Atmospheric Seeing and Adaptive Optics

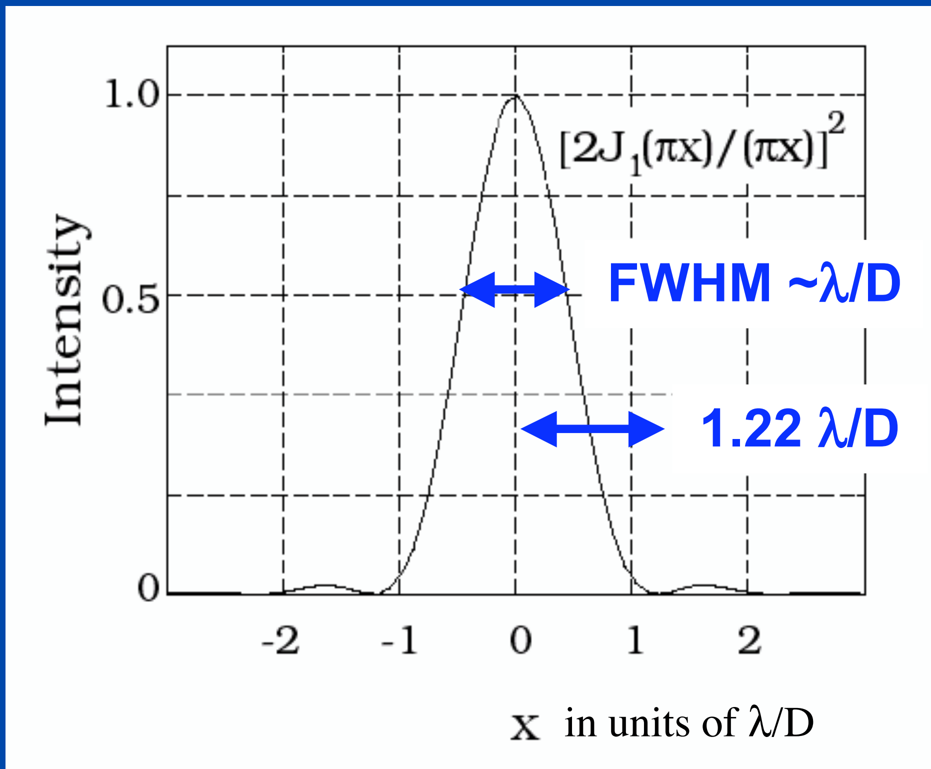
Adaptive Optics

- Technique to “fix” the errors on the wavefront introduced by the atmosphere
- Besides astronomy, also has applications to *vision science* - the study of the living eye.
- Many of the following slides taken from course on Adaptive Optics given by Claire Max from the *Center for Adaptive Optics* at University of California, Santa Cruz (http://www.ucolick.org/~max/289C/)

Angular resolution of telescopes

- Diffraction limit of an 8 m telescope is ~0.01 arcsec in optical and 0.05 arcsec in near-IR
- Atmosphere typically limits this to ~0.5 arcsec
- Important consequences for detecting faint, point-like sources - especially in the IR where the sky background is high.
- Difficult to build seeing-limited instruments for large telescopes

Imaging through a perfect telescope



Point Spread Function (PSF): intensity profile from point source

With no turbulence, FWHM is diffraction limit of telescope, $\vartheta \sim \lambda / D$

Example:

λ / D = 0.02 arc sec for λ = 1 μm, D = 10 m

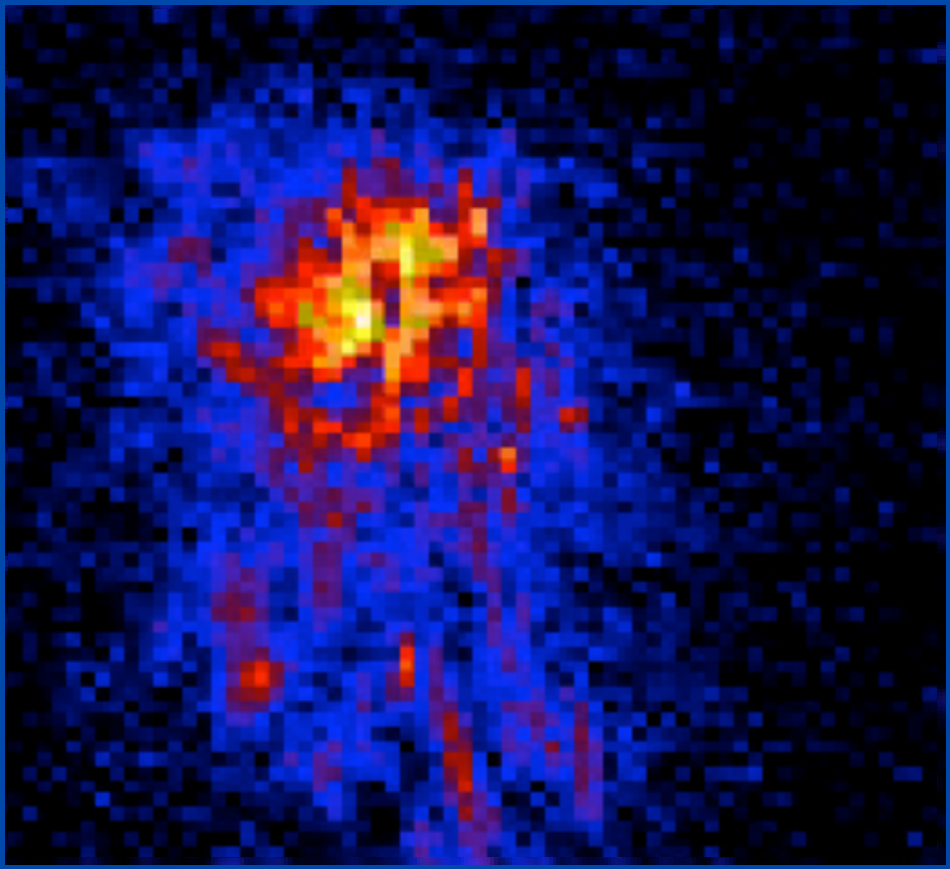
With turbulence, image size gets much larger (typically 0.5 - 2 arc sec)

Credit: Claire E. Max, UCSC

Credit: Claire E. Max, UCSC

Turbulence changes rapidly with time

Image is spread out into speckles



Centroid jumps around (image motion)

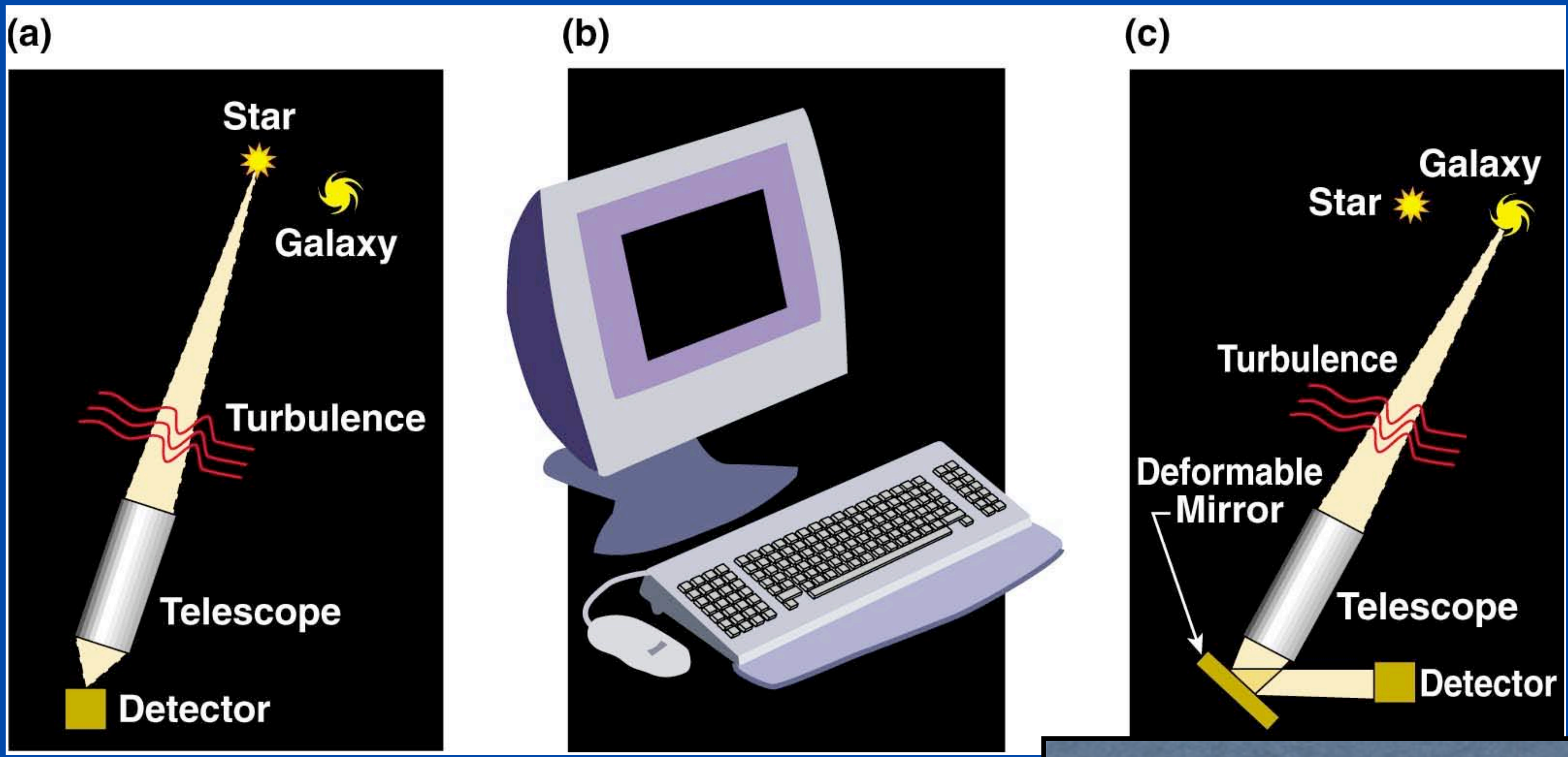
"Speckle images": sequence of short snapshots of a star, taken at Lick Observatory using the IRCAL infra-red camera

How does adaptive optics help? (cartoon approximation)

Measure details of blurring from "guide star" near the object you want to observe

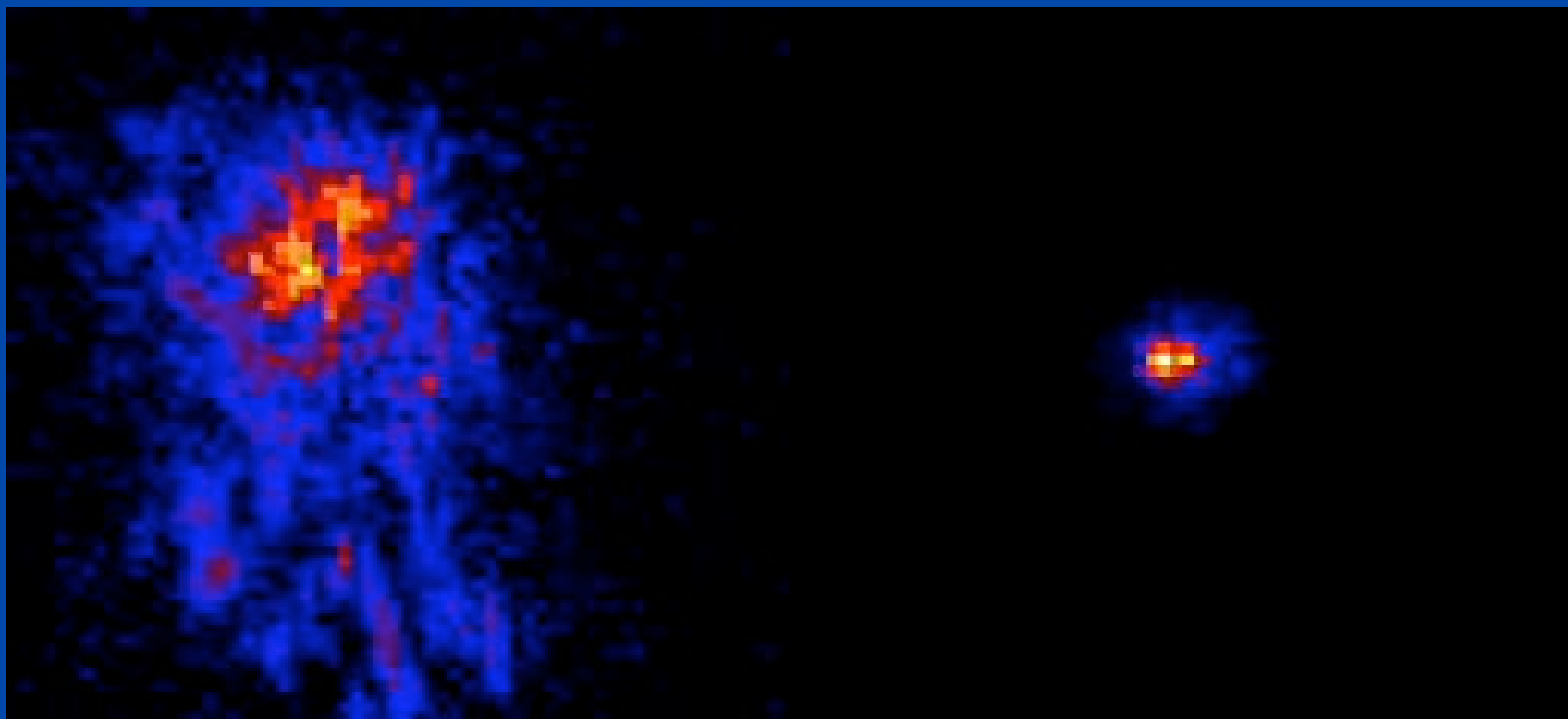
Calculate (on a computer) the shape to apply to deformable mirror to correct blurring

