Astronomical Magnitudes and Photometry

Magnitudes

- Introduced by Greek astronomers (probably first Hipparchus); used by Ptolemy in the *Almagest* around 150 A.D.
- Scale from 1 - 6, where 6 is the faintest (visible to the naked eye)
- Extension to fainter stars required more precise definition:
- N. Pogson (1856, MNRAS 17, 12) proposed to use a "light ratio" of 2.512 between successive magnitude steps - still used today (5 mag = factor 100 in flux)
- *Absolute* magnitude (Kapteyn 1902; Publ. Gron. 11, 1): Apparent magnitude a star would have for a parallax of 0.1" (D=10 pc)

Magnitudes

- The fluxes (F_1 and F_2) and apparent magnitudes (m_1 and m_2) of two objects are related as:

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right)$$

- If the zero-point (zp) of the scale is known, then

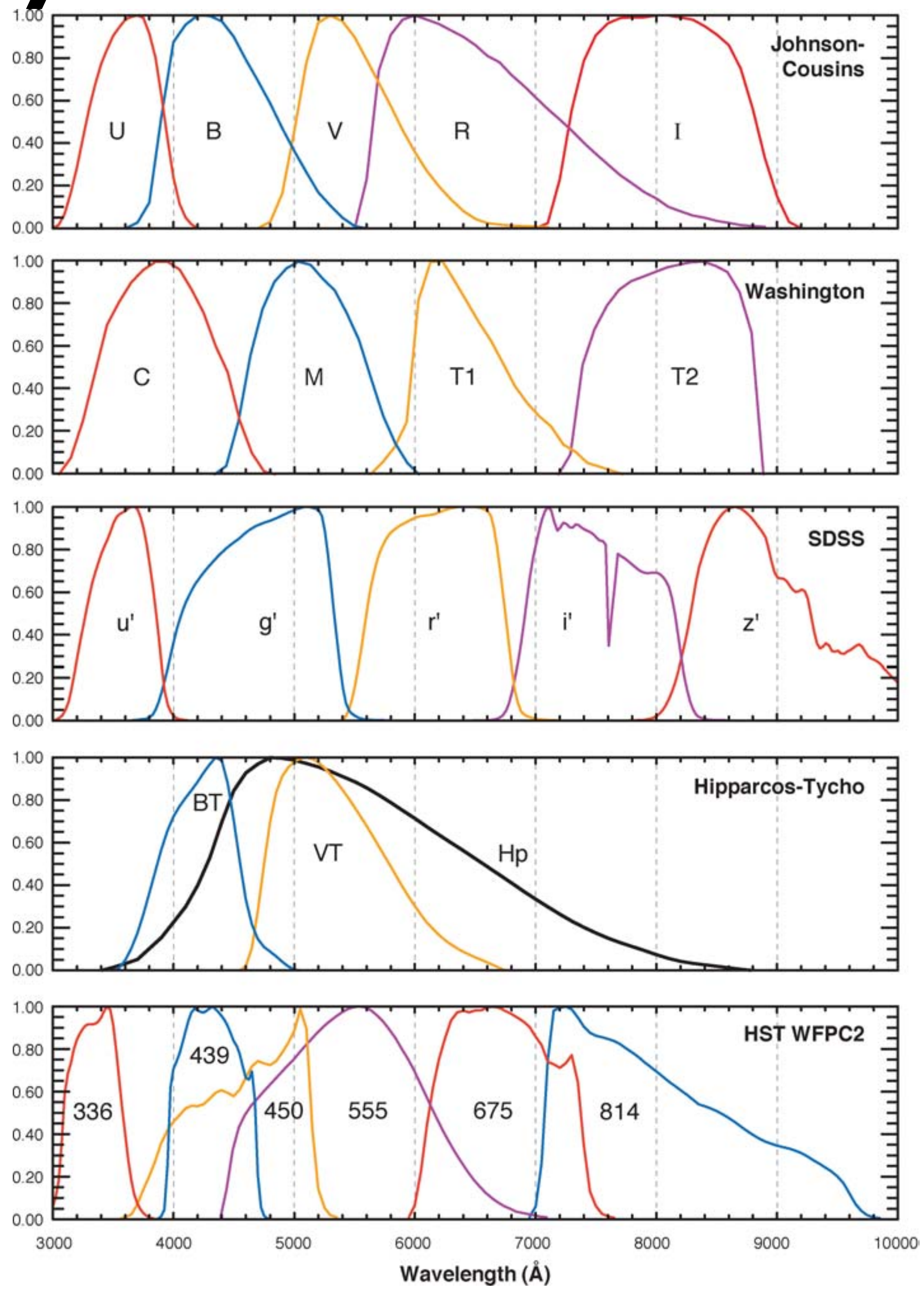
$$m = -2.5 \log_{10} F + \mathrm{zp}$$

- The star Vega is often used as a reference: m(Vega) $\equiv$ 0

- Sun: V = -26.7
- Full moon: V = -12.6
- Venus: V = -4.7
- Brightest star (Sirius): V = -1.47
- Faintest stars visible to naked eye: V=6
- Faintest objects detected in Hubble Ultra Deep Field: V ~ 29.5

Photometric Systems

- Magnitude systems: Defined by sets of *standard stars*. E.g. UBVRI, roughly normalised to Vega.
- Observations must be transformed from the *instrumental* system of the observer to the *standard* system.



Bessell 2005

UBV system

Filter	Definition	λ_{eff} [nm]	$\Delta\lambda$ [nm]
U	Corning 9863	365	70
B	Corning 5030 + Schott GG 13	440	100
V	Corning 3384	550	90

Defined by Johnson & Morgan (1953), using telescope with aluminised mirrors and RCA 1P21 photomultiplier tube.
U-band filter blue cut-off defined by atmosphere! *Difficult to reproduce*.

UBV Filter system

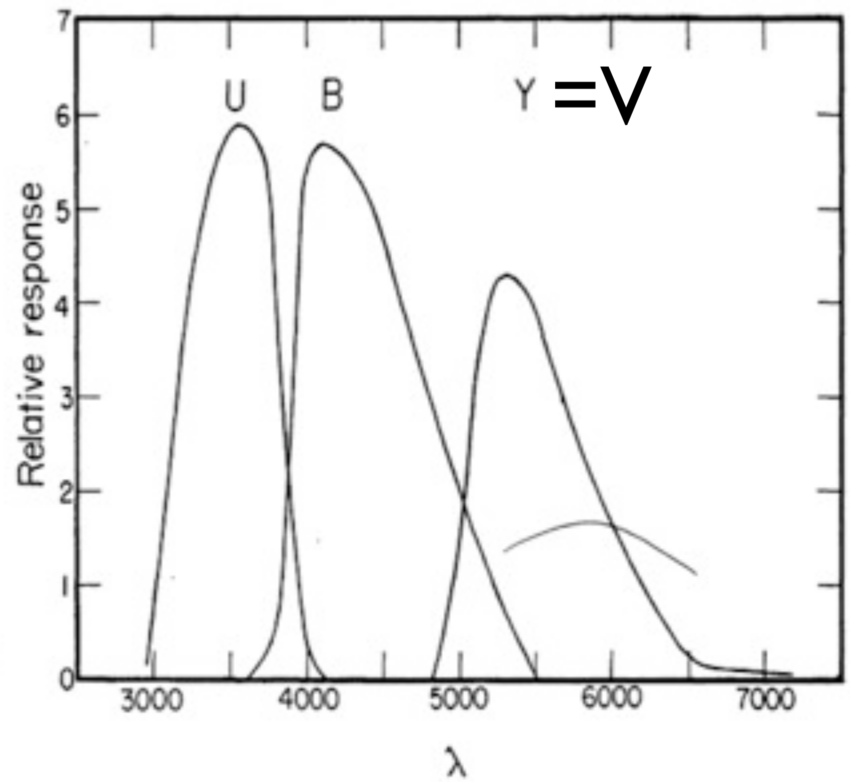
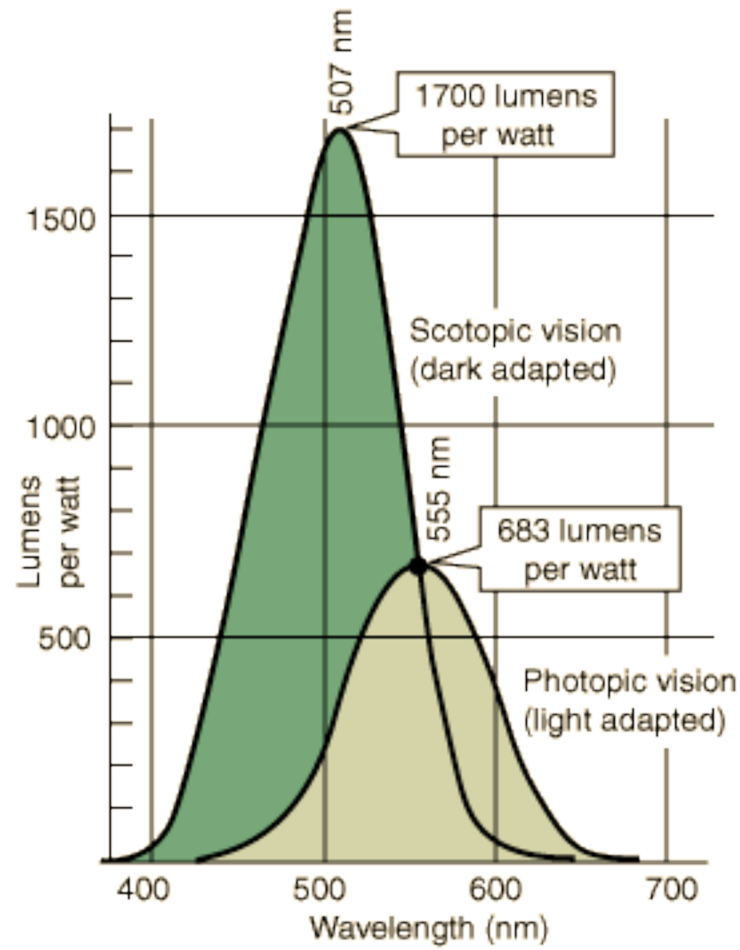


FIG. 1.—Response of the photometer to equal energy at all wave lengths

Johnson & Morgan 1951

Sensitivity of human eye



Colours

- Colours defined analagously to magnitudes, e.g.

$$B - V = -2.5 \log_{10} \left(\frac{F_B}{F_V} \right) + \mathrm{zp_{B-V}}$$

$$U - B = -2.5 \log_{10} \left(\frac{F_U}{F_B} \right) + \mathrm{zp_{U-B}}$$

Atmospheric extinction

z = zenith distance

X = *airmass* (≙ 1 at z=0) = 1/cos z = sec z (for z < 60°)

Observed flux: $F_{\rm obs} = F_0 \exp(-\tau_0 X)$

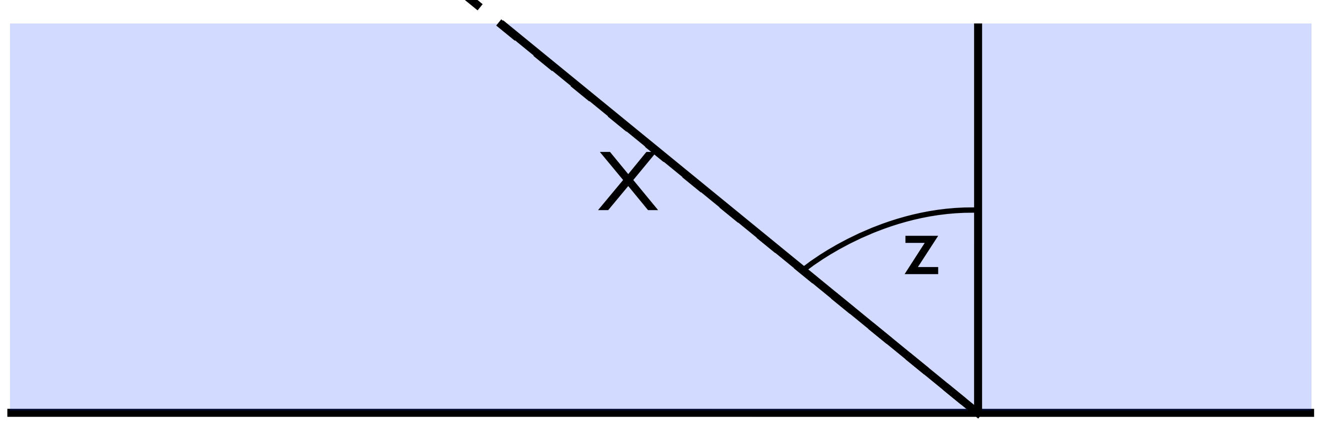
τ_0 = optical depth of atmosphere at z=0

