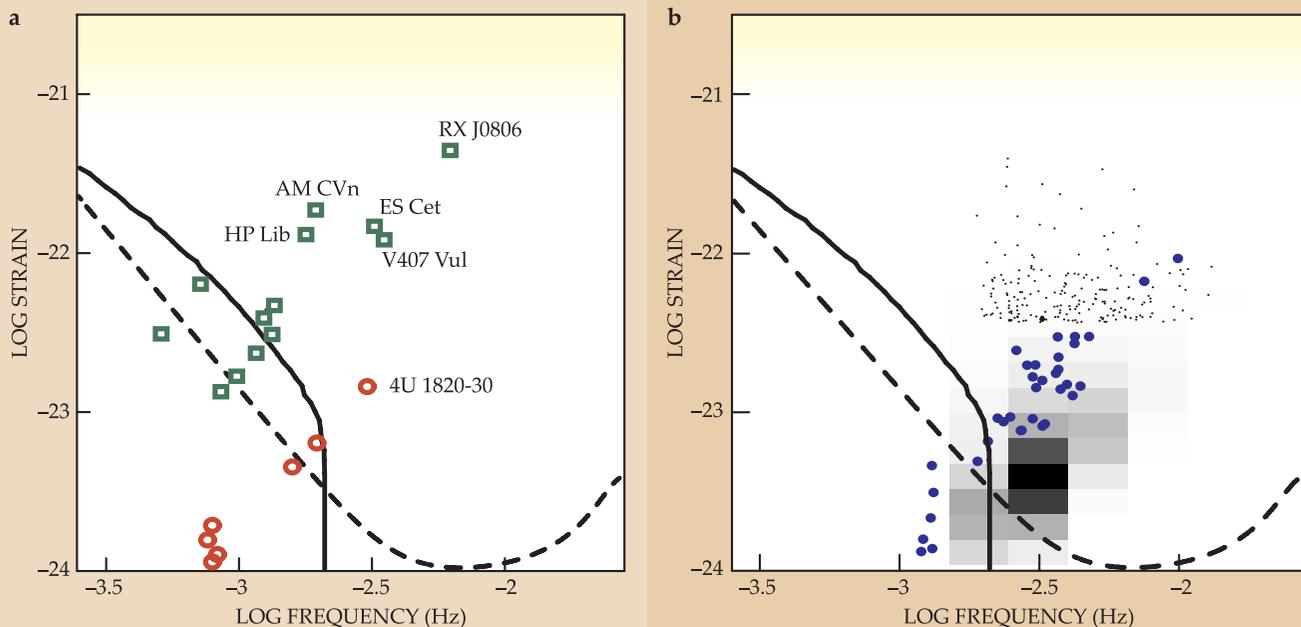


Figure 4. Gravitational-wave signals in *LISA*, the *Laser Interferometer Space Antenna*. (a) The expected strain (relative change in length of *LISA*'s interferometric arm) as a function of frequency for many of the known ultracompact AM CVn (green boxes) and x-ray (red circles) binaries. (b) Signals expected for the entire galaxy, based on theoretical calculations. The blue dots indicate 35 ultracompact x-ray binaries. The more than 10 000 AM CVn systems are indicated by gray-scale rectangles, with the darker shades indicating a greater density of points; the 200 highest-strain systems are shown explicitly. In both panels, the dashed line shows *LISA*'s sensitivity, and the solid line gives the background noise due to unresolved double white dwarfs.

throughout the Milky Way galaxy, it will provide a unique tool for studying galactic structure. The challenge will be to complement *LISA*'s detection of thousands of binaries with conventional astronomical observations using optical and x-ray instruments.

The ultimate ultracompact binaries?

Two objects have received special attention in the past few years because they show repeating signals with periods of 5.4 and 9.5 minutes. In some interpretations, those times are the remarkably small orbital periods of binaries in which two white dwarfs are separated by about a quarter of the Earth–Moon distance.

Called RX J0806.3+1527 and V407 Vul, the two systems were discovered as periodic x-ray sources. Intense optical- and x-ray-telescope monitoring by Tod Strohmayer (Goddard), GianLuca Israel (Astronomical Observatory of Rome), Gavin Ramsay, and their coworkers have shown that the periods are getting shorter for both objects. Those decreasing periods suggest that the two objects comprise detached white dwarfs moving toward each other as they evolve toward their minimum period. However, the sources are strong x-ray emitters, often a sign of accretion. A good deal of debate and uncertainty attends the two systems,¹³ and several competing theories purport to explain them.