Light from both guide star and astronomical object is reflected from deformable mirror; distortions are removed



Credit: Claire E. Max, UCSC

Infra-red images of a star, from Lick Observatory adaptive optics system



No adaptive optics With adaptive optics

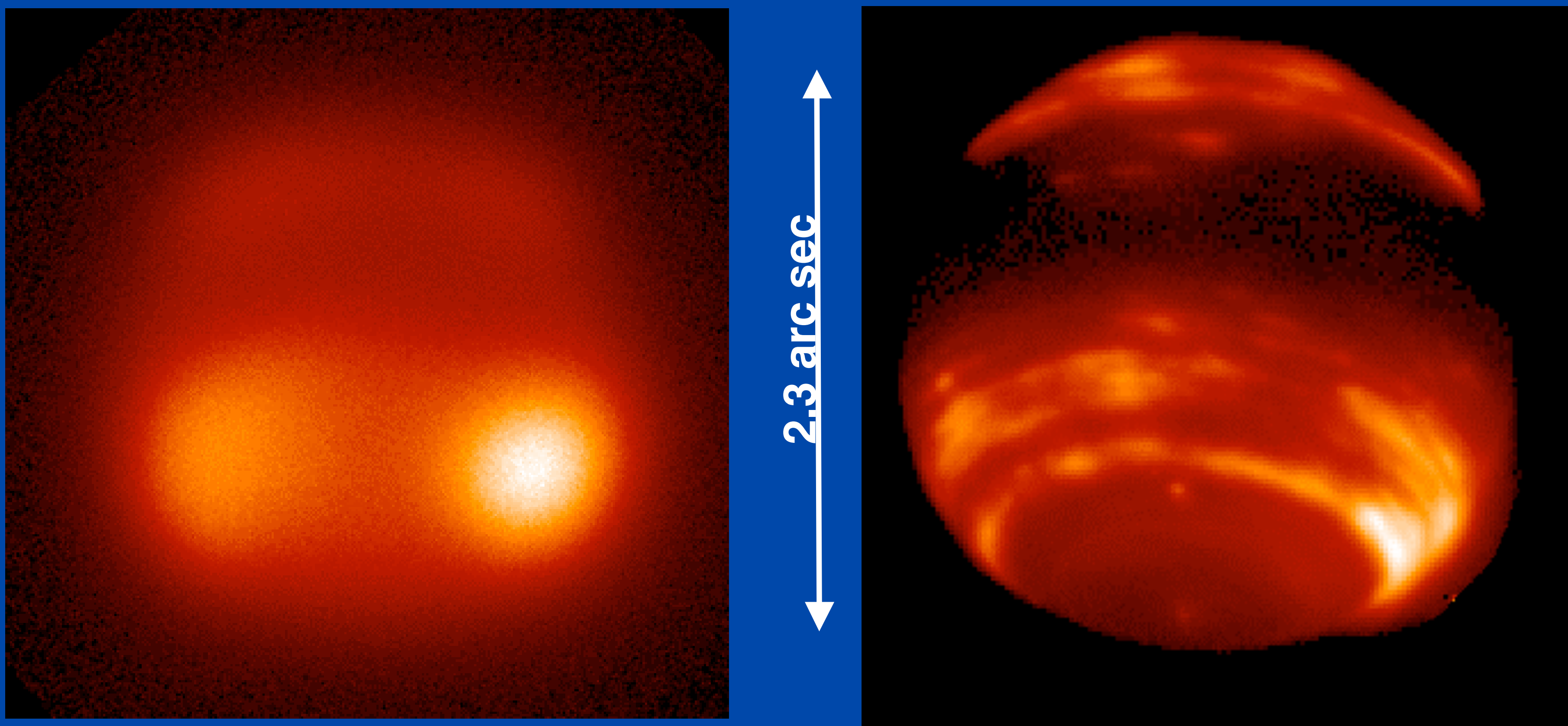
Note: "colors" (blue, red, yellow, white) indicate increasing intensity

Credit: Claire E. Max, UCSC

Neptune in infra-red light (1.65 microns)