F_0 = Flux outside atmosphere

Observed mag.: $m_{\rm obs} = -2.5 \log_{10} \left[F_0 \exp(-\tau_0 X)\right] + {\rm const}$

$$= m_0 - 2.5 \log_{10} \left[\exp(-\tau_0 X)\right]$$

$$= m_0 + \frac{2.5}{\ln 10} \tau_0 X \qquad = m_0 + kX$$

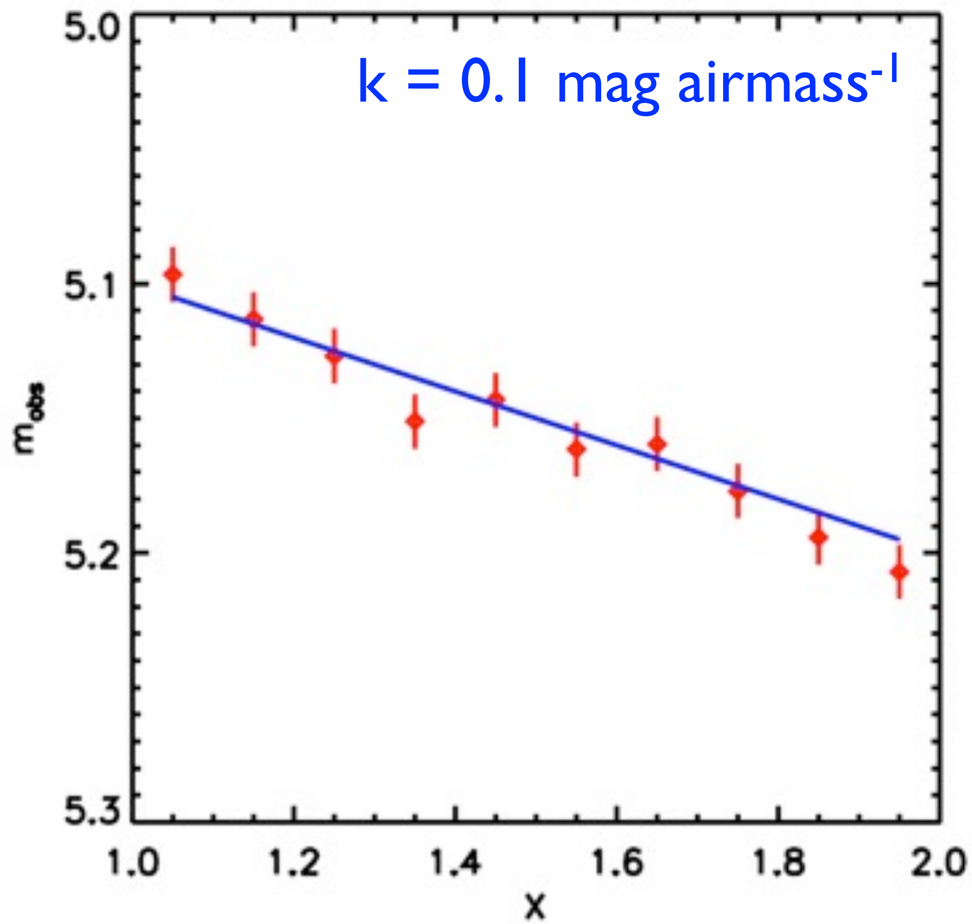


$$m_0 = m_{\rm obs} - kX$$

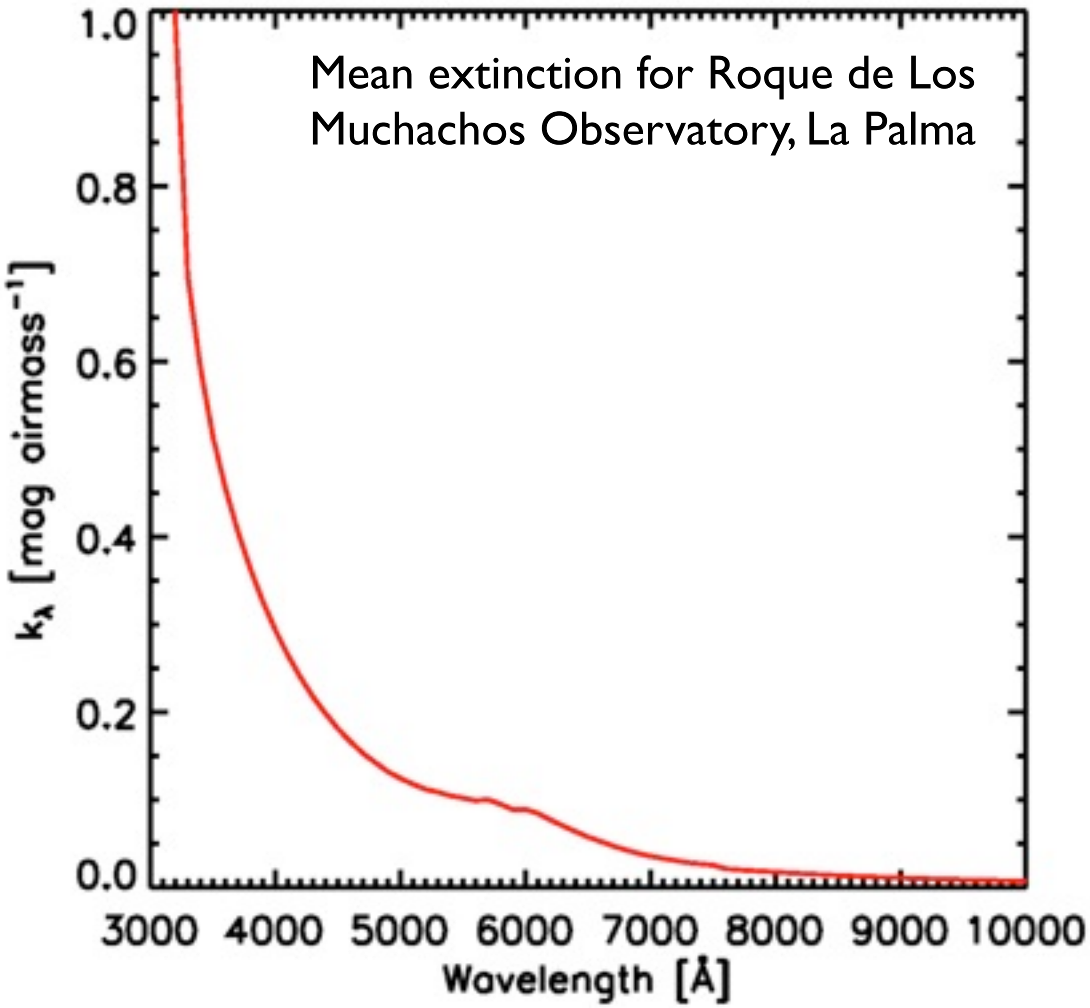
k = *extinction coefficient*

How to determine k:

- Since $m_{obs} = m_0 + k X$, the extinction coefficient k can be measured by observing a star at different airmass values and plotting m_{obs} versus X.



Atmospheric extinction



Extinction is wavelength-dependent.

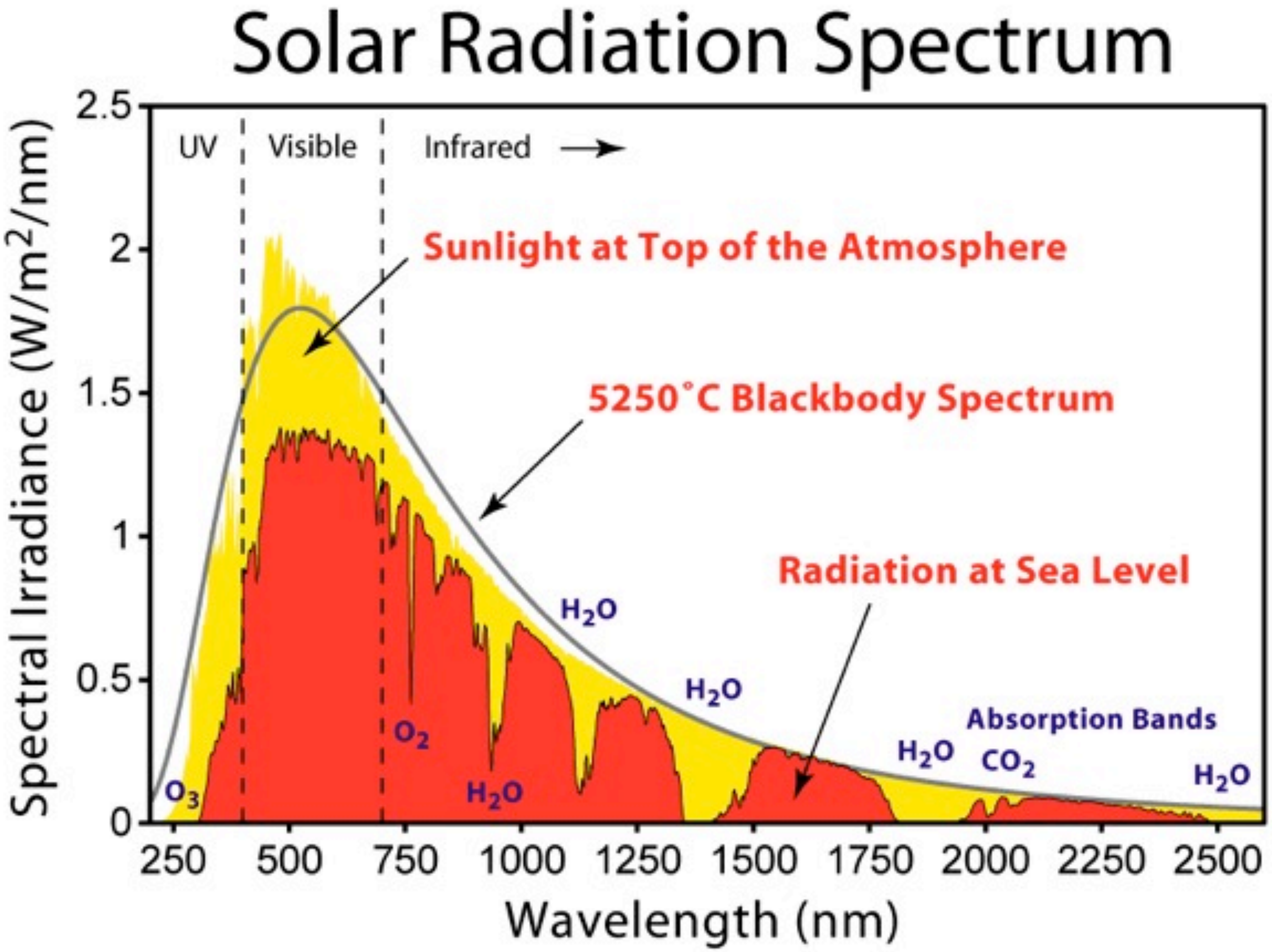
Typical values:

k_U = 0.4 mag airmass^{-1}

k_B = 0.2 mag airmass^{-1}

k_V = 0.1 mag airmass^{-1}

k_λ increases strongly below ~3400 Å (atmospheric cut-off)



Near-IR extensions to the UBV system must take into account atmospheric transmission.

Extensions to UBV

Filter	R	I	J	K	L	M	N	Q
λ_{eff} [nm]	700	900	1250	2200	3400	4900	10200	20000
$\Delta\lambda$ [nm]	220	240	380	480	700	300	5000	5000

Extensions to original UBV system defined by Johnson in 1960s

Filter	R	I	Z	J	H	K	L	M
λ_{eff} [nm]	638	797	908	1220	1630	2190	3450	4750
$\Delta\lambda$ [nm]	160	149	96	213	307	390	472	460

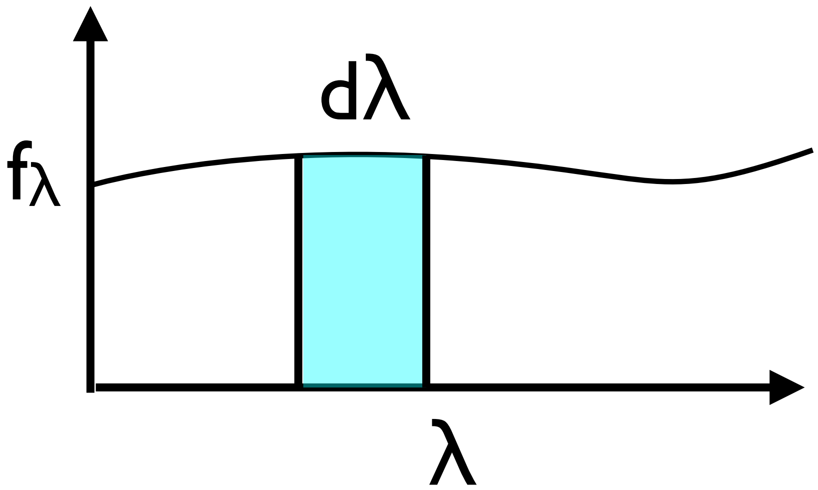
Johnson-Cousins-Glass filters, optimised for modern detectors

Beware of different filter definitions - most people now use the Johnson-Cousins-Glass system.