A model put forward by Kinwah Wu (University College London) and colleagues is the most exotic. It suggests that RX J0806 and V407 Vul are both made from a pair of white dwarfs in which one spirals through the magnetic field of the other. Induction causes x-ray emission at the footpoints of the magnetic field—that is, at the points where the magnetic field lines that pass through the spiraling dwarf enter the magnetic companion star. Wu and company must invoke a complex magnetic field configuration to explain the x-ray light curves. Moreover, the x-ray-active phases in their model are short-

lived, and the period changes much more rapidly than is observed. A revised model suggested by Simone Dall'Osso (Astronomical Observatory of Rome) and colleagues remedies those problems and can explain the relatively low x-ray emission of RX J0806, but it is compatible with the stronger emission of V407 Vul only if the magnetic white dwarf is rather rapidly spinning. Such rapid rotation would, in my opinion, indicate a previous phase of accretion that is not envisaged in their model.

In a second model, the two objects are AM CVn systems undergoing a special kind of mass transfer called direct impact accretion. Normally in binaries containing compact objects, transferred mass wraps around the accreting compact star and, as illustrated in figure 1a, interacts with itself to form a flat accretion disk in the plane of the orbit. But white dwarfs orbiting with periods on the order of a few minutes are extremely close to each other at the beginning of the mass transfer process, and that proximity allows for a second possibility. The larger, lower-mass star begins to lose mass to its companion in a stream, but the companion is large enough and close enough that the stream lands directly on its surface, as illustrated in figure 1b. In his seminal 1984 paper on double white dwarfs, Ronald Webbink (University of Illinois at Urbana-Champaign) realized the possibility of such direct impact.¹⁴ Then, as so often happens in science, the idea was reinvented—in this case by me and also by Thomas Marsh and Daniel Steeghs (Harvard-Smithsonian Center for Astrophysics) who together coined the term “direct impact accretor.” Astrophysicists have identified a few objects that might be in the direct impact phase. If RX J0806 and V407 Vul are directly accreting, then the observed rate of change in their periods means that the mass transfer in the two systems is lower than expected, if only temporarily.

A third idea is that the observed periods are not orbital periods at all, but are instead spin periods of accreting white

dwarfs in binaries with orbital periods of a few hours.¹⁵ In this model, the absence of an observed orbital period is explained by proposing that observers see the two systems face-on. That is, the only orbital motion is in the plane of the sky, and thus it remains invisible. One drawback to the idea is that both RX J0806 and V407 Vul lack spectral features that are normally observed in the types of binaries posited in the model. A second problem is the required finely tuned head-on configurations.

The study of ultracompact binaries combines such classic astronomical topics as stellar and binary evolution with new developments—for example, in gravitational-wave astronomy. It also highlights the importance of technological excellence. Only with the biggest optical telescopes can observers see ultracompact binaries in detail, and *LISA* scientists will have to overcome immense technological challenges to get their detector to work. For the near future, I hope the impetus of recent discoveries will continue and the imminent dedicated surveys will reveal much more. I also hope that theoretical developments keep pace with observational results, in particular those pertaining to the common-envelope phase and the formation of ultracompact binaries in general. With luck, the next article to appear in *PHYSICS TODAY* on ultracompact binaries will paint a completely different picture than this one does.

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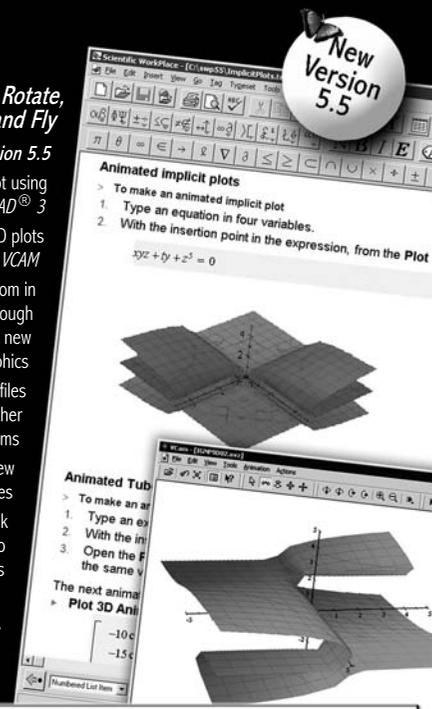
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