Without adaptive optics

With Keck adaptive optics

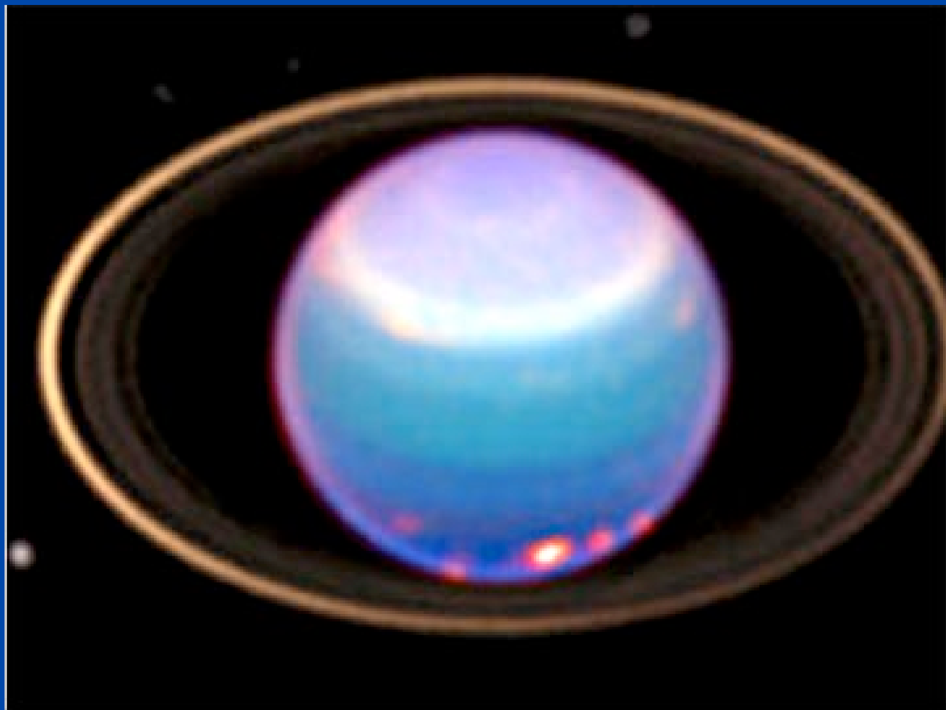


May 24, 1999

June 27, 1999

Credit: Claire E. Max, UCSC

Uranus with Hubble Space Telescope and Keck AO



HST, Visible

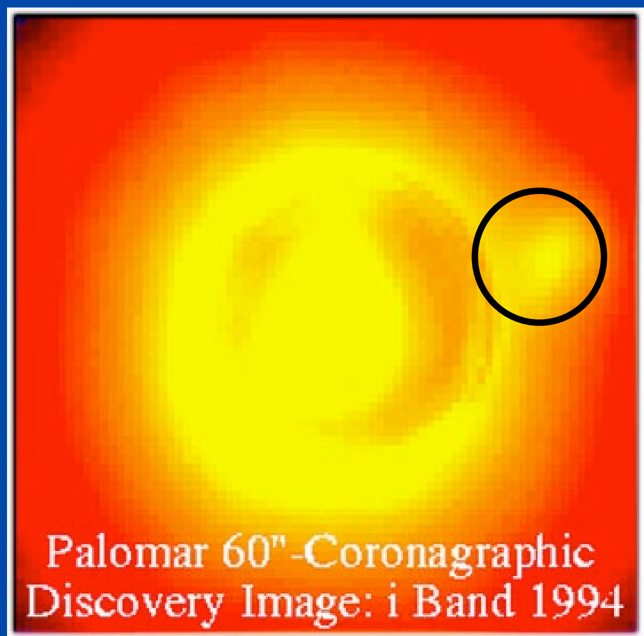
Keck AO, IR

Lesson: Keck in near IR has ~ same resolution as Hubble in visible

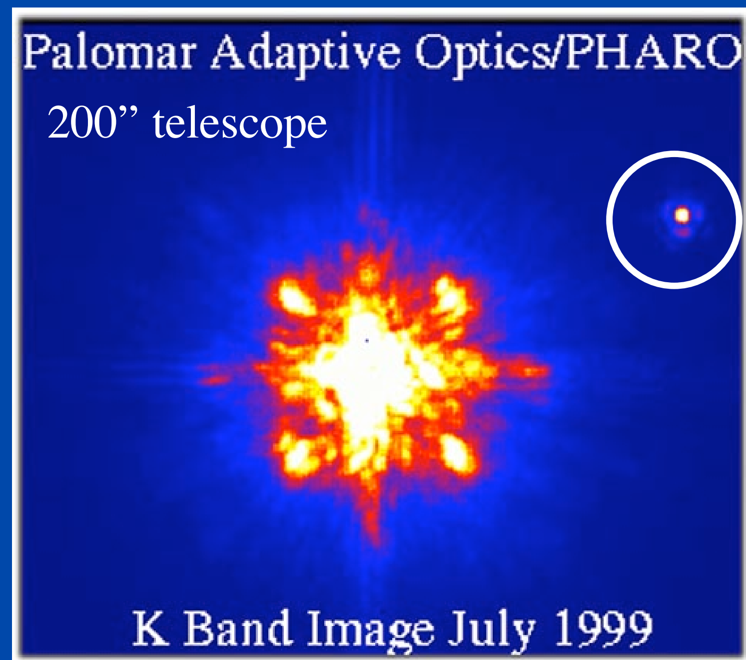
Credit: Claire E. Max, UCSC

Adaptive optics makes it possible to find faint companions around bright stars

Two images from Palomar of a brown dwarf companion to GL 105



No AO



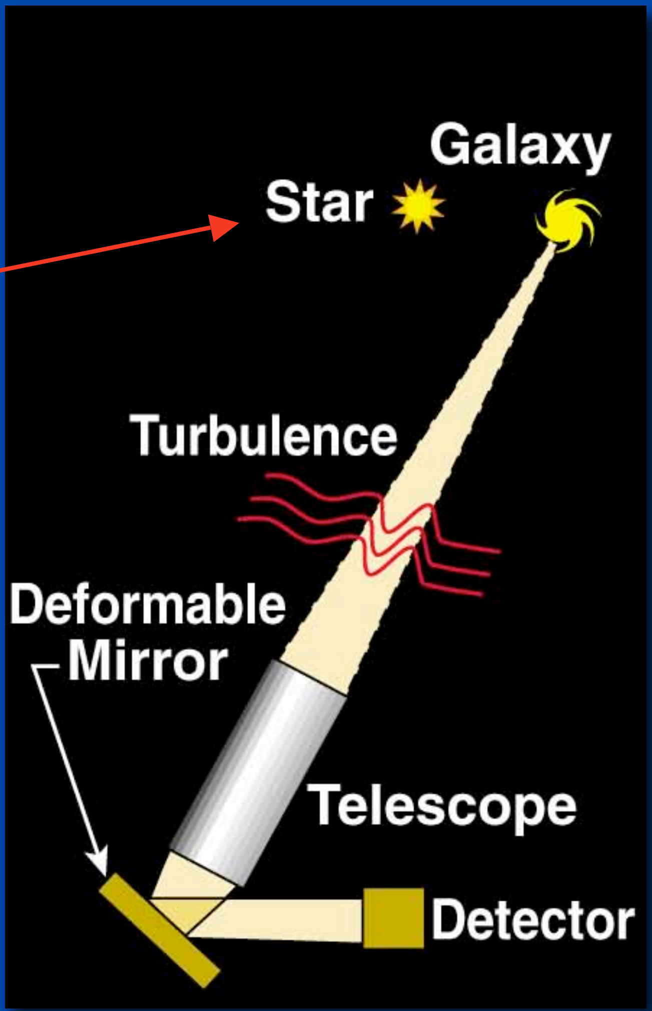
With AO

Credit: David Golimowski

Credit: Claire E. Max, UCSC

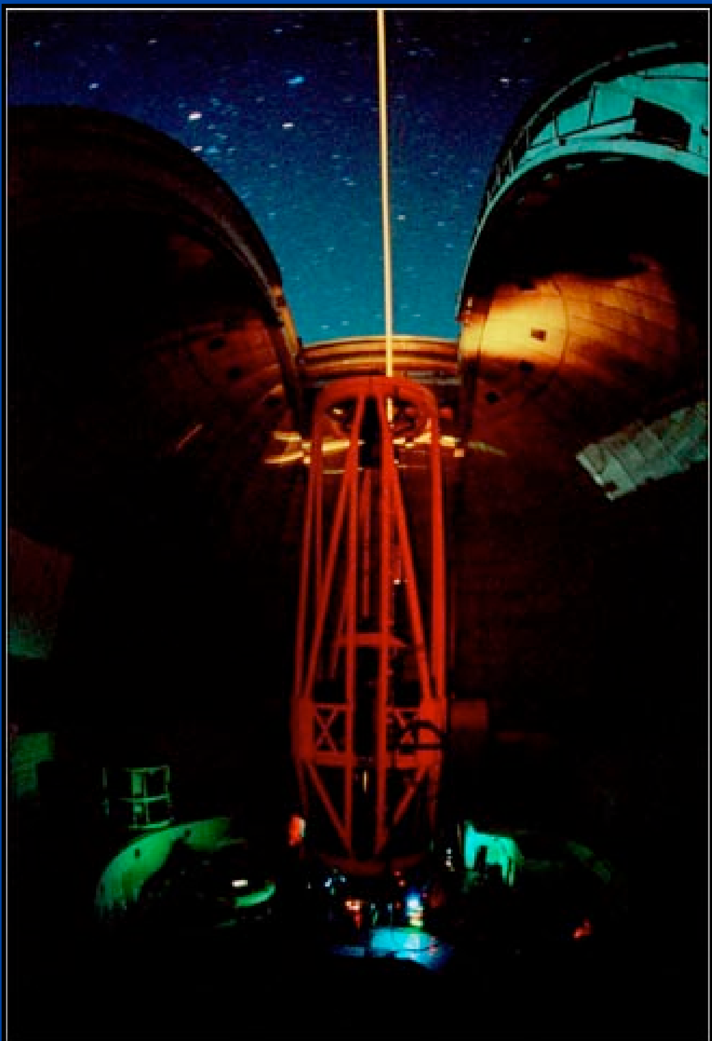
If there's no close-by "real" star, create one with a laser

- Use a laser beam to create artificial "star" at altitude of 100 km in atmosphere



Credit: Claire E. Max, UCSC

Laser guide stars are operating at Lick, Keck, Gemini North, VLT Observatories



Lick Observatory

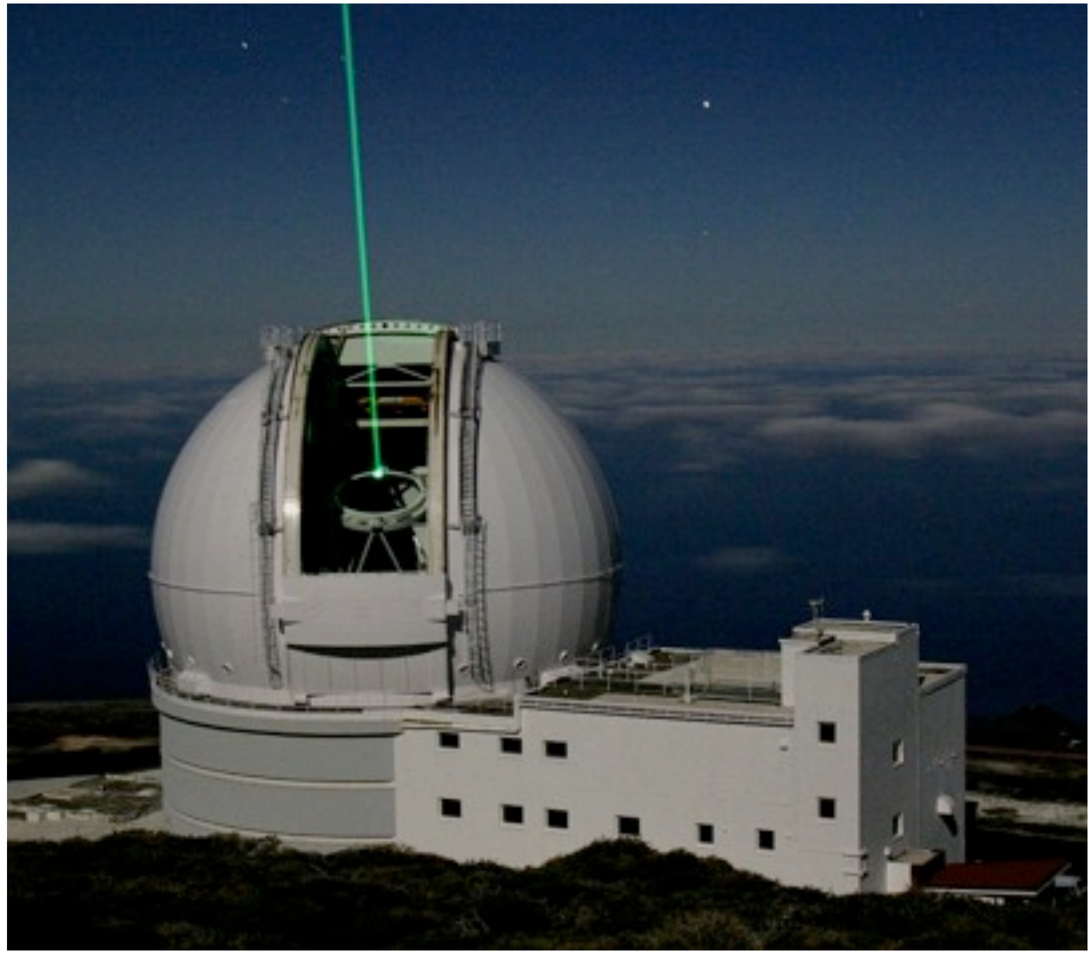
Keck Observatory

Credit: Claire E. Max, UCSC

VLT, Paranal, Chile

First Light of the VLT Laser Giude Star

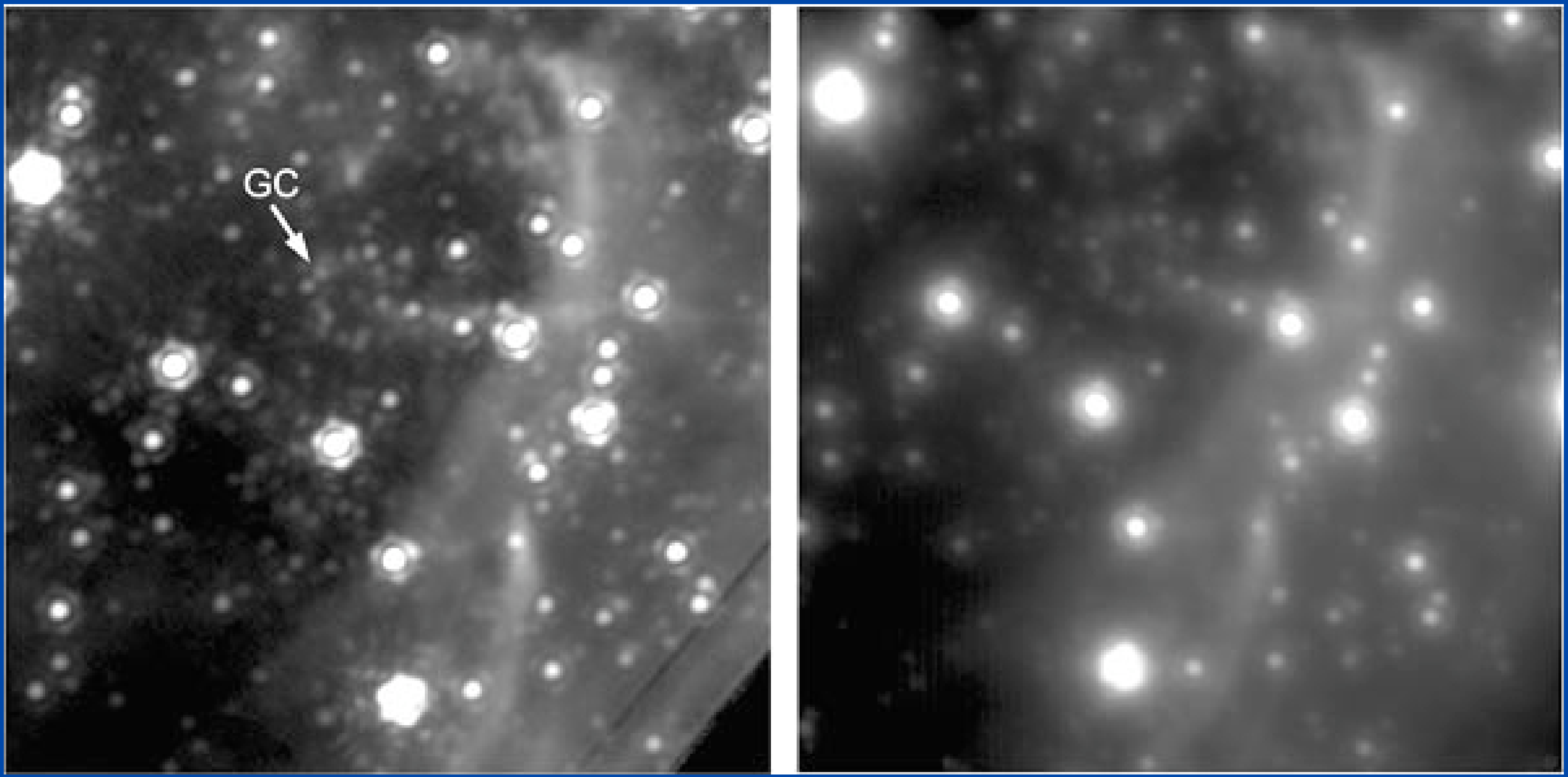
ESO PR Photo 07a/06 (23 February 2006) © ESO



William Herschel Telescope, La Palma

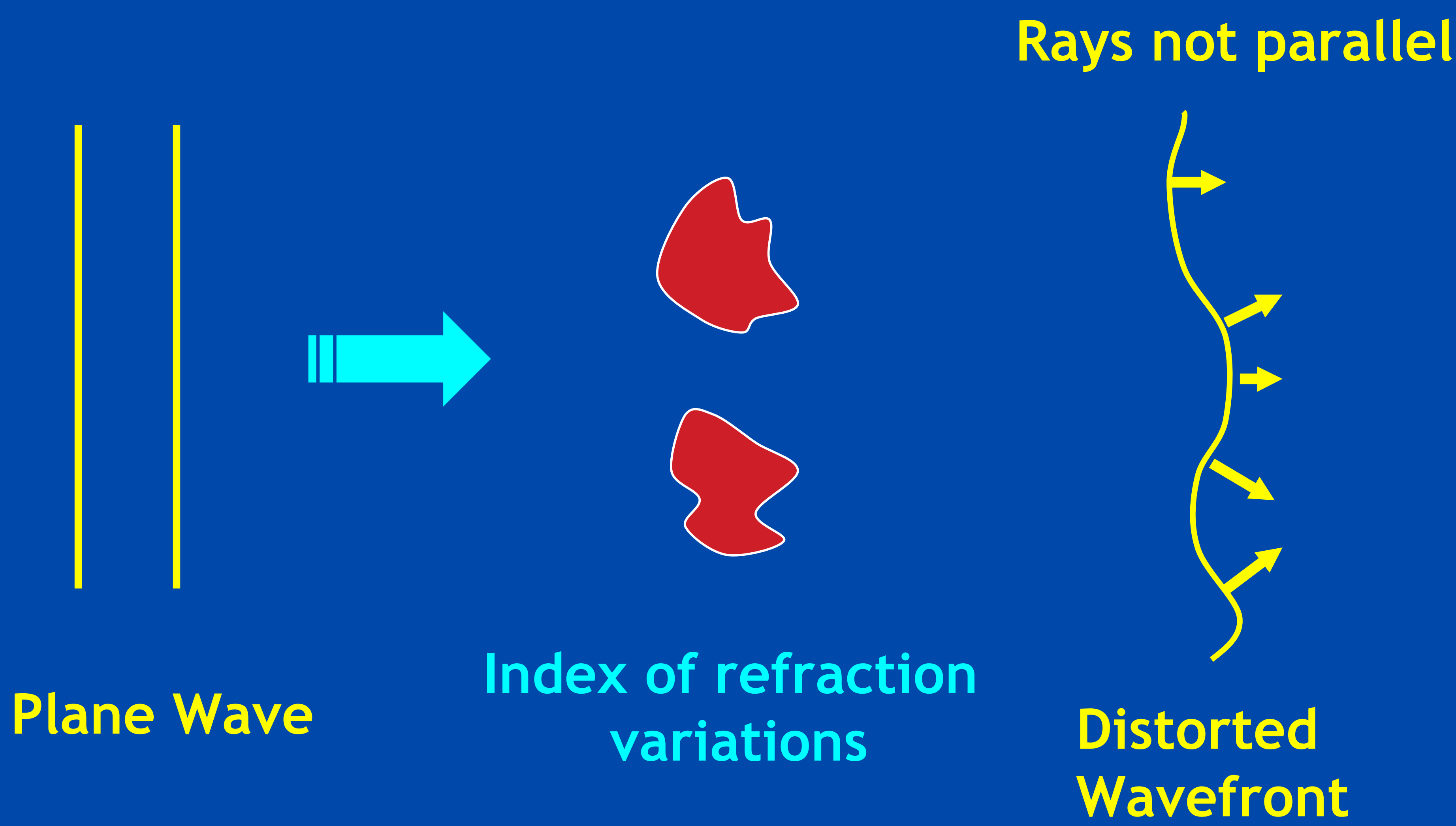
Galactic Center with Keck laser guide star