Defining the zero-points

1. Normalise to flux of a particular star (or stellar type). Typically A0 star (Vega), e.g. UBV system:
   - $V_{Vega}=0$ and $B_{Vega}=0$, $U_{Vega}=0$
     (actually: V = 0.03 for Vega; Bessell et al. 1998)
2. Normalise to flat spectral energy distribution, either as a function of frequency or wavelength.
   - E.g. ABmag system (used by Sloan, HST):
     $ABmag = -2.5 \log_{10} f_\nu - 48.60$ $[f_\nu] = [\text{ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}]$
     $m_u = -2.5 \log_{10} f_u - 48.6$,
     $m_g = -2.5 \log_{10} f_g - 48.6$, etc..
   - STmag system (used by HST):
     $STmag = -2.5 \log_{10} f_\lambda - 21.10$ $[f_\lambda] = [\text{ergs cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1}]$

F(λ) versus F(ν)



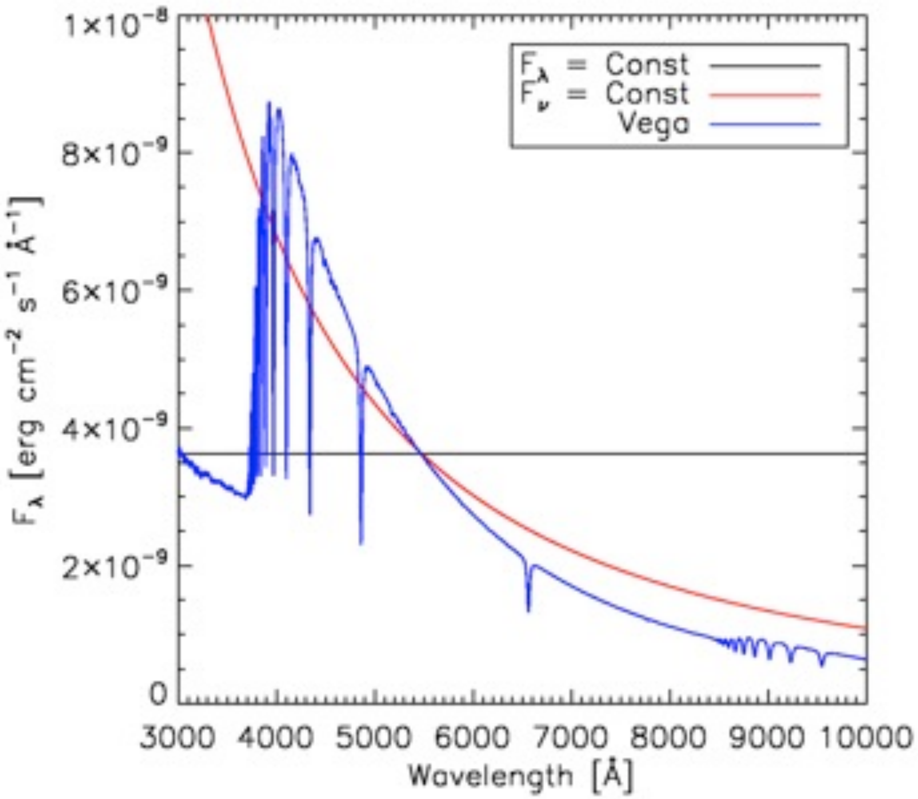
$$f = f_\nu d\nu$$
$$f = f_\lambda d\lambda$$

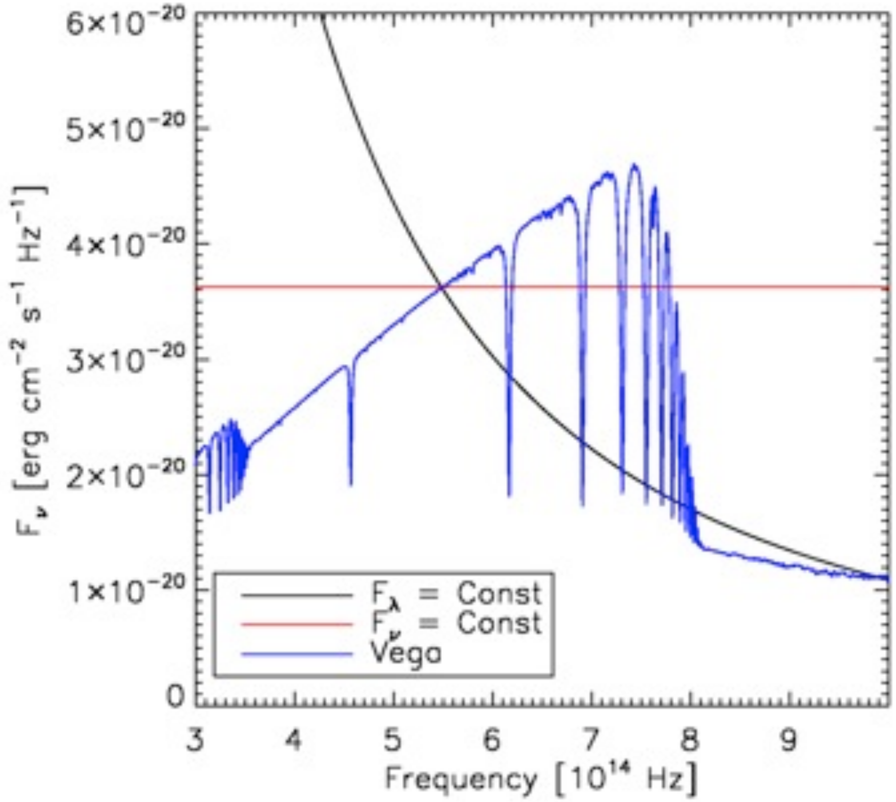
$$f_\nu d\nu = f_\lambda d\lambda$$

$$\frac{f_\nu}{f_\lambda} = \frac{d\lambda}{d\nu}$$

$$\lambda = c/\nu$$
$$d\lambda/d\nu = -c/\nu^2$$

$$\boxed{\frac{f_\nu}{f_\lambda} = c/\nu^2 = \lambda^2/c}$$



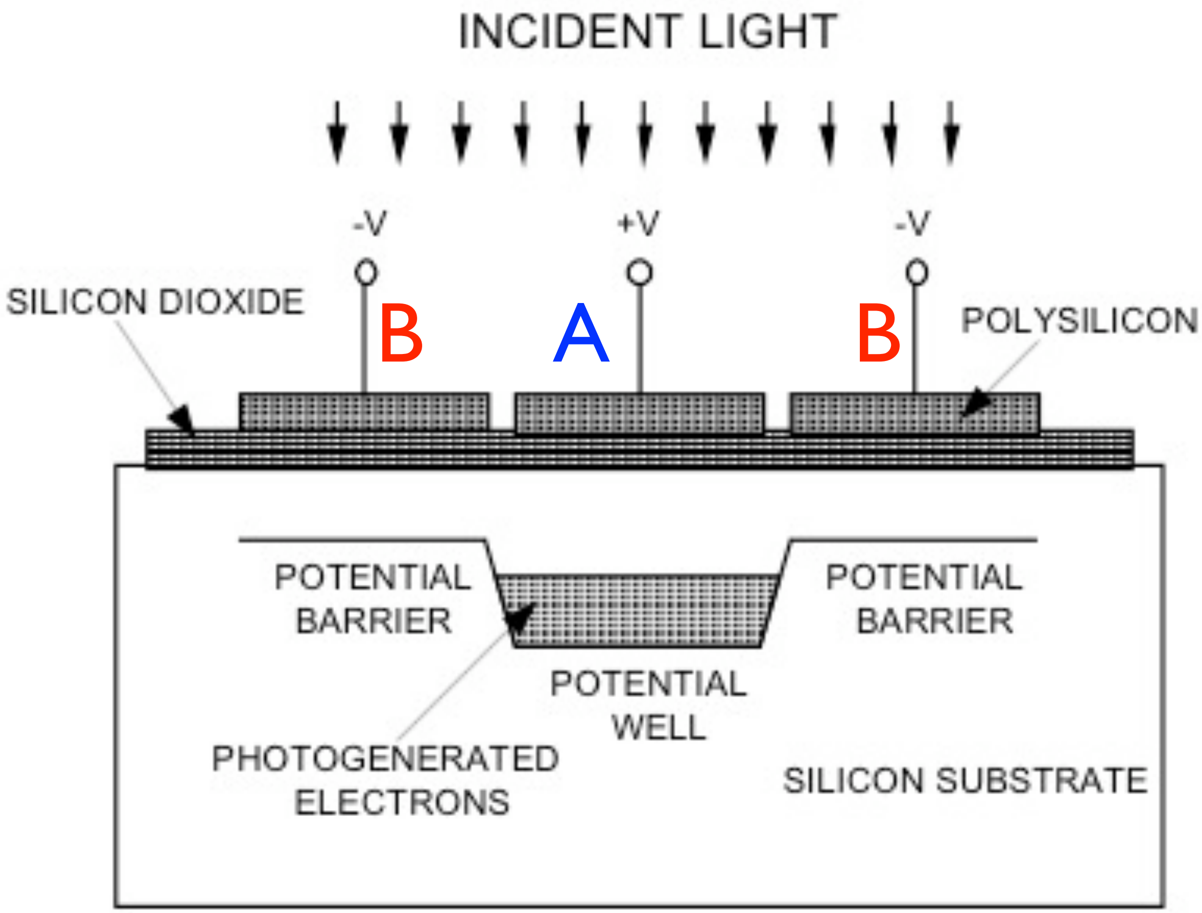


All magnitude systems normalised to same flux at V-band (550 nm)

Detectors for photometry

- **Eye** (Cheap, but relative accuracy only ~0.1 - 0.5 mag; no permanent record of "data")
- **Photographic plates** (relative accuracy ~0.01 mag; can be exposed for a long time to record faint objects; provide permanent record of data; basically no limit on size. However, response is non-linear, calibration difficult, not very sensitive (Q.E.~1%))
- **Photomultipliers** (photon-counting, linear, moderate Q.E. (~25%), but can only measure one object at the time.)
- **Charge-Coupled Devices (CCDs)** (linear, high Q.E. (up to ~90%), now available in large formats and can be mosaic'ed. Detector of choice in the UV/optical for most applications)
- **Infrared detectors** (various semiconductors; behaviour similar to CCDs although smaller and linearity / noise characteristics not generally as good)

The CCD detector

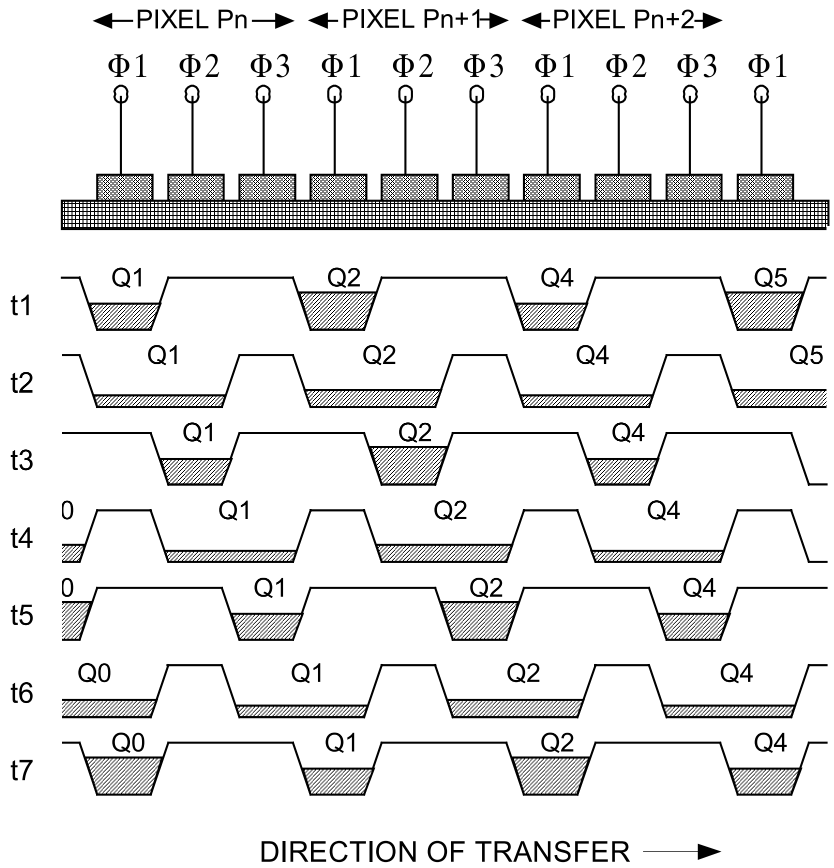


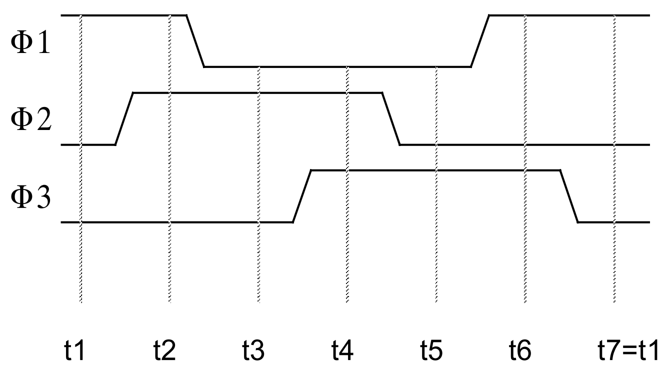
Schematic illustration of a single CCD pixel

Electron-hole pairs generated via photoelectric effect during integration.