Keck laser guide star AO   Best natural guide star AO



Credit: Claire E. Max, UCSC

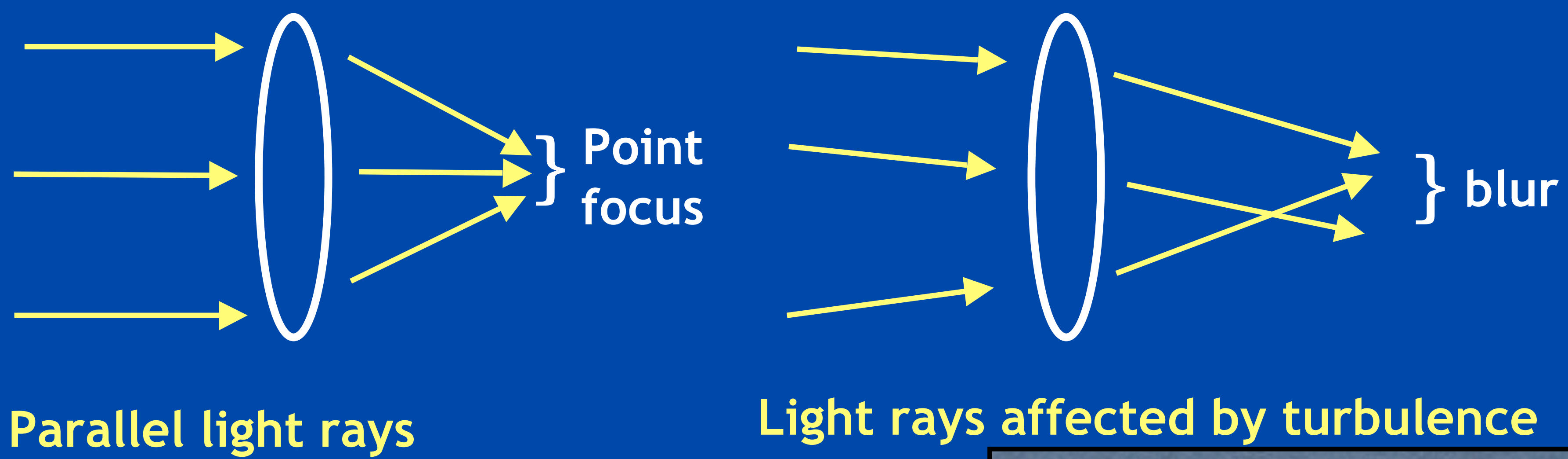
Atmospheric perturbations cause distorted wavefronts



Credit: Claire E. Max, UCSC

Optical consequences of turbulence

- Temperature fluctuations in small patches of air cause changes in index of refraction (like many little lenses)
- Light rays are refracted many times (by small amounts)
- When they reach telescope they are no longer parallel
- Hence rays can't be focused to a point:



Credit: Claire E. Max, UCSC

Fluctuations in index of refraction are due to temperature fluctuations

- **Refractivity of air**

$$N \equiv (n-1)\times 10^{6} = 77.6\left(1+\frac{7.52\,10^{-3}}{\lambda^{2}}\right)\times\left(\frac{P}{T}\right)$$

where P = pressure in millibars, T = temp. in K, λ in microns
n **= index of refraction. Note VERY weak dependence on λ**

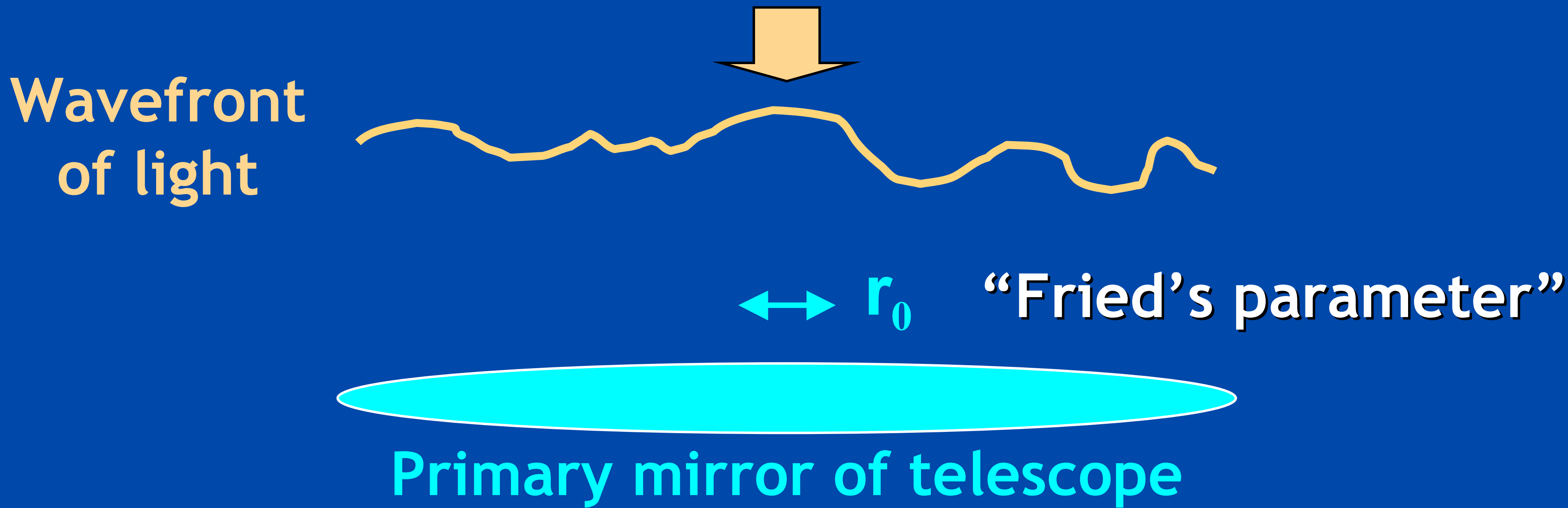
- **Temperature fluctuations → index fluctuations**

$$\delta N \cong -77.6\times(P/T^{2})\,\delta T$$

(pressure is constant, because velocities are highly sub-sonic -- pressure differences are rapidly smoothed out by sound wave propagation)

Credit: Claire E. Max, UCSC

Characterize turbulence strength by quantity r_0

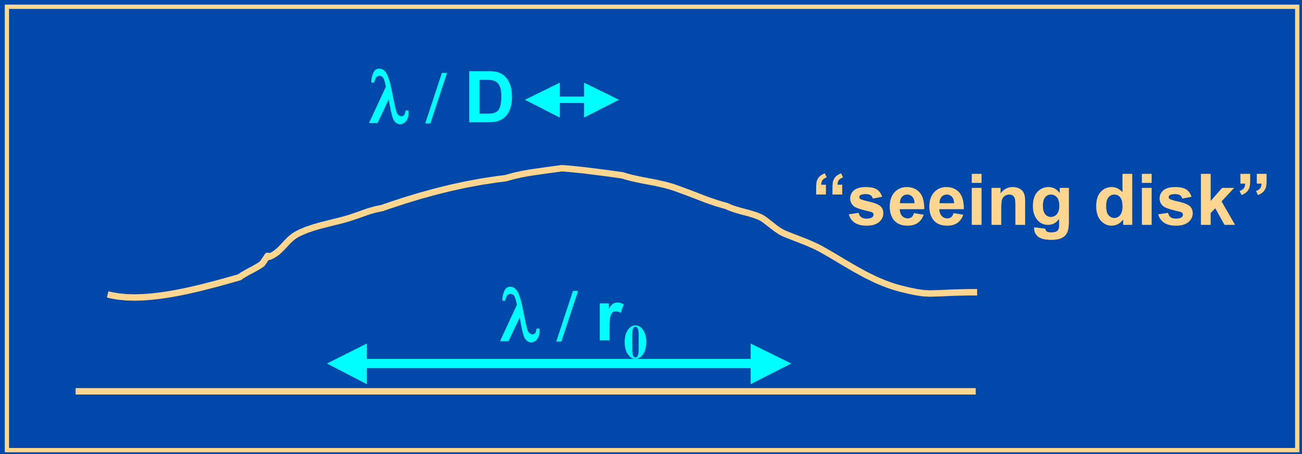


- "Coherence Length" r_0 : distance over which optical phase distortion has mean square value of 1 rad^2 (r_0 ~ 15 - 30 cm at good observing sites)
- Easy to remember: r_0 = 10 cm ⇔ FWHM = 1 arc sec at λ = 0.5μm

Credit: Claire E. Max, UCSC

Effect of turbulence on image size

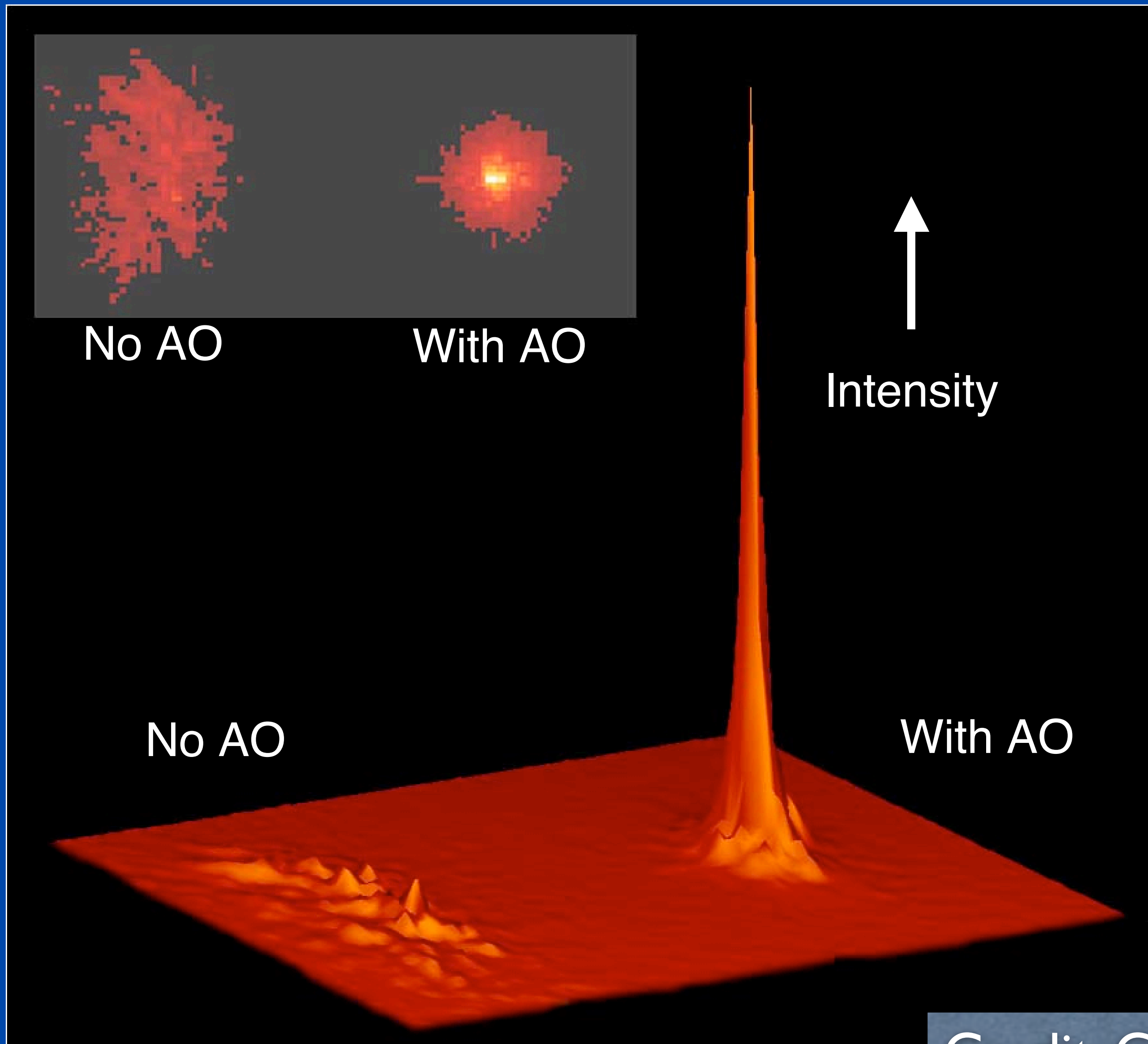
- If telescope diameter D >> r_0 , image size of a point source is $\lambda / r_0 >> \lambda / D$



- r_0 is diameter of the circular pupil for which the diffraction limited image and the seeing limited image have the same angular resolution.
- $r_0 \approx$ 10 inches at a good site. So any telescope larger than this has no better spatial resolution!