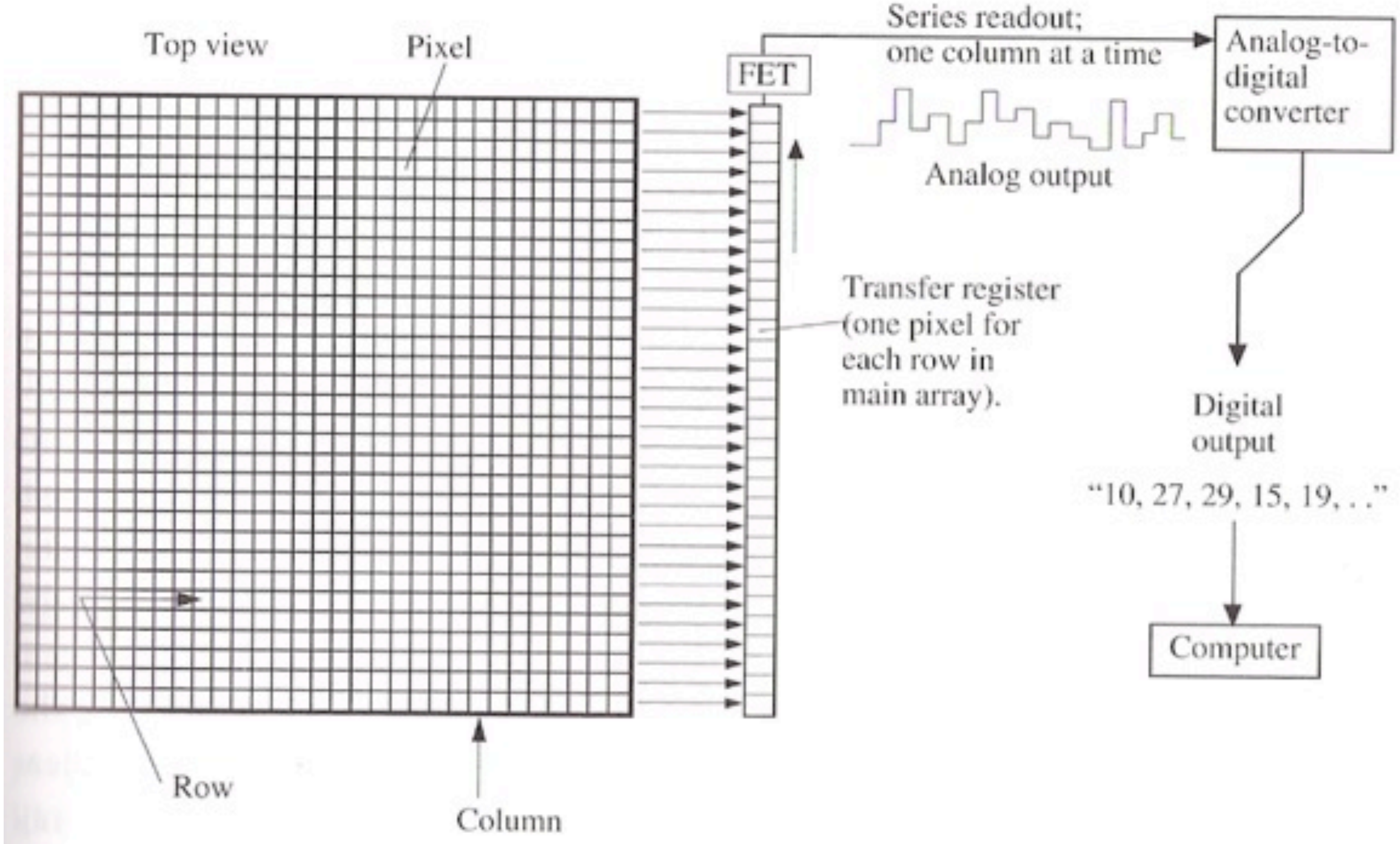
Kept in place by positive charge at "A" and negative charge at "B".

CCD Primer, Eastman Kodak (2001)





At the end of the exposure, the charges are shifted across the CCD by cycling the voltages on electrodes.

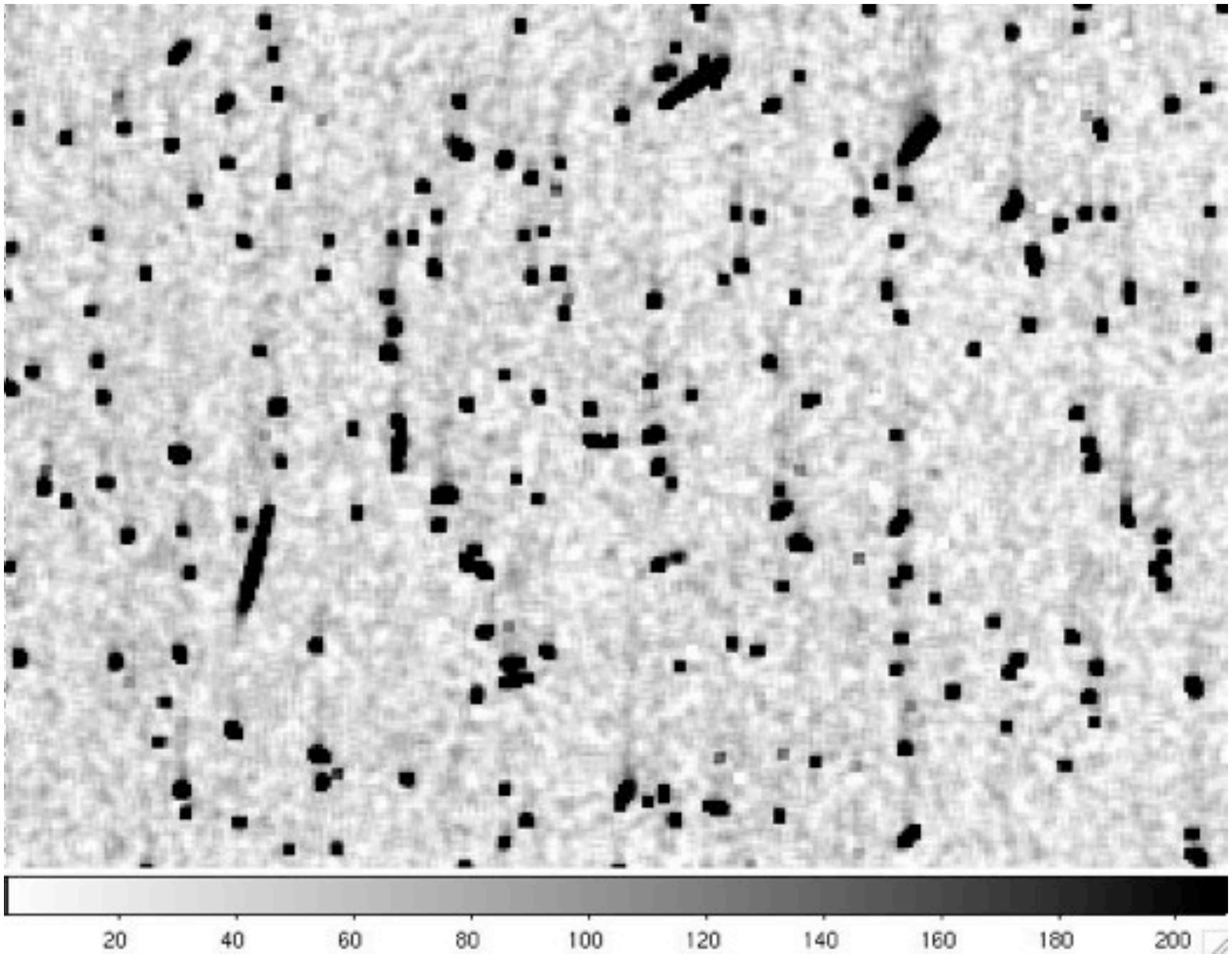


CCD "read out" one column at the time, pixel by pixel. Charges converted to a voltage and then to 16-bit "Digital Numbers" via an analogue-to-digital converter.

CCD Characteristics - I

- Typical CCD: 2048x2048 pixels, i.e. 4096 transfers needed. Very high pixel-to-pixel *Charge Transfer Efficiency* (CTE) required!
- If CTE = 99.9%: Only $0.999^{4098} < 2\%$ of charge left after read-out!
- CTE > 99.9999% required to preserve >99% of charge
- Can be an issue for space-based detectors: CTE degrades over time due to radiation damage.