Credit: Claire E. Max, UCSC

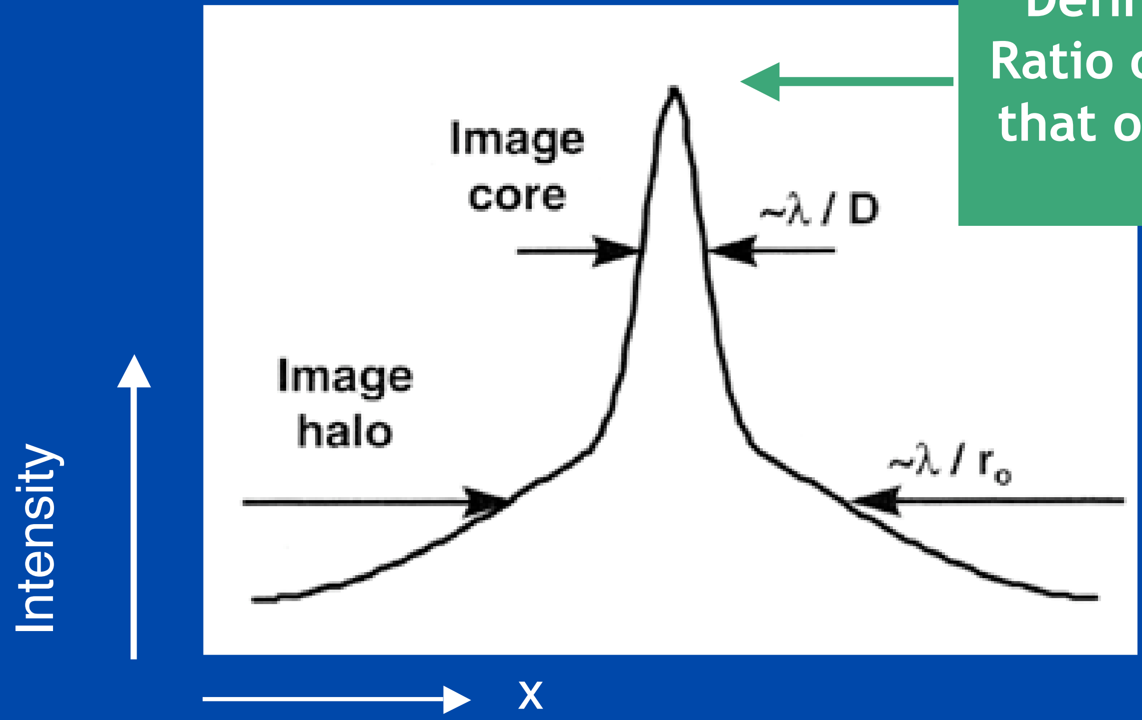
Adaptive optics increases peak intensity of a point source



Lick Observatory

Credit: Claire E. Max, UCSC

AO produces point spread functions with a "core" and "halo"



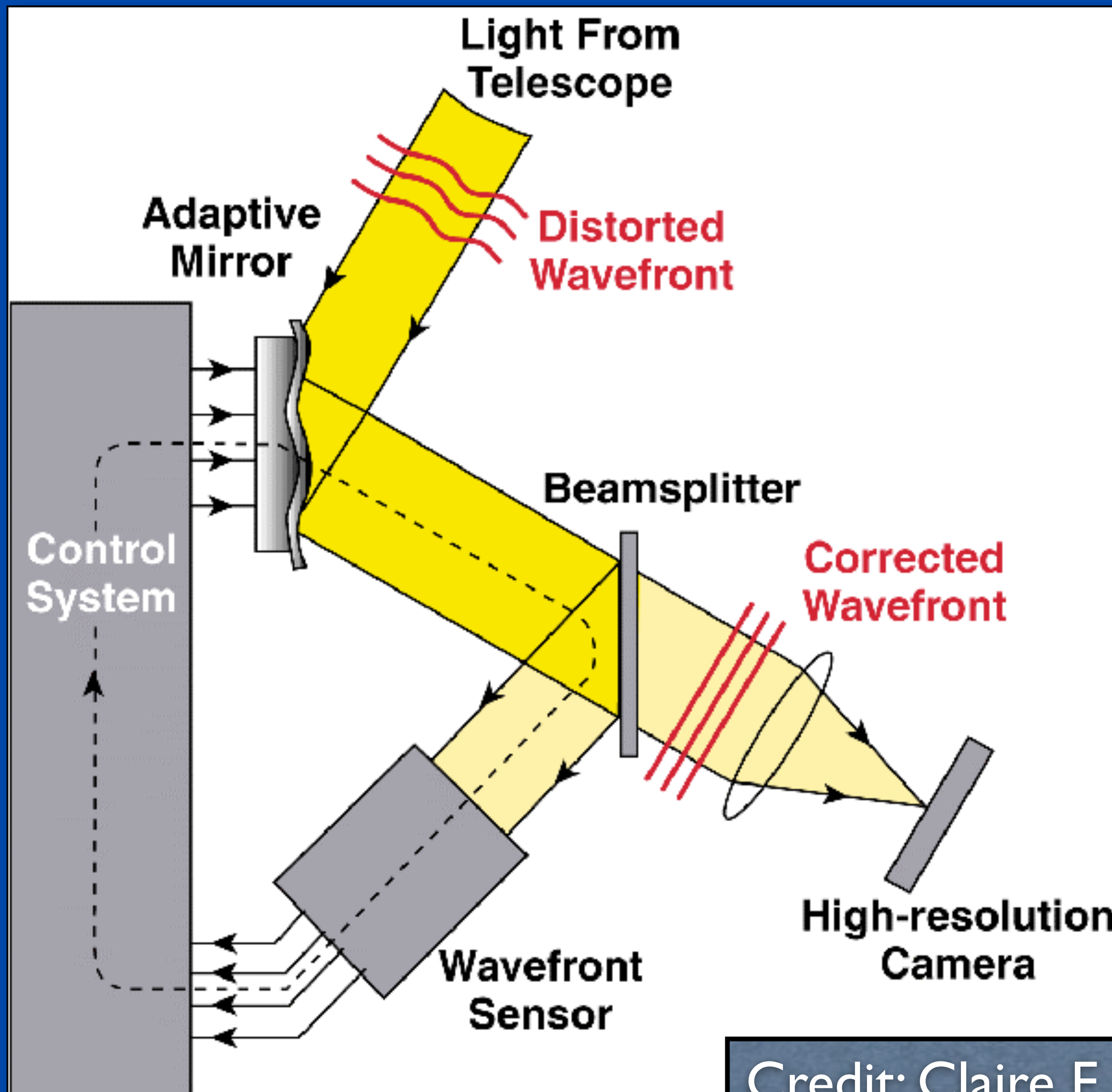
Definition of "Strehl": Ratio of peak intensity to that of "perfect" optical system

- When AO system performs well, more energy in core
- When AO system is stressed (poor seeing), halo contains larger fraction of energy (diameter ~ r_0)
- Ratio between core and halo varies during night

Credit: Claire E. Max, UCSC

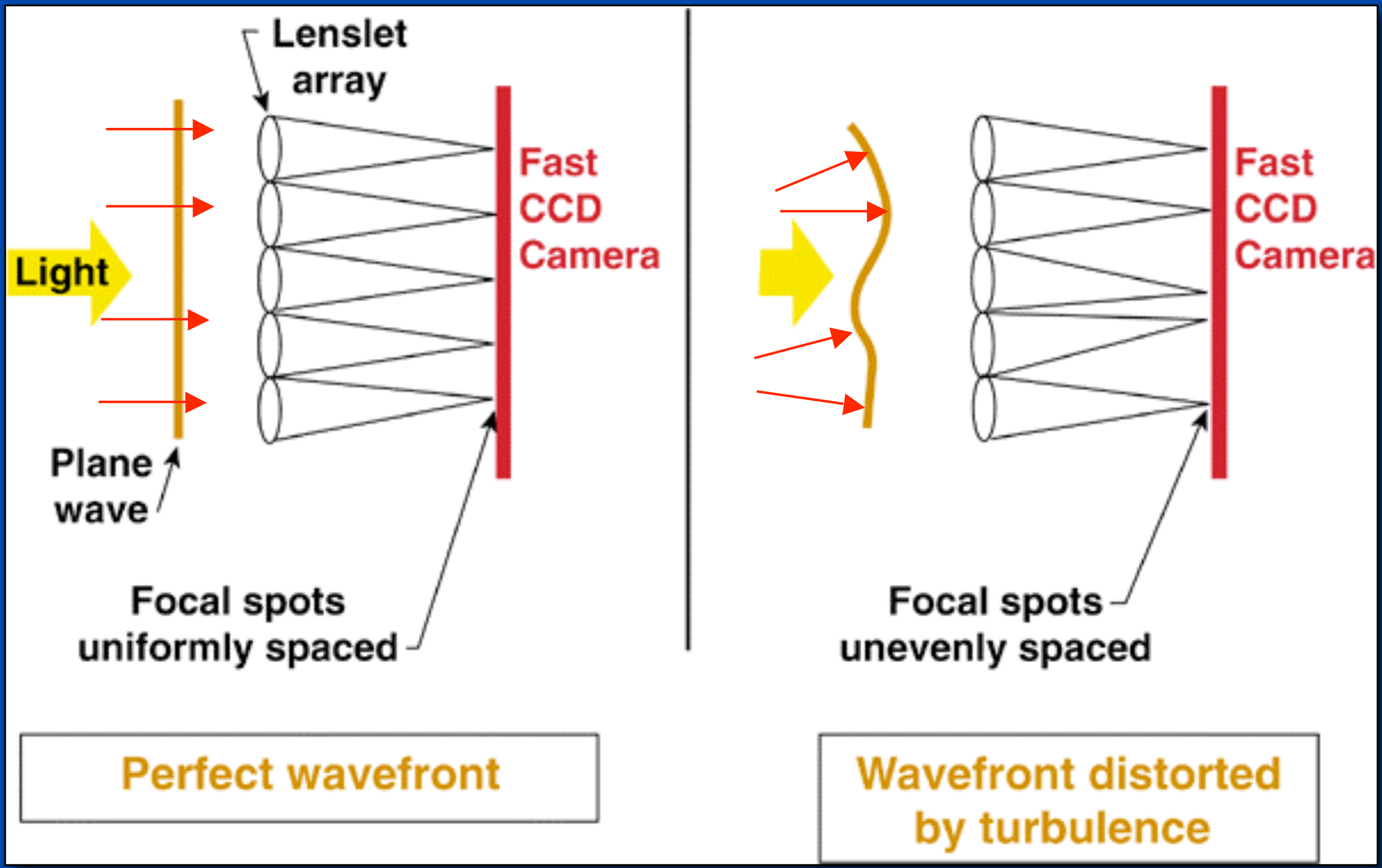
Schematic of adaptive optics system

Feedback loop: next cycle corrects the (small) errors of the last cycle



Credit: Claire E. Max, UCSC

How to measure turbulent distortions (one method among many)



Shack-Hartmann wavefront sensor

Credit: Claire E. Max, UCSC

Shack-Hartmann wavefront sensor measures local "tilt" of wavefront

- Divide pupil into subapertures of size ~ r_0
  - Number of subapertures α $(D / r_0)^2$

- Lenslet in each subaperture focuses incoming light to a spot on the wavefront sensor's CCD detector

- Deviation of spot position from a perfectly square grid measures shape of incoming wavefront

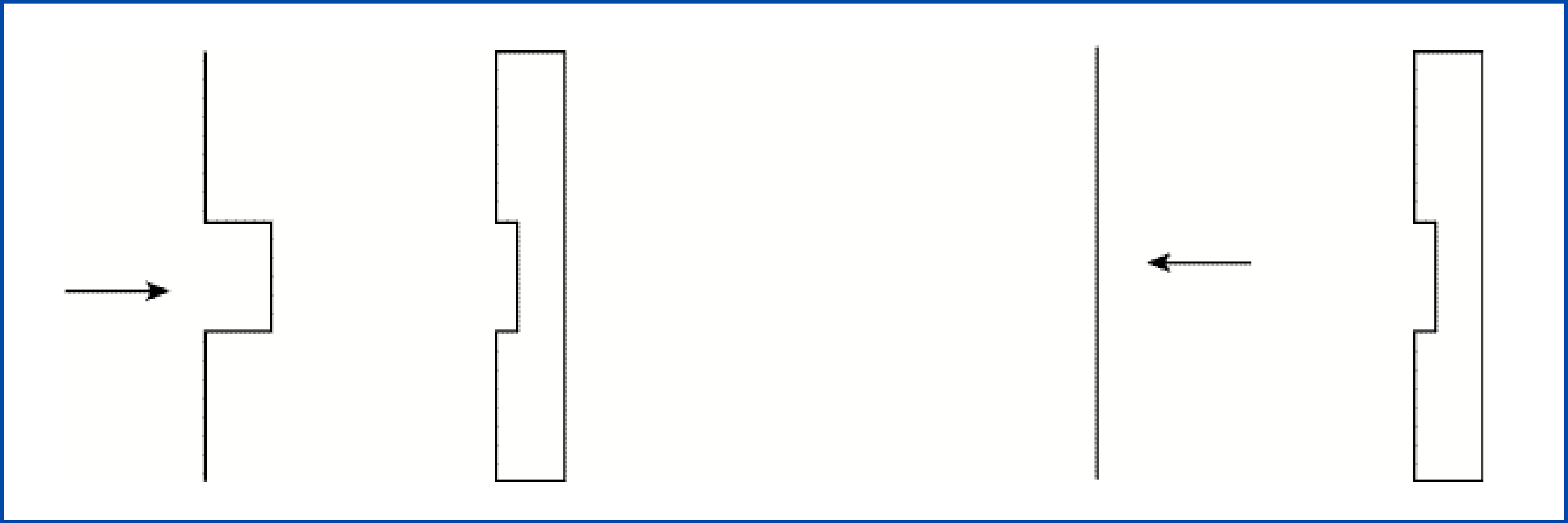
- Wavefront reconstructor computer uses positions of spots to calculate voltages to send to deformable mirror