Radiation damage in HST image

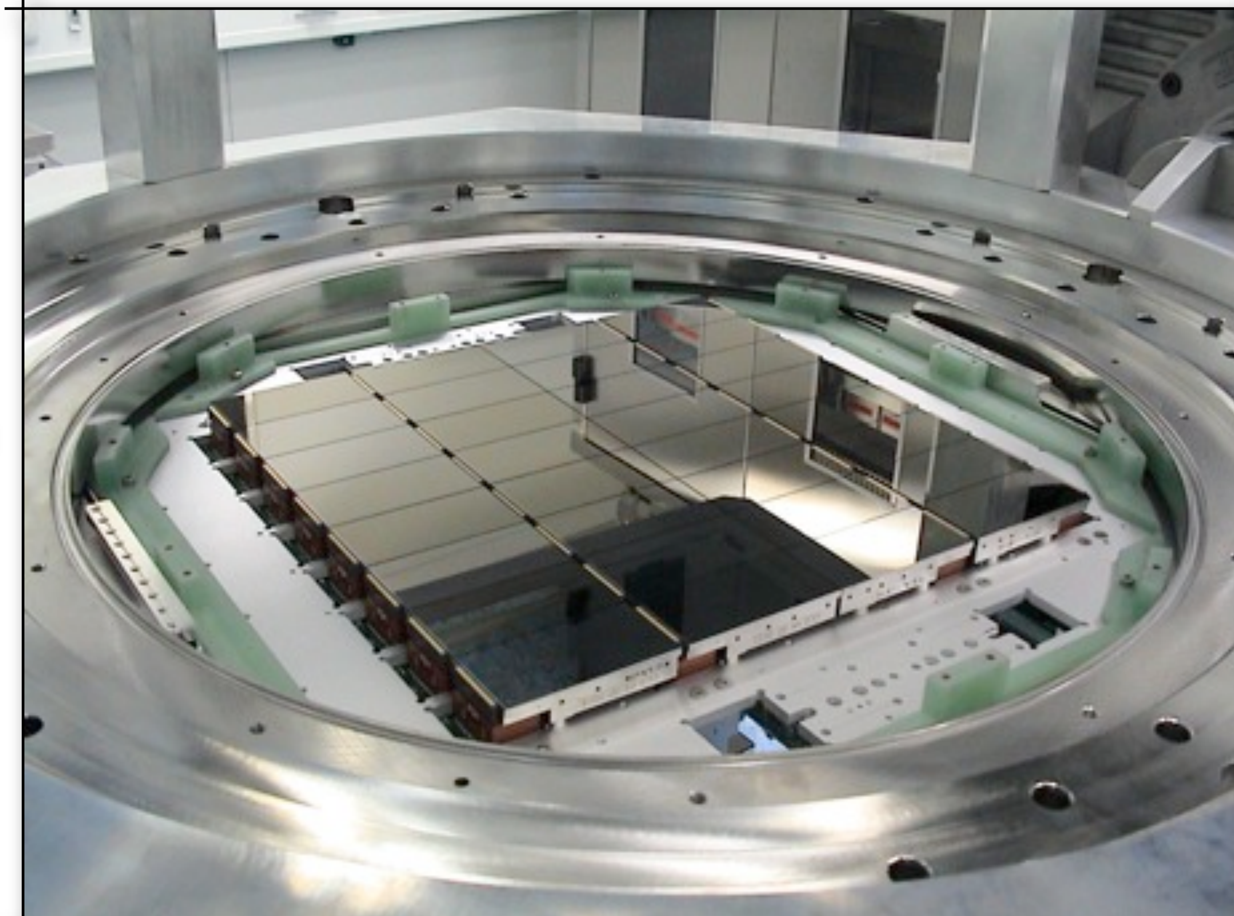


Note the vertical "trails": Charge "left behind" during CCD read-out

CCD Characteristics - II

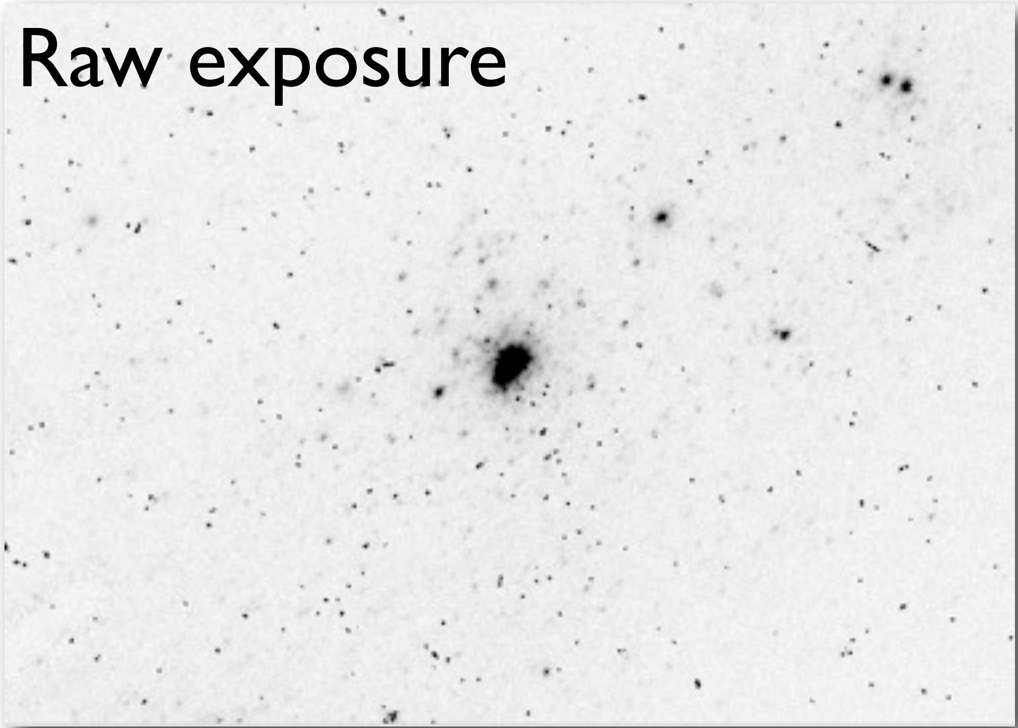
- High quantum efficiency: >90% of photons create electron-hole pairs.
- Large dynamic range - "full well capacity" typically $\sim 10^5$ electrons
- Linear response - simple conversion between "counts" and flux/intensity
- Spectral range ~300 nm - 1 μm
- Largest current sizes used in astronomy typically ~2048x4096 pixels, but several CCDs can be mounted together in mosaic cameras.

OmegaCam on the ESO 2.6 m VST ("VLT survey telescope"):
Mosaic of 32 CCDs of 2048x4096 pixels, total 16k x16 k (=256 Megapixels).
Field of view = 1x1 degree.



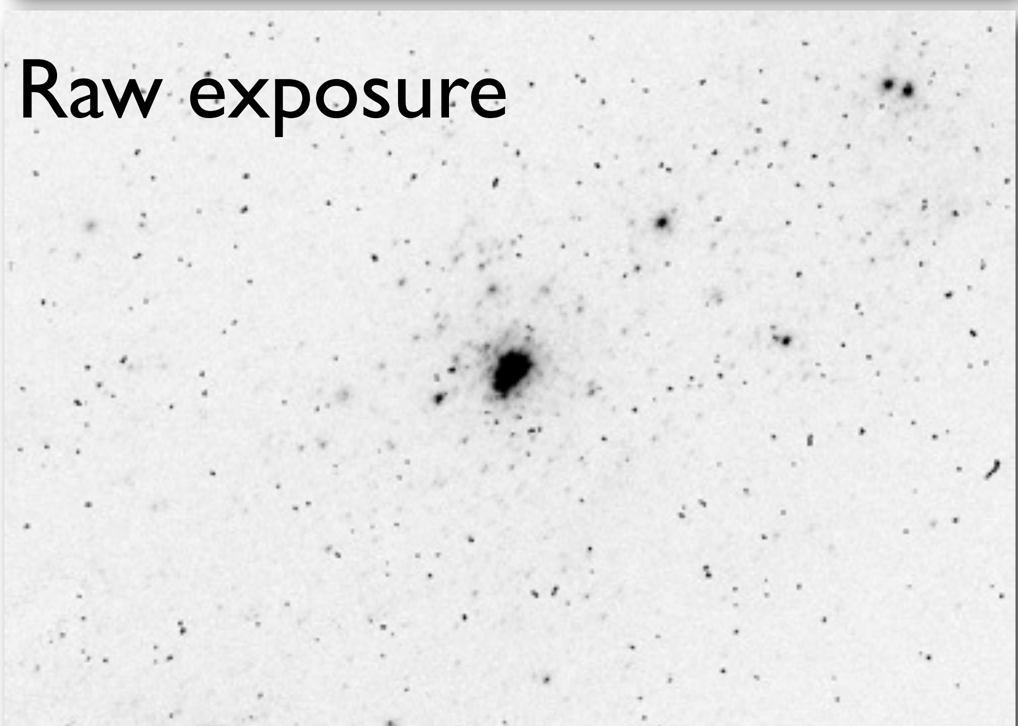
CCD Characteristics - III

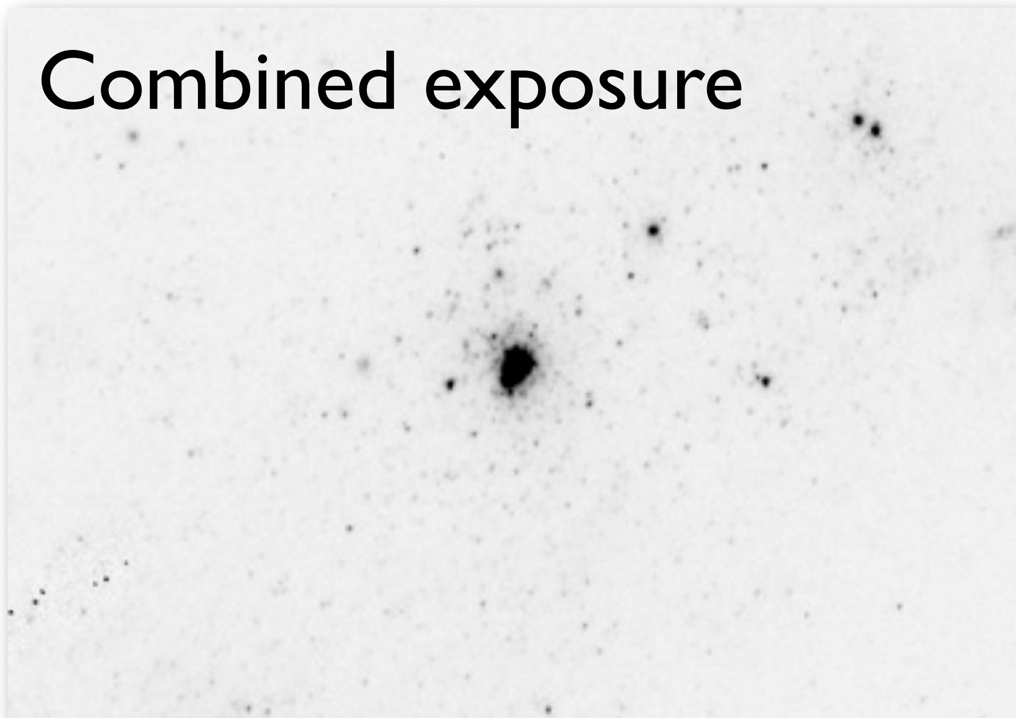
- Images typically stored as "FITS" files with 16-bit "Digital Number" data values (0 - 65535)
- To convert DN to actual electrons per pixel:
  - Subtract "Bias" level
  - Multiply by "Gain" factor
- All CCD images suffer from a (typically small) random "*read-out noise*". For modern CCDs this is usually only a few electrons per pixel.
- *Dark current* also present - can usually be reduced to negligible levels by cooling CCD with liquid Nitrogen
- CCDs are also sensitive to *cosmic rays* - can be filtered out if multiple exposures are taken.



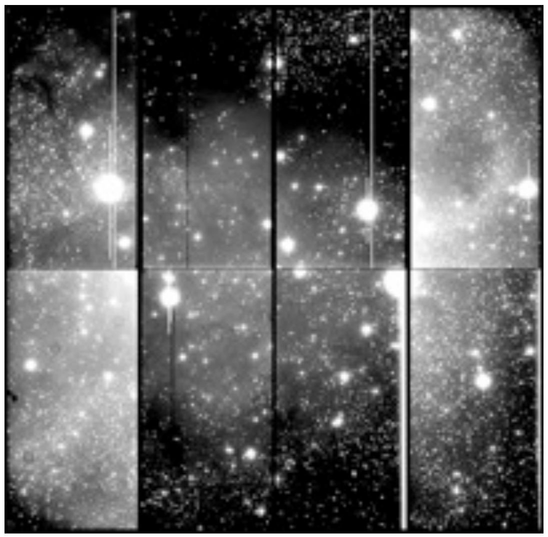
Left: raw exposures (220 s each) from HST.

Below: combination of 3 exposures. Note that cosmic rays have disappeared.





Flat-fielding of CCD images



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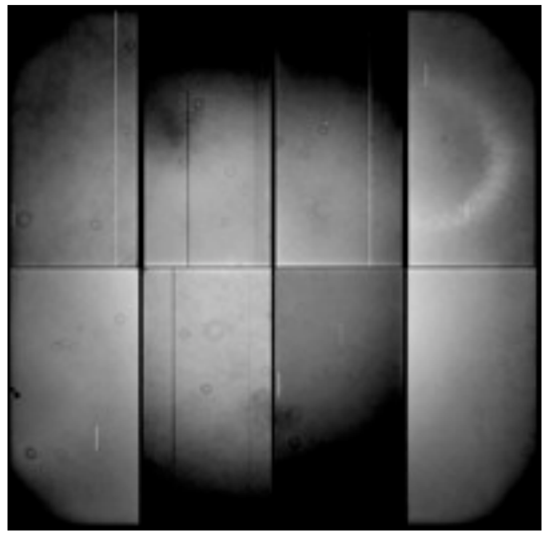

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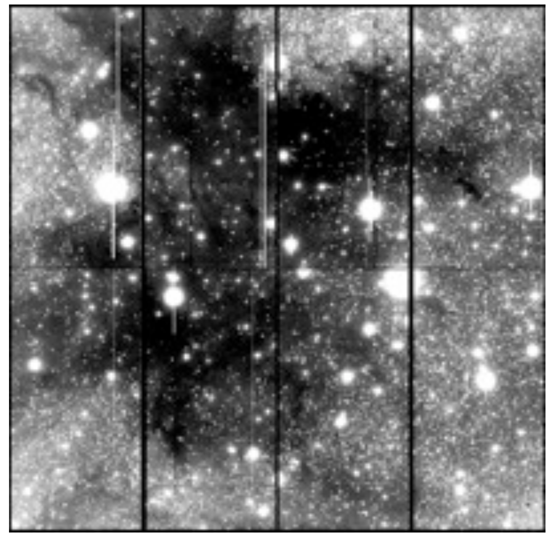
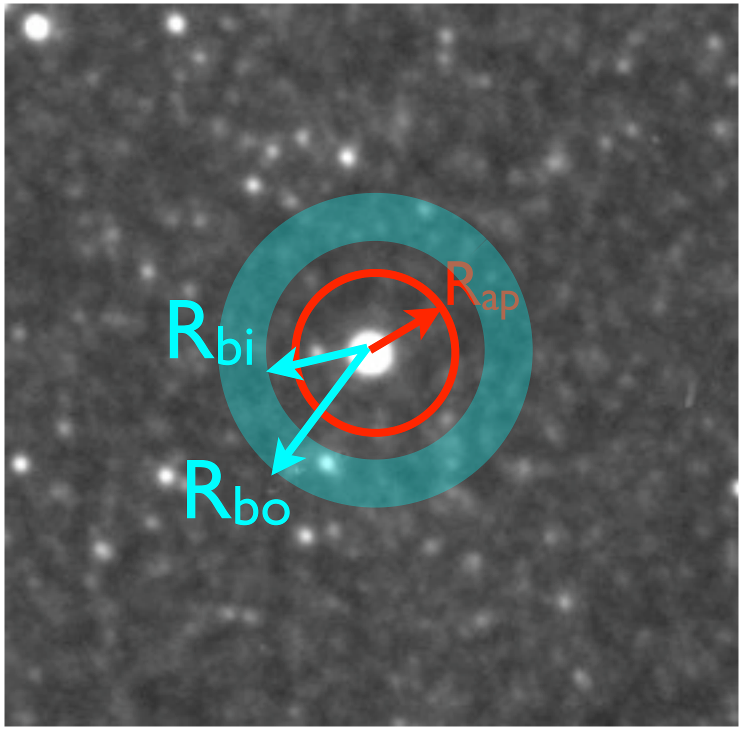


- Not all pixels have the same sensitivity. Correct by dividing with a uniformly illuminated image - *flatfield* - e.g. of the twilight sky.