Credit: Claire E. Max, UCSC

How a deformable mirror works (idealization)

BEFORE

AFTER



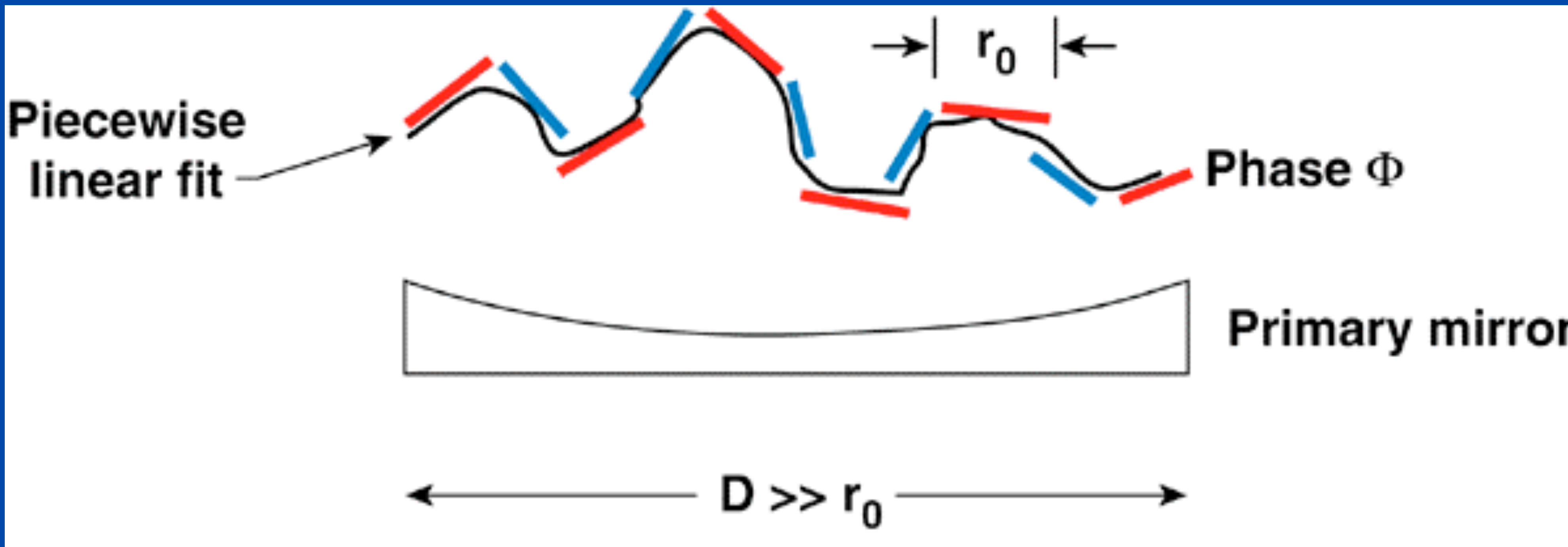
Incoming Wave with Aberration

Deformable Mirror

Corrected Wavefront

Credit: Claire E. Max, UCSC

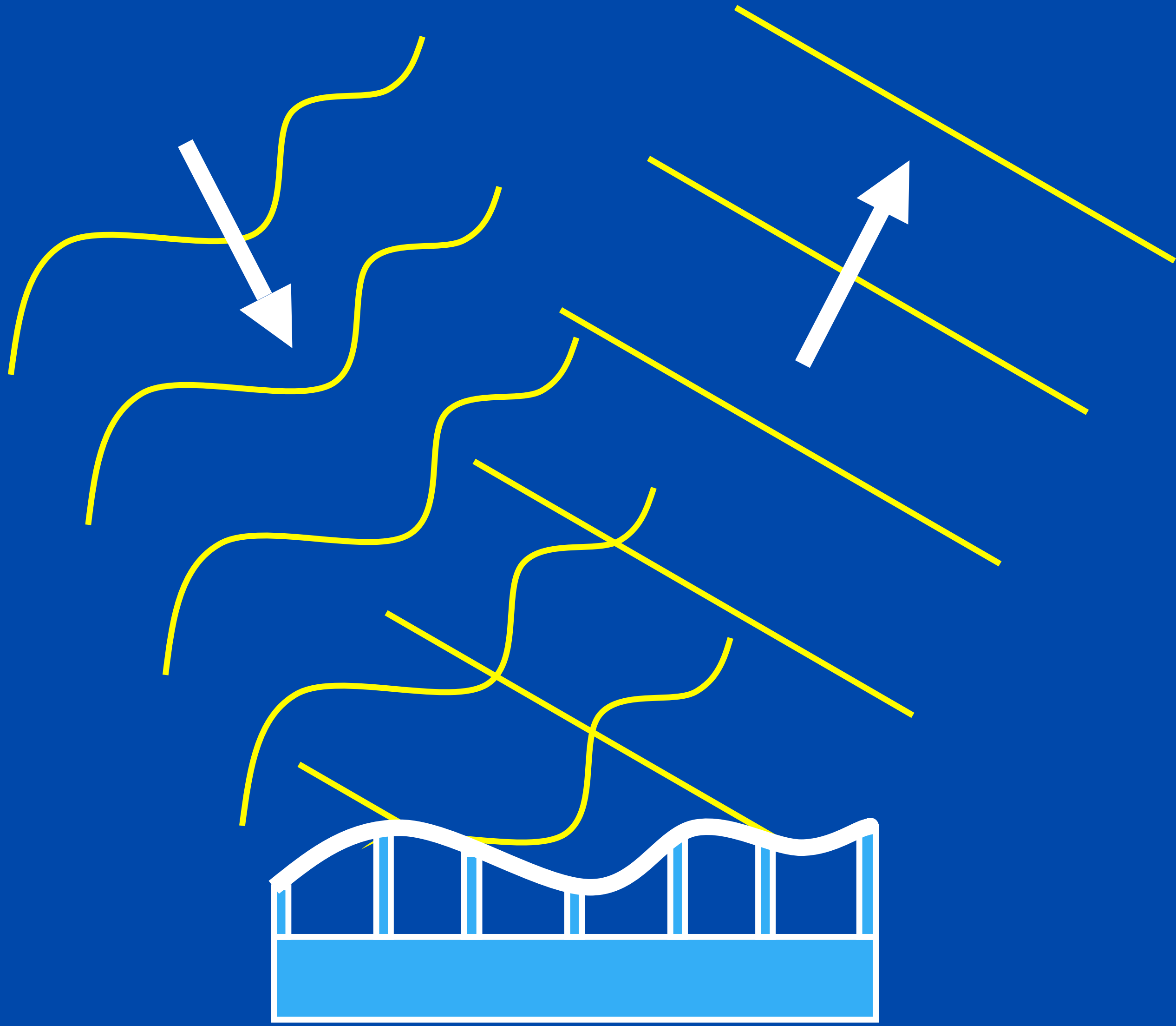
Real deformable mirrors have smooth surfaces



- In practice, a small deformable mirror with a thin bendable face sheet is used
- Placed <u>after</u> the main telescope mirror

Credit: Claire E. Max, UCSC

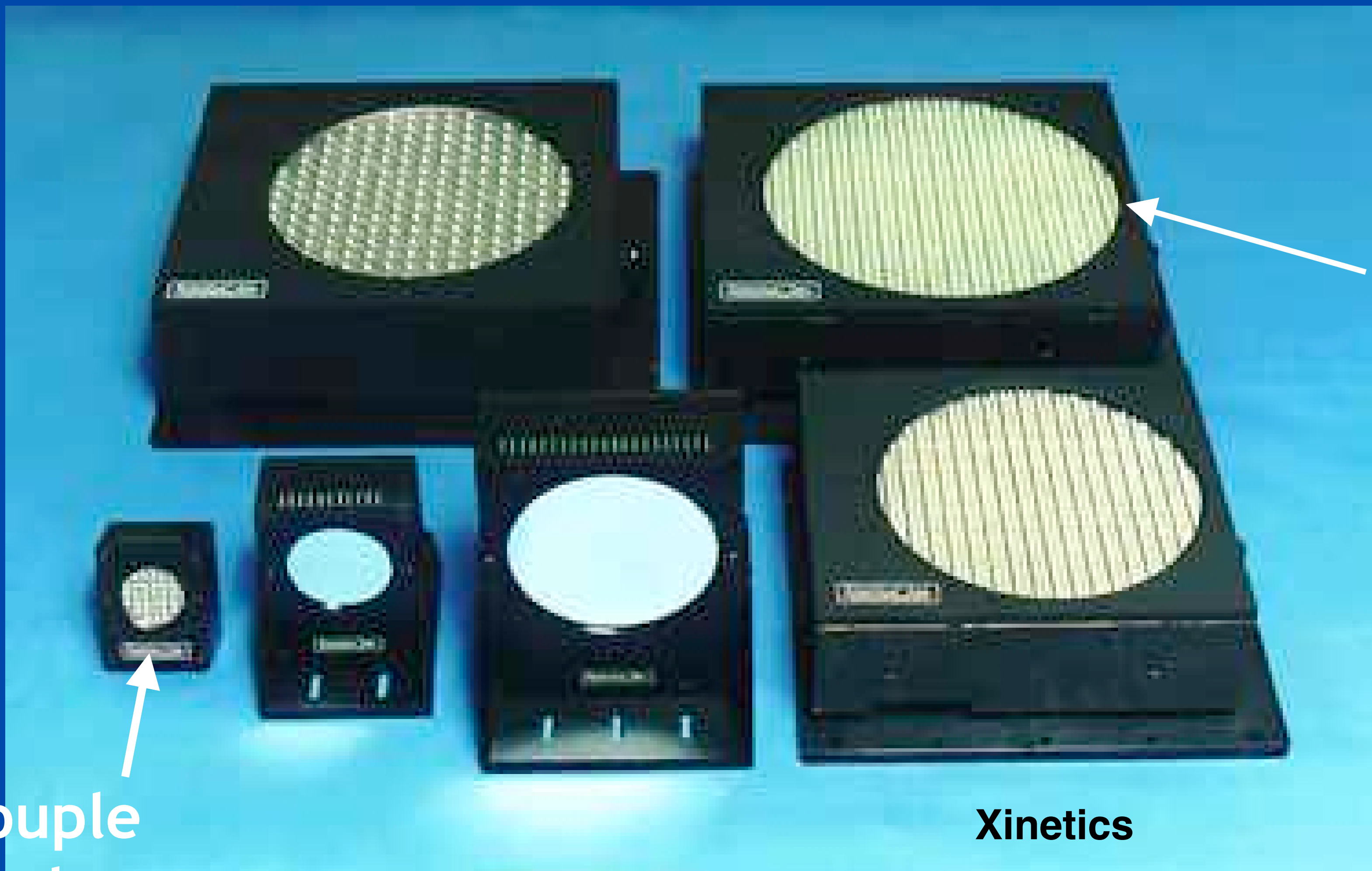
Deformable Mirror for real wavefronts



Credit: Claire E. Max, UCSC

Deformable mirrors come in many sizes

- Range from 13 to > 900 actuators (degrees of freedom)



Credit: Claire E. Max, UCSC

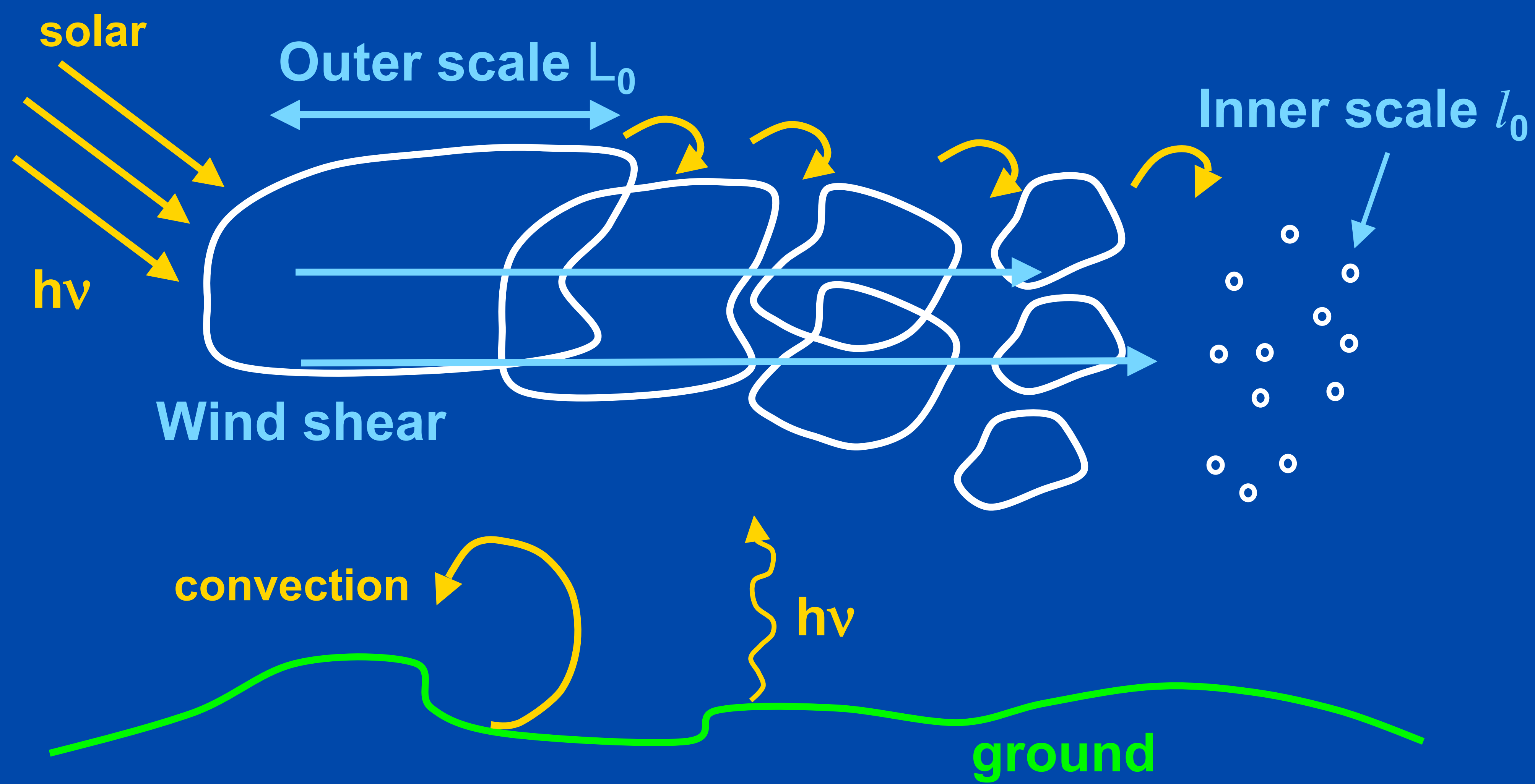
What determines r_0 and hence the number of actuators needed?

Leonardo da Vinci's view of turbulence

Drawing of a turbulent flow by Leonardo da Vinci (1452–1519), who recognized that turbulence involves a multitude of eddies at various scales.

Credit: Claire E. Max, UCSC

Kolmogorov turbulence, cartoon



Credit: Claire E. Max, UCSC

Kolmogorov turbulence, in words

- Assume energy is added to system at largest scales - "outer scale" L_0
- Then energy cascades from larger to smaller scales (turbulent eddies "break down" into smaller and smaller structures).
- Size scales where this takes place: "Inertial range".
- Finally, eddy size becomes so small that it is subject to dissipation from viscosity. "Inner scale" l_0
- L_0 ranges from 10's to 100's of meters; l_0 is a few mm

Credit: Claire E. Max, UCSC

The Kolmogorov turbulence model, derived from dimensional analysis

- u = velocity, $\dot{\varepsilon}$ = energy dissipation rate per unit mass, ν = viscosity, l_0 = inner scale, ℓ = local spatial scale
- Energy/mass = $u^2/2 \propto u^2$
- Energy dissipation *rate* per unit mass:

$$\dot{\varepsilon} \sim \frac{u^2}{t} \sim \frac{u^2}{(l/u)} = \frac{u^3}{l}$$

Solve for u: $u \sim (\dot{\varepsilon} l)^{1/3}$

Energy per unit mass $\propto u^2 \sim (\dot{\varepsilon} l)^{2/3}$

Credit: Claire E. Max, UCSC

Derive the Kolmogorov Power Spectrum using dimensional analysis

- 1-D power spectrum of velocity fluctuations: $k = 2\pi / \ell$

$$\Phi(k)\Delta k \propto u^2 \propto \left(\dot{\varepsilon} l\right)^{2/3} \propto \dot{\varepsilon}^{2/3} k^{-2/3}$$

For dimensional analysis, divide by k:

$$\Phi(k) \sim k^{-5/3}$$

- 3-D power spectrum: divide by k^3

$$\Phi(k) k^2 \Delta k \propto \dot{\varepsilon}^{2/3} k^{-2/3}$$

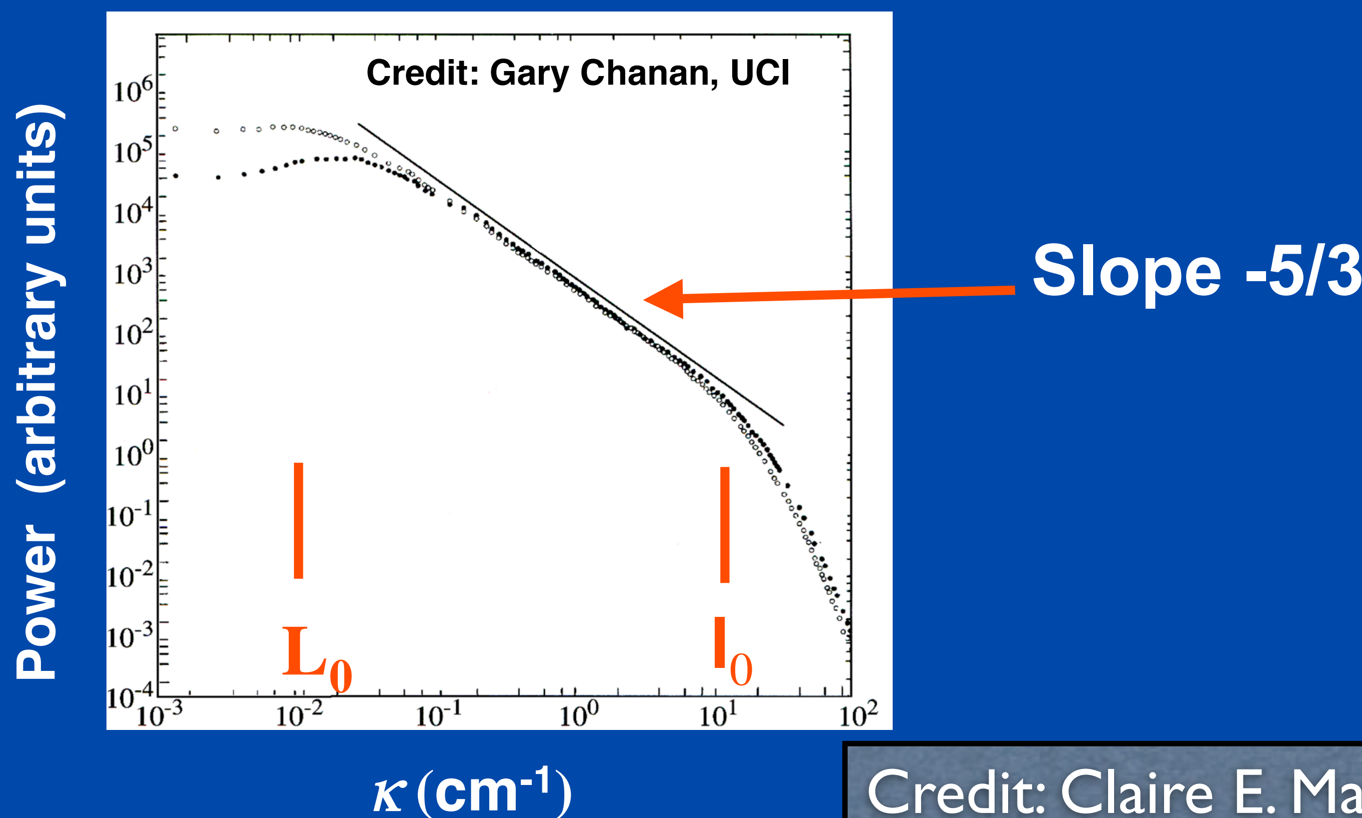
$$\Phi(k) \sim k^{-11/3}$$

- For rigorous calculation: see V. I. Tatarski, 1961, “Wave Propagation in a Turbulent Medium”, McGraw-Hill, NY

Credit: Claire E. Max, UCSC

Lab experiments agree

- Air jet, 10 cm diameter (Champagne, 1978)
- Assumptions: turbulence is homogeneous, isotropic, stationary in time



Credit: Claire E. Max, UCSC

Structure functions are used a lot in AO. What are they?

- Mean values of meteorological variables change over minutes to hours. Examples: T, p, humidity
- If $f(t)$ is a non-stationary random variable,

  $F_t(\tau) = f(t + \tau) - f(t)$ is a difference function that is stationary for small τ.

- Structure function is measure of intensity of fluctuations of $f(t)$ over a time scale $\leq \tau$:

$$D_f(\tau) = \langle [F_t(\tau)]^2 \rangle = \langle [f(t + \tau) - f(t)]^2 \rangle$$

Credit: Claire E. Max, UCSC

Structure function for atmospheric fluctuations

- Scaling law we derived earlier for Kolmogorov turbulence:

$$u^2 \sim \dot{\varepsilon}^{2/3} \ell^{2/3} \sim r^{2/3}$$

- Heuristic derivation: Velocity structure function ~ u^2

$$D_u(r) = \langle [\, u(x) - u(x + r)\,]^2 \rangle = C_u^2 \; r^{2/3}$$

here C_u^2 = constant to clean up the "look" of the equation

Credit: Claire E. Max, UCSC

What about temperature and index of refraction fluctuations?

- Temperature fluctuations are carried around passively by the velocity field (for incompressible fluids).
- So *T* and *N* have structure functions similar to *u* :

$$D_T(r) = \langle [\, T(x) - T(x+r) \,]^2 \rangle = C_T^2 \; r^{2/3}$$

$$D_N(r) = \langle [\, N(x) - N(x+r) \,]^2 \rangle = C_N^2 \; r^{2/3}$$

Credit: Claire E. Max, UCSC

Atmospheric Turbulence: Main Points

- The dominant locations for index of refraction fluctuations that affect astronomers are the atmospheric boundary layer and the tropopause
- Atmospheric turbulence (mostly) obeys Kolmogorov statistics
- Kolmogorov turbulence is derived from dimensional analysis (heat flux in = heat flux in turbulence)
- Structure functions derived from Kolmogorov turbulence are $\propto r^{2/3}$
- All else will follow from these points!

Credit: Claire E. Max, UCSC

Several equivalent meanings for r_0

- Define r_0 as telescope diameter where optical transfer functions of the telescope and atmosphere are equal
- r_0 is separation on the telescope primary mirror where phase correlation has fallen by 1/e
- $(D/r_0)^2$ is approximate number of speckles in short-exposure image of a point source
- D/r_0 sets the required number of degrees of freedom of an AO system
- Can you think of others?

Credit: Claire E. Max, UCSC

From Structure Functions to r_0

- Use that the change in phase of a wave propagating through medium with refractive index *n(**x**,h)* is

$$\Delta\phi(\mathbf{x}) = k \int_h^{h+\Delta h} n(\mathbf{x}, h)\, dh \qquad k \equiv \frac{2\pi}{\lambda}$$

- Knowing $D_n(r) = C_n^2\, r^{2/3}$, it follows (OA sec 4.4.1) that the structure function for the *phase* is

$$D_\phi(r) = 2.914 k^2 r^{5/3} \int_0^\infty C_n^2(h) dh$$

so

$$r \propto D_\phi^{3/5}\, k^{-6/5} \left(\int_0^\infty C_n^2 dh \right)^{-3/5} \propto D_\phi^{3/5}\, \lambda^{6/5} \left(\int_0^\infty C_n^2 dh \right)^{-3/5}$$

The Fried Parameter r_0

For Kolmogorov turbulence, we then have:

$$r_0 \propto \lambda^{6/5} (\sec z)^{-3/5} \left(\int_0^H C_N^2(h) dh \right)^{-3/5}$$

where $\sec z \equiv 1/\cos z$

and $C_N^2 = (77.6P/T^2)^2 C_T^2$

is an indicator of the "turbulence strength" and follows from amplitude of temperature variations, C_T^2.

Note: Eq. 1.1.73 in Kitchin for r_0 is incorrect.