Counting Digital Numbers



Science CCD image (e.g. Johnson V)

Measure:

DN_{ap} counts in aperture

DN_{bkg} counts in background

Counts from star:

$DN_{star} = DN_{ap} - DN_{bkg} * R_{ap}^2 / (R_{bo}^2 - R_{bi}^2)$

Instrumental magnitude:

$m_{V,i} = -2.5 \log DN_{star}/T_{exp} - k_V*X$

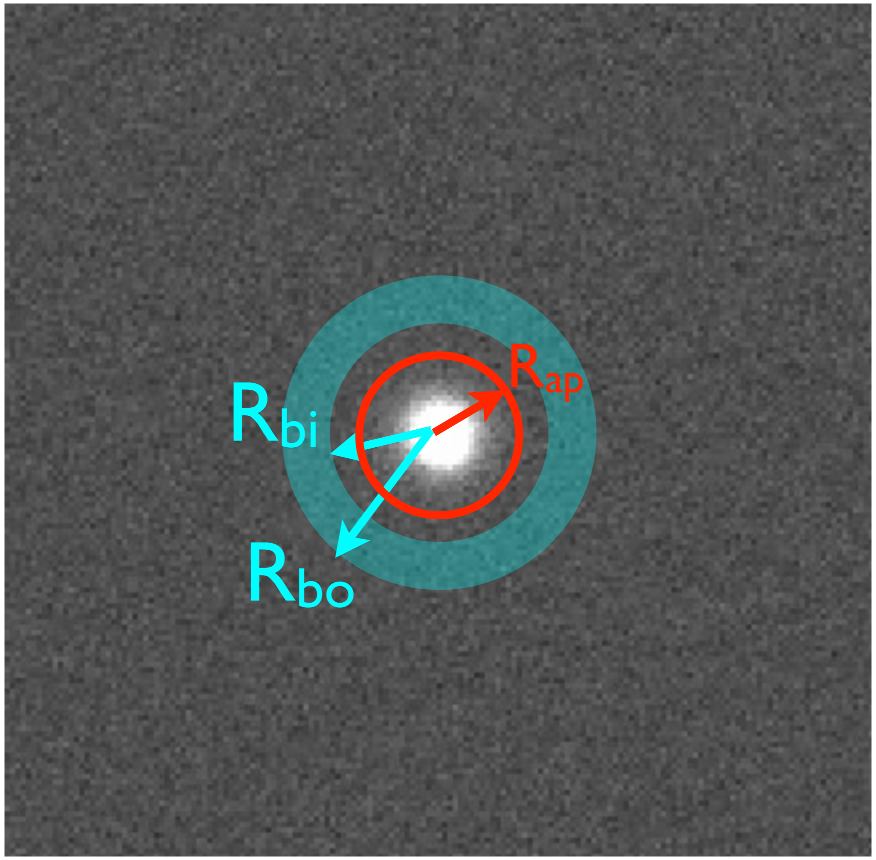
Standard magnitude:

$m_V = m_{V,i} + z_V$

T_{exp} = exposure time

z_V = zero-point

Standard star



To determine z_V, observe standard star with known magnitude $m_{V,std}$:

Instrumental magnitude:

$m_{V,i} = -2.5 \log DN_{star}/T_{exp} - k_V*X$

Zero-point:

$z_V = m_{V,std} - m_{V,i}$

UBVRI standard stars

Landolt (1992)

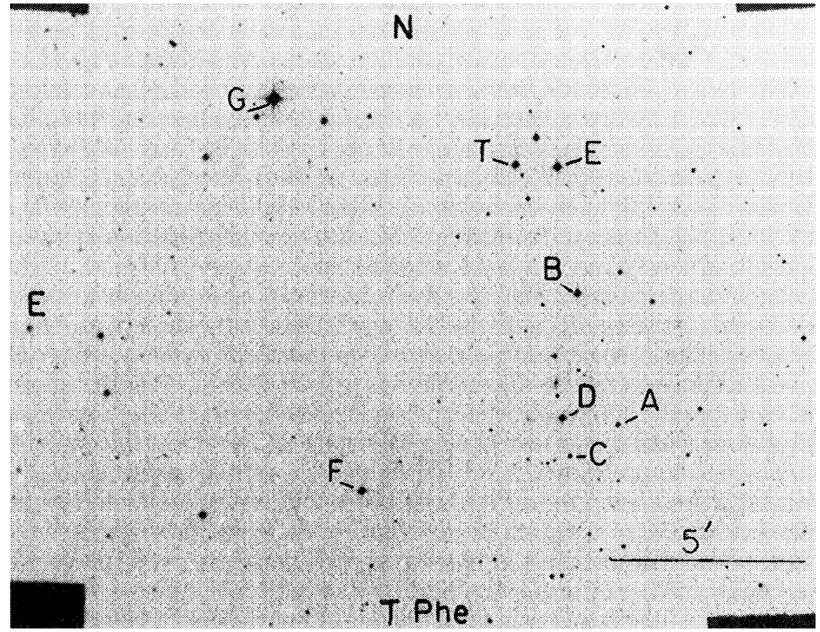
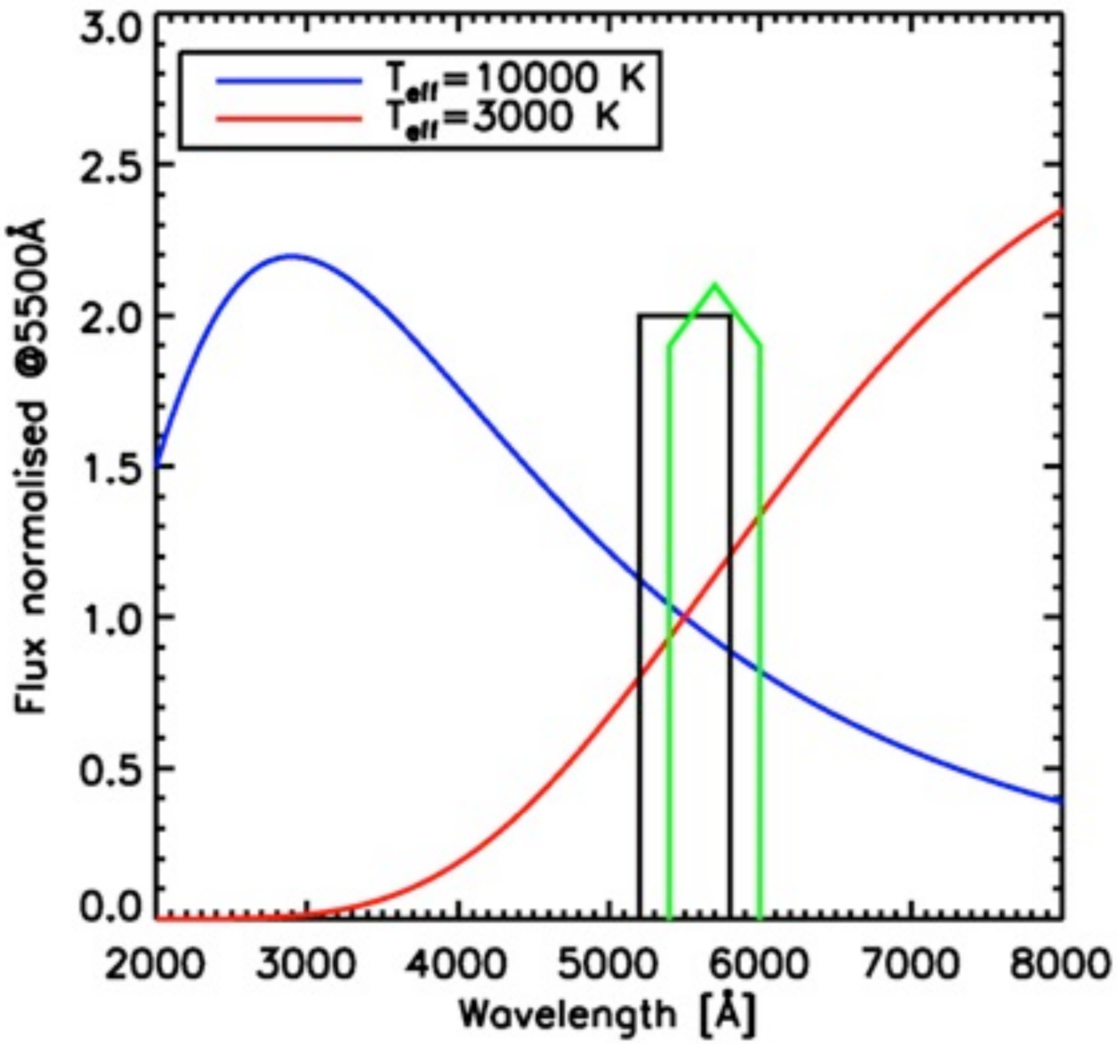


TABLE 2. UBVRI standard stars.

											Mean Errors of the Mean						
Star	α(2000)	δ(2000)	V	B-V	U-B	V-R	R-I	V-I	n	m	V	B-V	U-B	V-R	R-I	V-I	Notes
TPHE A	00:30:09	-46 31 22	14.651	0.793	0.380	0.435	0.405	0.841	29	12	0.0028	0.0046	0.0071	0.0019	0.0035	0.0032	
TPHE B	00:30:16	-46 27 55	12.334	0.405	0.156	0.262	0.271	0.535	29	17	0.0115	0.0026	0.0039	0.0020	0.0019	0.0035	1
TPHE C	00:30:17	-46 32 34	14.376	-0.298	-1.217	-0.148	-0.211	-0.360	39	23	0.0022	0.0024	0.0043	0.0038	0.0133	0.0149	
TPHE D	00:30:18	-46 31 11	13.118	1.551	1.871	0.849	0.810	1.663	37	23	0.0033	0.0030	0.0118	0.0015	0.0023	0.0030	
TPHE E	00:30:19	-46 24 36	11.630	0.443	-0.103	0.276	0.283	0.564	34	8	0.0017	0.0012	0.0024	0.0007	0.0015	0.0019	
TPHE F	00:30:50	-46 33 33	12.474	0.855	0.532	0.492	0.435	0.926	5	3	0.0004	0.0058	0.0161	0.0004	0.0040	0.0036	
TPHE G	00:31:05	-46 22 43	10.442	1.546	1.915	0.934	1.085	2.025	5	3	0.0004	0.0013	0.0036	0.0004	0.0009	0.0009	
PG0029+024	00:31:50	+02 38 26	15.268	0.362	-0.184	0.251	0.337	0.593	5	2	0.0094	0.0174	0.0112	0.0161	0.0125	0.0067	
PG0039+049	00:42:05	+05 09 44	12.877	-0.019	-0.871	0.067	0.097	0.164	4	3	0.0020	0.0030	0.0055	0.0035	0.0055	0.0045	
92 309	00:53:14	+00 46 02	13.842	0.513	-0.024	0.326	0.325	0.652	2	1	0.0035	0.0057	0.0028	0.0014	0.0035	0.0014	
92 235	00:53:16	+00 36 18	10.595	1.638	1.984	0.894	0.911	1.806	5	2	0.0058	0.0045	0.0098	0.0031	0.0045	0.0067	
92 322	00:53:47	+00 47 33	12.676	0.528	-0.002	0.302	0.305	0.608	2	1	0.0007	0.0049	0.0028	0.0014	0.0007	0.0007	
92 245	00:54:16	+00 39 51	13.818	1.418	1.189	0.929	0.907	1.836	21	8	0.0028	0.0079	0.0301	0.0024	0.0024	0.0028	
92 248	00:54:31	+00 40 15	15.346	1.128	1.289	0.690	0.553	1.245	4	2	0.0255	0.0160	0.0955	0.0215	0.0145	0.0175	
92 249	00:54:34	+00 41 05	14.325	0.699	0.240	0.399	0.370	0.770	17	8	0.0049	0.0085	0.0114	0.0046	0.0065	0.0073	
92 250	00:54:37	+00 38 56	13.178	0.814	0.480	0.446	0.394	0.840	20	9	0.0022	0.0034	0.0074	0.0022	0.0022	0.0029	
92 330	00:54:44	+00 43 26	15.073	0.568	-0.115	0.331	0.334	0.666	2	1	0.0141	0.0297	0.0163	0.0304	0.0000	0.0304	
92 252	00:54:48	+00 39 23	14.932	0.517	-0.140	0.326	0.332	0.666	41	18	0.0033	0.0055	0.0082	0.0047	0.0072	0.0068	
92 253	00:54:52	+00 40 20	14.085	1.131	0.955	0.719	0.616	1.337	39	17	0.0032	0.0062	0.0221	0.0027	0.0043	0.0050	
92 335	00:55:00	+00 44 13	12.523	0.672	0.208	0.380	0.338	0.719	2	1	0.0007	0.0028	0.0049	0.0000	0.0014	0.0014	
92 339	00:55:03	+00 44 11	15.579	0.449	-0.177	0.306	0.339	0.645	19	8	0.0087	0.0117	0.0126	0.0117	0.0197	0.0177	2
92 342	00:55:10	+00 43 14	11.613	0.436	-0.042	0.266	0.270	0.538	48	34	0.0013	0.0012	0.0023	0.0013	0.0009	0.0016	
92 188	00:55:10	+00 23 12	14.751	1.050	0.751	0.679	0.573	1.254	14	6	0.0096	0.0187	0.0551	0.0051	0.0043	0.0088	2
92 409	00:55:14	+00 56 07	10.627	1.138	1.136	0.734	0.625	1.361	5	3	0.0031	0.0027	0.0085	0.0022	0.0027	0.0018	
92 410	00:55:15	+01 01 49	14.984	0.398	-0.134	0.239	0.242	0.484	27	13	0.0058	0.0064	0.0083	0.0052	0.0102	0.0117	

Complications..

- Generally, the filter system used by the *observer* is not a perfect match to the *standard* system.
- The zero-point will depend on the spectrum of the star in question.



Standard V filter

Some Observer's V filter

A simple offset is not enough to transform from observer's to standard system.

$V_{std} = V_i + F(SED)$,

SED = Spectral Energy Distribution

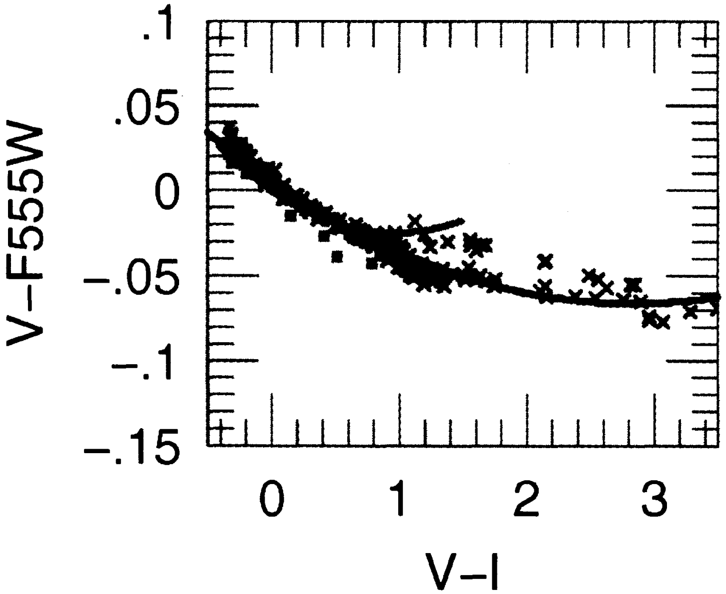
E.g. $V_{std} = V_i + z_V + c_V\ (B\text{-}V)$

$B_{std} = B_i + z_B + c_B\ (B\text{-}V)$

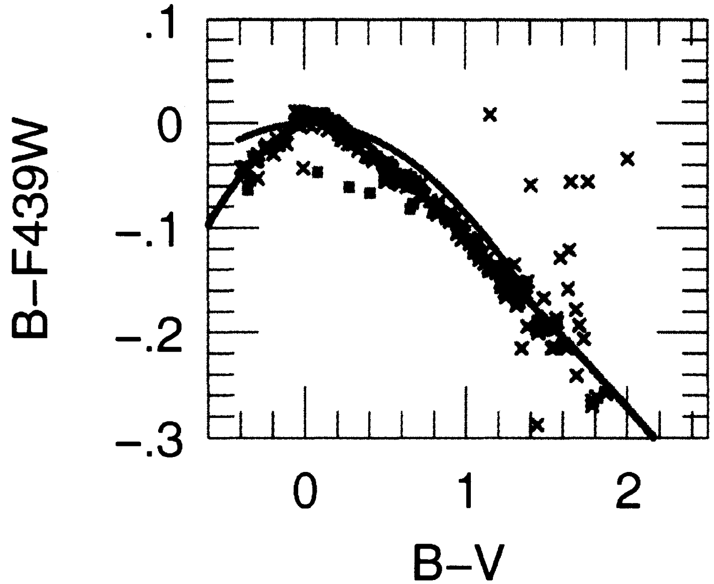
etc..

Real examples - WFPC2

HST/WFPC2 F555W



HST/WFPC2 F439W



$$B = m_{\mathrm{F439W}} + 20.070 + 0.003 \times (B{-}V) - 0.088 \times (B{-}V)^2$$

$$V = m_{\mathrm{F555W}} + 21.725 - 0.060 \times (B{-}V) + 0.033 \times (B{-}V)^2$$

Holtzman et al. (1995)

Error estimation, S/N, limiting magnitudes

Error on magnitude

$$(\sigma m)^2 = \left(\frac{\partial m}{\partial F}\right)^2 (\sigma F)^2$$

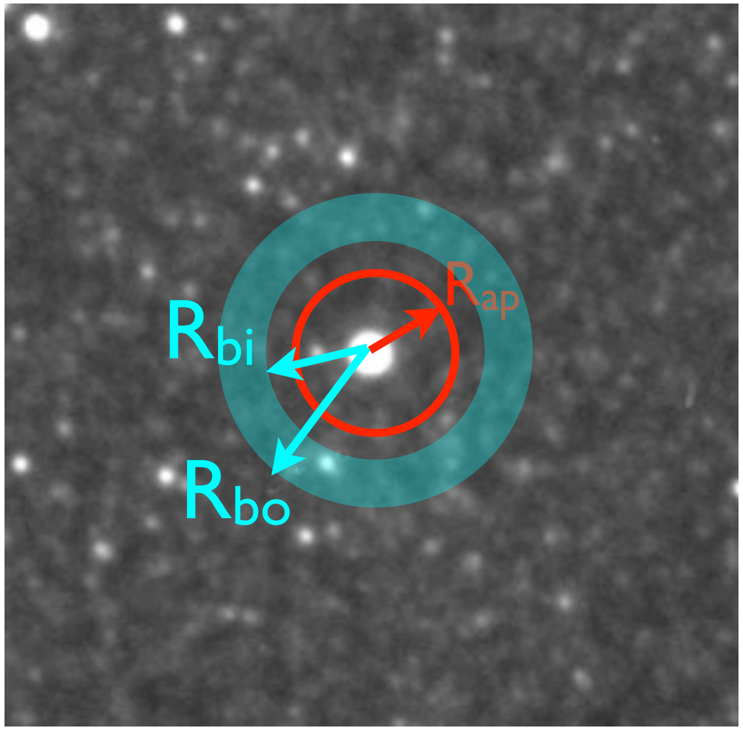
F = flux

$$m = -2.5 \log F + Z$$

$$\frac{\partial m}{\partial F} = -2.5 \frac{\partial \log F}{\partial F} = -\frac{2.5}{\ln 10} \frac{1}{F}$$

$$\sigma m = \frac{2.5}{\ln 10} \frac{\sigma F}{F} \approx 1.09 \frac{\sigma F}{F}$$

Good approximation:
Error on magnitude is about equal to *relative* error on linear measured quantity (Flux, counts, etc.)



DN ("Digital Numbers") from star:

$$\mathrm{DN_{star}} = \mathrm{DN_{ap}} - \mathrm{DN_{bkg}} \times A_{\mathrm{ap}}/A_{\mathrm{bkg}}$$

A_{ap} and A_{bkg} = Area of aperture and background

S/N = (Signal)/(Noise)

$$= \mathrm{DN_{star}}/\sigma(\mathrm{DN_{star}})$$

Including Poisson noise from background:

$$[\sigma(\mathrm{DN_{star}})]^2 = [\sigma(\mathrm{DN_{ap}})]^2 + [\sigma(\mathrm{DN_{bkg}} \times A_{\mathrm{ap}}/A_{\mathrm{bkg}})]^2$$
$$= [\sigma(\mathrm{DN_{ap}})]^2 + [\sigma(\mathrm{DN_{bkg}}) \times A_{\mathrm{ap}}/A_{\mathrm{bkg}}]^2$$

$$\mathrm{DN_{bkg}} \propto A_{\mathrm{bkg}} \Rightarrow \sigma(\mathrm{DN_{bkg}}) \propto \sqrt{A_{\mathrm{bkg}}}$$

Random error on background can be minimised by using large background annulus.

$$(\sigma DN_{star})^2 = (\sigma DN_{ap})^2 + (A_{ap}/A_{bkg}\ \sigma DN_{bkg})^2$$

First term:

$$(\sigma DN_{\rm ap})^2 = (\sigma DN_{\rm sky})^2 + (\sigma DN_{\rm star})^2$$

DN_{sky} = sky counts within aperture

$$\sigma(DN_{\rm sky}) = \boxed{\sigma(\rm bkg)\sqrt{A_{\rm ap}}}$$

σ(bkg) = std dev of background

$$DN_{\rm star} = N_{\rm star}/{\rm Gain} \ \Rightarrow$$

N_{star} = Number of photons from star

$$\sigma(DN_{\rm star}) = \sqrt{N_{\rm star}}/{\rm Gain} = \boxed{\sqrt{DN_{\rm star}/{\rm Gain}}}$$

Poisson stat.

$$(\sigma DN_{\rm ap})^2 = A_{\rm ap}(\sigma(\rm bkg))^2 + DN_{\rm star}/{\rm Gain}$$

Second term:

$$DN_{\rm bkg} = A_{\rm bkg}\langle{\rm bkg}\rangle$$

<bkg> = mean background level

$$\sigma(DN_{\rm bkg}) = A_{\rm bkg}\sigma(\langle{\rm bkg}\rangle) = A_{\rm bkg}\frac{\sigma(\rm bkg)}{\sqrt{A_{\rm bkg}}} = \sigma(\rm bkg)\sqrt{A_{\rm bkg}}$$

$$(\sigma DN_{\rm star})^2 = DN_{\rm star}/{\rm Gain} + (\sigma({\rm bkg}))^2 A_{\rm ap} + (\sigma({\rm bkg}))^2 A_{\rm ap}^2/A_{\rm bkg}$$

Photon noise from star

Background noise
in aperture

Uncertainty on mean
sky level due to
noise in sky annulus

$$\sigma m = 1.09 \frac{\sqrt{DN_{\rm star}/{\rm Gain} + (\sigma({\rm bkg}))^2 A_{\rm ap} + (\sigma({\rm bkg}))^2 A_{\rm ap}^2/A_{\rm bkg}}}{DN_{\rm star}}$$

Trade-offs:

- Smaller aperture -> less background noise. If aperture *too small*, some fraction of signal from star will be lost and it becomes increasingly critical to have star exactly centered in aperture
- Larger background annulus -> better determined background. However, if background annulus *too large* then non-uniform background may cause problems.

Exposure time estimation

Exposure time estimation

- Goal: to estimate for a given telescope+instrument
  1. the limiting magnitude (for a given S/N) in a given amount of integration time
  2. The integration time required to reach a given limiting magnitude
- Requires estimates of expected counts (from source) and background noise

Assuming no uncertainty on *mean* sky level:

$$S/N = \frac{N_{\rm star}}{\sqrt{N_{\rm star} + N_{\rm sky} + \sigma^2_{\rm instr}}}$$

N_{star} = photons detected from star
N_{sky} = photons from sky background in aperture
σ_{instr} = instrumental noise (e.g. read-out noise, dark current)

Flux to counts:

Photon energy: $E = h\nu = hc/\lambda$

Flux density to [photons m^{-2} Å^{-1} s^{-1}]: $N_p = f_\lambda \frac{\lambda}{hc}$

Number of photons in bandpass
(where T_λ is the transmission of bandpass) $N_{p,bp} = \frac{1}{hc}\int_{\lambda_1}^{\lambda_2} T_\lambda f_\lambda \lambda \, d\lambda$

Can be approximated by

$$N_{p,bp} \approx \frac{f_\lambda \lambda_{\rm eff}}{hc} T_{\rm sys} \Delta\lambda = \frac{\Delta\lambda}{\lambda_{\rm eff}} \frac{f_\nu}{h} T_{\rm sys}$$

For T_{sys} = system throughput, $\Delta\lambda$ = width of filter, λ_{eff} = effective wavelength

Estimating counts:

$$N_{p,bp} \approx= \frac{\Delta\lambda}{\lambda_{\rm eff}} \frac{f_\nu}{h} T_{\rm sys}$$

Number of detected photons:

$$N_{\rm det} = N_{\rm p,bp} A_{\rm tel} t_{\rm exp}$$

A_{tel} = telescope collecting area
t_{exp} = exposure time

ABSOLUTE CALIBRATION OF PHOTOMETRY

Filter band	λ_o	Absolute flux density for mag = 0.00 F_ν
U	0.36 μ	1.81×10^{-23} W m^{-2} hz^{-1}
B	0.44 μ	4.26×10^{-23}
V	0.55 μ	3.64×10^{-23}
R_C	0.64 μ	3.08×10^{-23}
I_C	0.79 μ	2.55×10^{-23}
104	1.04 μ	2.00×10^{-23}
K	2.2 μ	6.49×10^{-24}

Bessell 1979

E.g. Johnson V (λ_{eff} = 550 nm, $\Delta\lambda$ = 88 nm)
$N_V = 10^{-0.4V}\ 8.8\times10^{9}\ T_{sys}\ A_{tel}\ t_{exp}$.

$T_{sys} = T_{atm}\ T_{opt}$ QE
T_{atm} = atmospheric transmission
T_{opt} = transmission of optical system
QE = Quantum efficiency of detector

$$S/N = \frac{N_{\rm star}}{\sqrt{N_{\rm star} + N_{\rm sky} + \sigma^2_{\rm instr}}}$$

$$N_{\rm star} = 10^{-0.4V_{\rm star}} \times 8.8 \times 10^9 \times T_{\rm atm} \times T_{\rm opt} \times {\rm QE} \times A_{\rm tel} \times t_{\rm exp}$$

$$N_{\rm sky} = \pi r^2_{\rm ap} 10^{-0.4\mu_{V,\rm sky}} \times 8.8 \times 10^9 \times T_{\rm atm} \times T_{\rm opt} \times {\rm QE} \times A_{\rm tel} \times t_{\rm exp}$$

r_{ap} = aperture radius in arcsec

$\mu_{V,sky}$ = sky brightness (in mag arcsec^{-2})

$$\sigma^2_{\rm instr} = ({\rm ron}^2 + n_{\rm dark} t_{\rm exp}) \pi r^2_{\rm ap} / {\rm scl}^2$$

ron = read-out noise (in e$^-$ pixel^{-1})

n_{dark} = dark current (in e$^-$ pixel^{-1} s^{-1})

scl = image scale (in arcsec pixel^{-1})

Example 1 (ground-based observation, 8 m telescope):

8 m telescope → A_{tel} = 50 m^2

k_V = 0.1, X=1.0 → T_{atm} = 0.91

t_{exp}=1000 s, T_{opt}=50%, QE=90%

Pixel scale = 0.2" $pixel^{-1}$

Read noise: ron= 5 e^- $pixel^{-1}$

Sky background = 22 mag $arcsec^{-2}$

Photometry of V=25 star in r_{ap}=1" aperture:

N_{star} = 18000

$N_{sky} = 286000 * \pi * r_{ap}^2 = 897000$

$\sigma_{instr}^2 = ron^2 * \pi * r_{ap}^2 / 0.2^2 = 1960$

Predicted S/N = 19 (noise dominated by sky)

Example 2 (Space-based observation, HST):

2.4 m telescope → A_{tel} = 18 m^2

T_{atm} = 1.00

t_{exp}=1000 s, QE * T_{opt}=40%,

Pixel scale = 0.045" $pixel^{-1}$

Read noise: ron= 5 e^- $pixel^{-1}$

Sky background = 22.5 mag $arcsec^{-2}$

Photometry of V=25 star in r_{ap}=0.2" aperture:

N_{star} = 1700

$N_{sky} = 17000*\pi*r_{ap}^2 = 2170$

$\sigma_{instr}^2 = ron^2 * \pi*r_{ap}^2 / 0.045^2 = 1551$

Predicted S/N = 23

Better spatial resolution (and darker background in space) makes HST competitive with the largest ground-based telescopes.

$$S/N = \frac{N_{\rm star}}{\sqrt{N_{\rm star} + N_{\rm sky} + \sigma^2_{\rm instr}}}$$

Typically, N_{sky} dominates over N_{star} and σ_{instr} for faint objects observed in broad-band filters.

$$N_{\rm star} \propto F_{\rm star} t_{\rm exp}$$

F_{star} = flux from star

$$N_{\rm sky} \propto I_{\rm sky} r^2_{\rm ap} t_{\rm exp}$$

I_{sky} = sky intensity

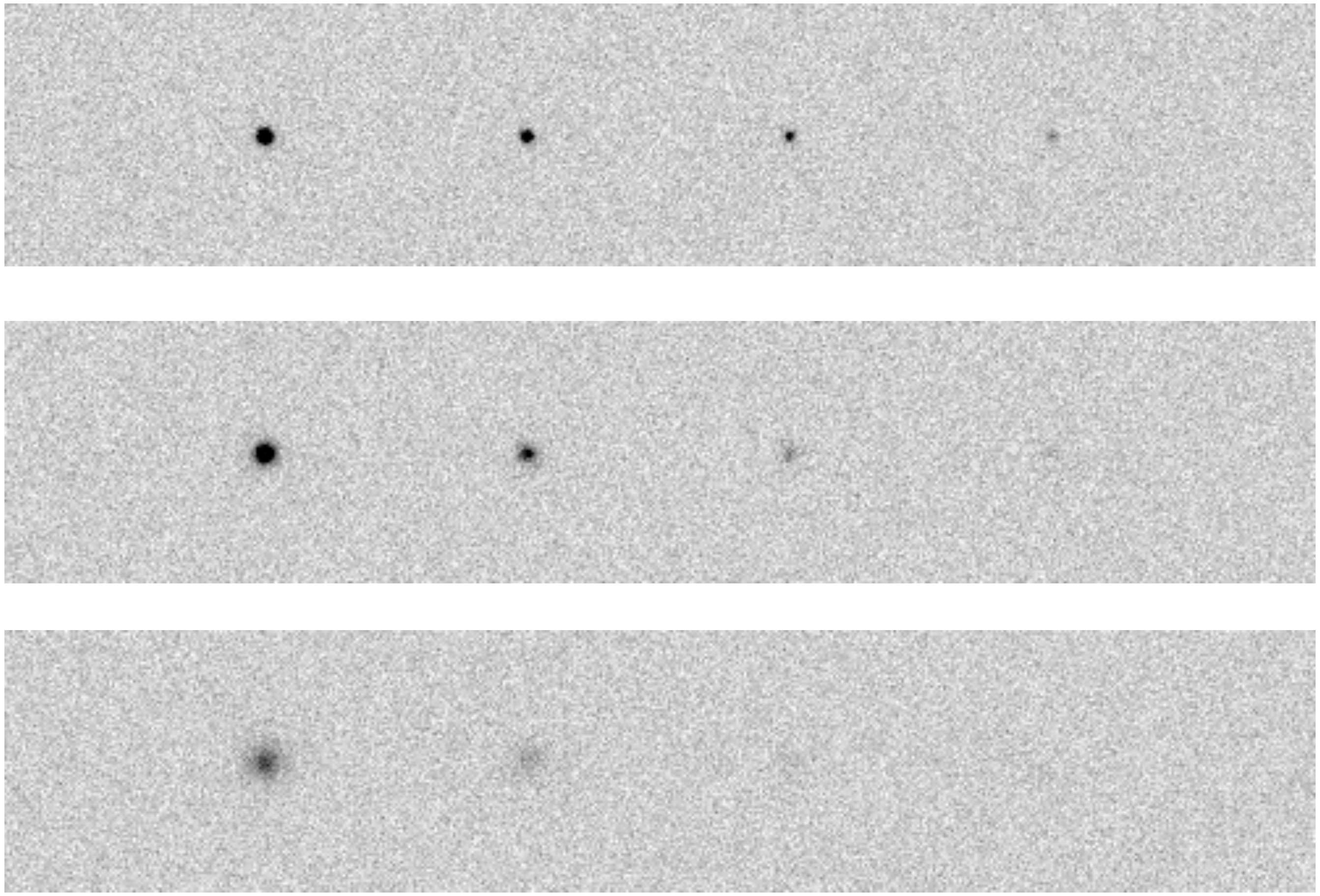
For sky-dominated observations:

$$S/N \propto \frac{F_{\rm star} t_{\rm exp}}{r_{\rm ap}\sqrt{I_{\rm sky} t_{\rm exp}}} = \frac{F_{\rm star}}{\sqrt{I_{\rm sky}}} \frac{\sqrt{t_{\rm exp}}}{r_{\rm ap}}$$

S/N is inversely proportional to r_{ap} for sky-dominated observations.

If r_{ap} increases by factor x (e.g. because of poor seeing), t_{exp} must increase by x^2 to reach same S/N!

Simulated 1000 s V-band exposures with an 8 m telescope

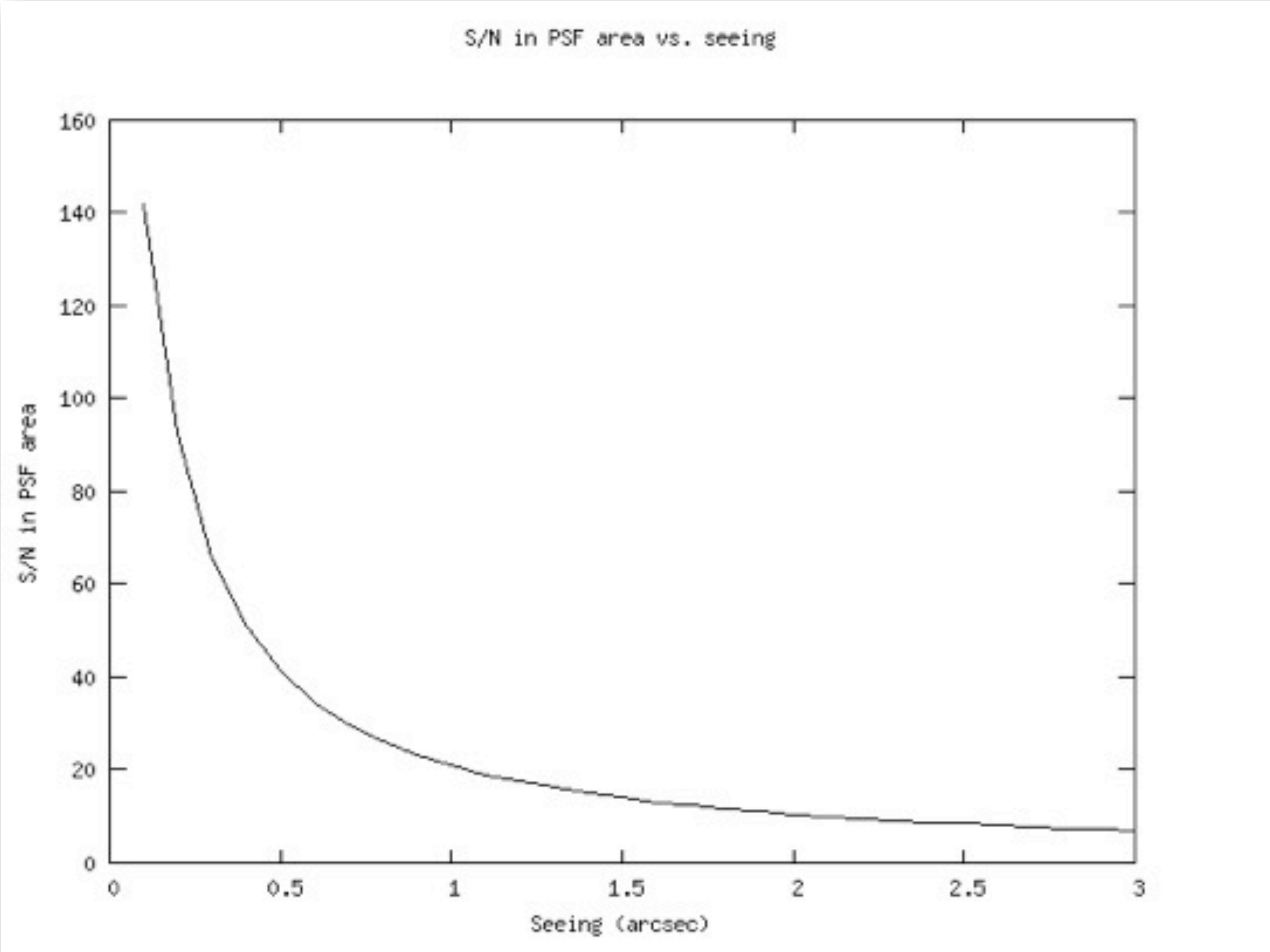


Seeing = 0.5”

Seeing = 1.0”

Seeing = 2.0”

V=23 V=24 V=25 V=26



Output from ESO/VLT FORS1 Exposure Time Calculator
(V=25, 3 days from new Moon, t_{exp} = 1000 s)