The Fried Parameter

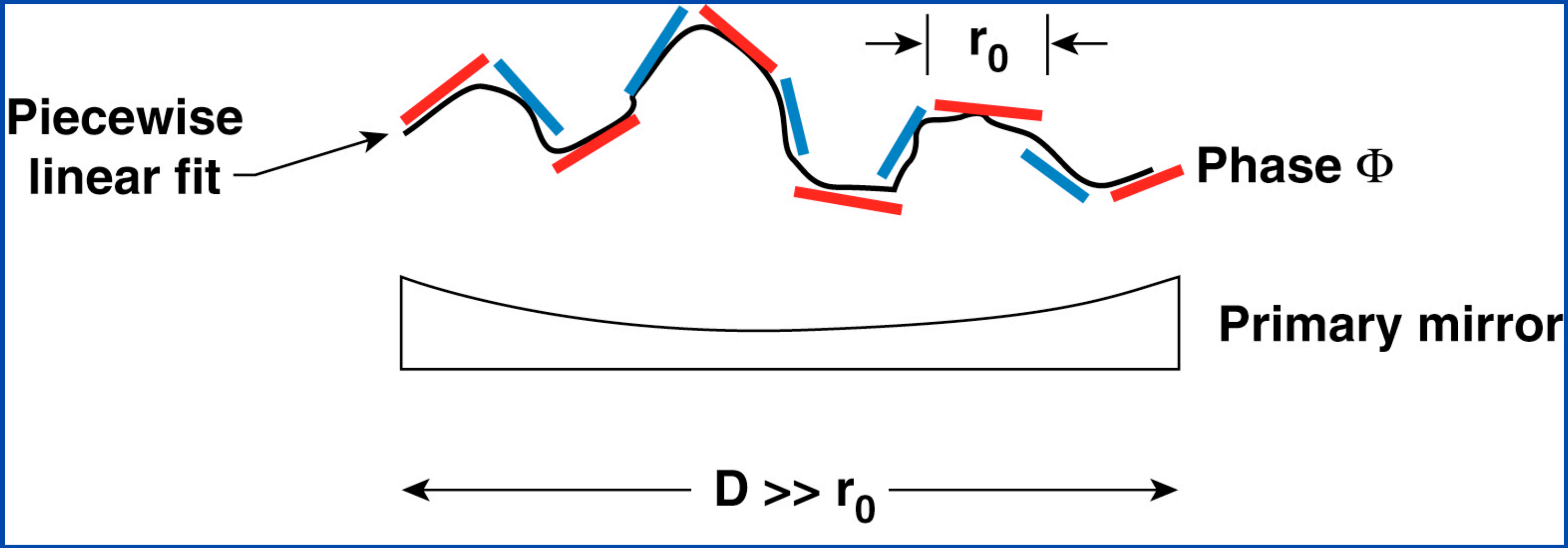
- Fried coherence length r_0 scales with:
  - wavelength: $r_0 \sim \lambda^{6/5}$
  - Inversely with airmass: $r_0 \sim \sec z^{-3/5}$
  - "Strength of turbulence" (integral of C_N^2 from ground to "top" of atmosphere)
- r_0 larger (AO correction easier) at
  - longer wavelengths
  - near zenith

Typical values of r_0

- Usually r_0 is given at a 0.5 micron wavelength for reference purposes. It's up to you to scale it by $\lambda^{-1.2}$ to evaluate r_0 at your favorite wavelength.
- At excellent sites such as Mauna Kea in Hawaii, r_0 at 0.5 micron is 10 - 30 cm. But there is a big range from night to night, and at times also within a night.

Credit: Claire E. Max, UCSC

r_0 sets the number of degrees of freedom of an AO system



- Divide primary mirror into "subapertures" of diameter r_0
- Number of subapertures ~ $(D / r_0)^2$ where r_0 is evaluated at the desired observing wavelength
- Example: Keck telescope, D=10m, r_0 ~ 60 cm at λ = 2 μm. $(D / r_0)^2$ ~ 280. Actual # for Keck : ~250.

Credit: Claire E. Max, UCSC

Next: All sorts of good things come from knowing r_0

- Timescales of turbulence

- Isoplanatic angle: AO performance degrades as astronomical targets get farther from guide star

Credit: Claire E. Max, UCSC

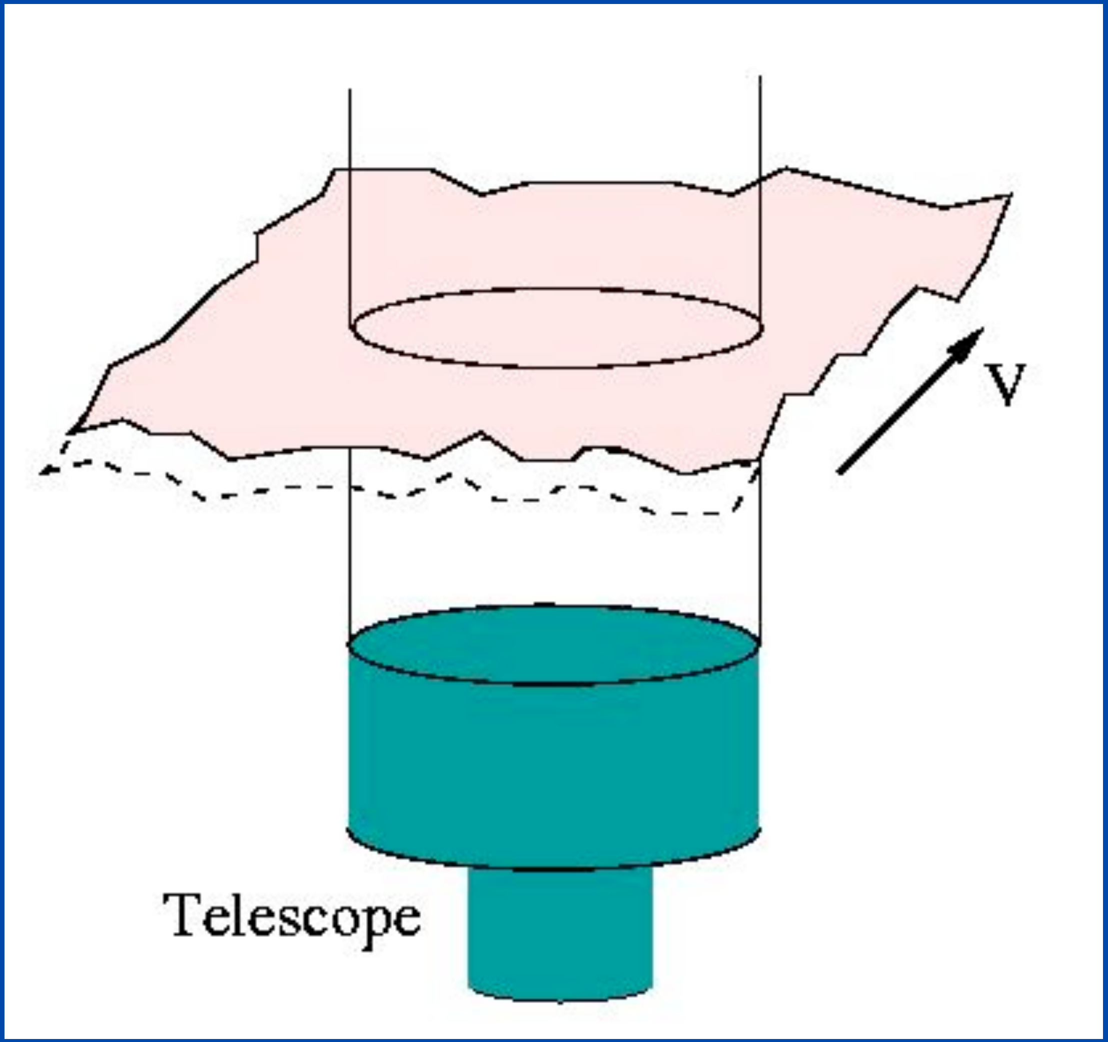
A simplifying hypothesis about time behavior

- Almost all work in this field uses "Taylor's Frozen Flow Hypothesis"
  - Entire spatial pattern of a random turbulent field is transported along with the wind velocity
  - Turbulent eddies do not change significantly as they are carried across the telescope by the wind
  - True if typical velocities within the turbulence are small compared with the overall fluid (wind) velocity
- Allows you to infer time behavior from measured spatial behavior and wind speed:

$$\frac{\partial \vec{u}}{\partial t} = -\vec{u} \bullet \nabla \vec{u}$$

Credit: Claire E. Max, UCSC

Cartoon of Taylor Frozen Flow



- From Tokovinin tutorial at CTIO:
- http://www.ctio.noao.edu/~atokovin/tutorial/

Credit: Claire E. Max, UCSC

Order of magnitude estimate

- Time for wind to carry frozen turbulence over a subaperture of size r_0 (Taylor's frozen flow hypothesis):

$$\tau_0 \sim r_0 / V$$

$1/\tau_0$ = "Greenwood frequency"

- Typical values:
  - λ = 0.5 μm, r_0 = 10 cm, V = 20 m/sec $\Rightarrow$ τ_0 = 5 msec
  - λ = 2.0 μm, r_0 = 53 cm, V = 20 m/sec $\Rightarrow$ τ_0 = 27 msec
  - λ = 10 μm, r_0 = 36 m, V = 20 m/sec $\Rightarrow$ τ_0 = 1.8 sec

- Determines how fast an AO system has to run

Credit: Claire E. Max, UCSC

AO Specifications

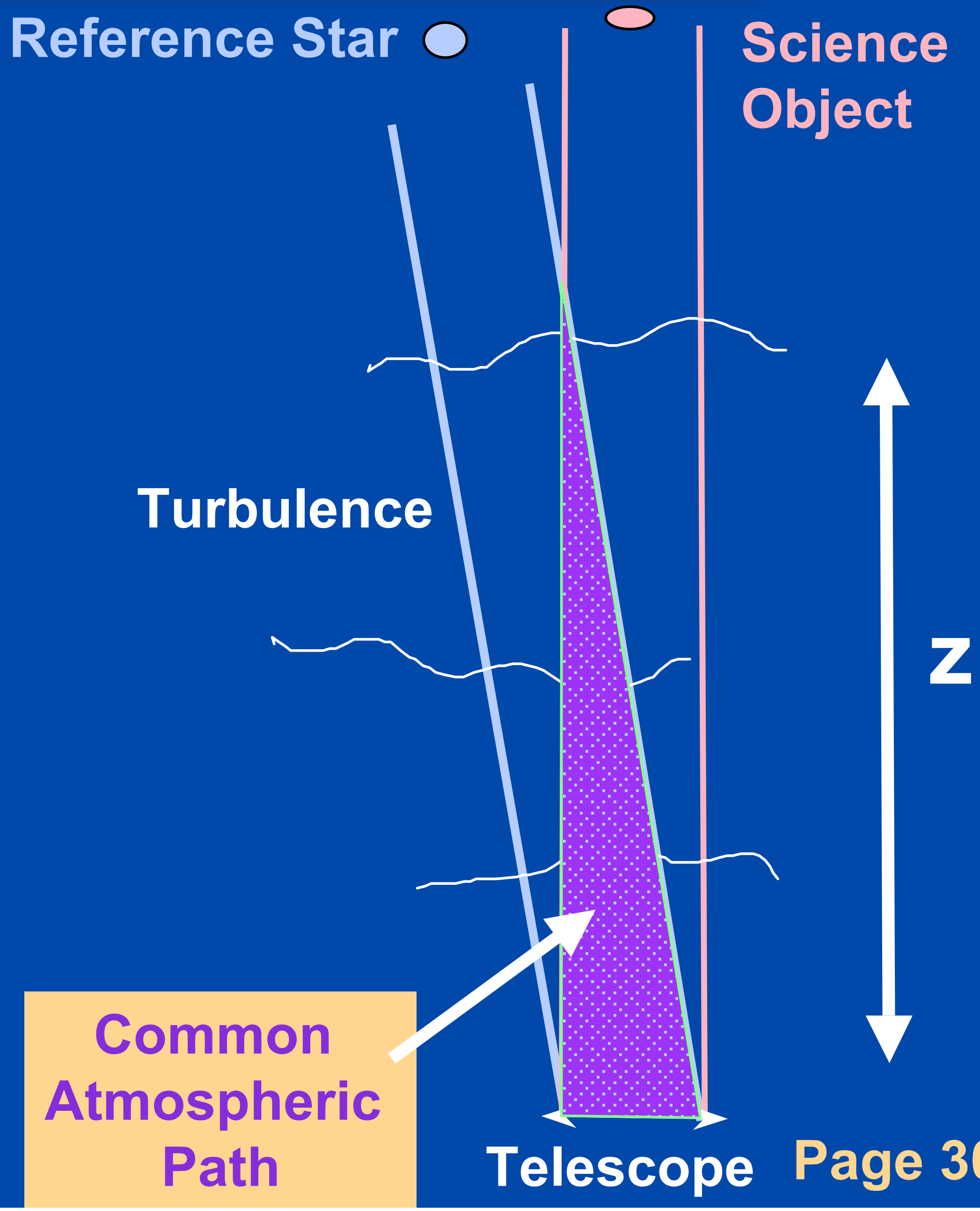
- E.g. 8 m telescope
  - Optical: r_0 ~ 10 cm.
    Number of actuators ~ $(D/r_0)^2$ = 6400
    Timescale ~ 5 msec, frequency = 200 Hz
  - IR: r_0 ~ 50 cm
    Number of actuators ~ 250
    Timescale ~ 26 msec, frequency = 40 Hz
- Greenwood frequency increases strongly at shorter wavelengths.

What determines how close the reference star has to be?

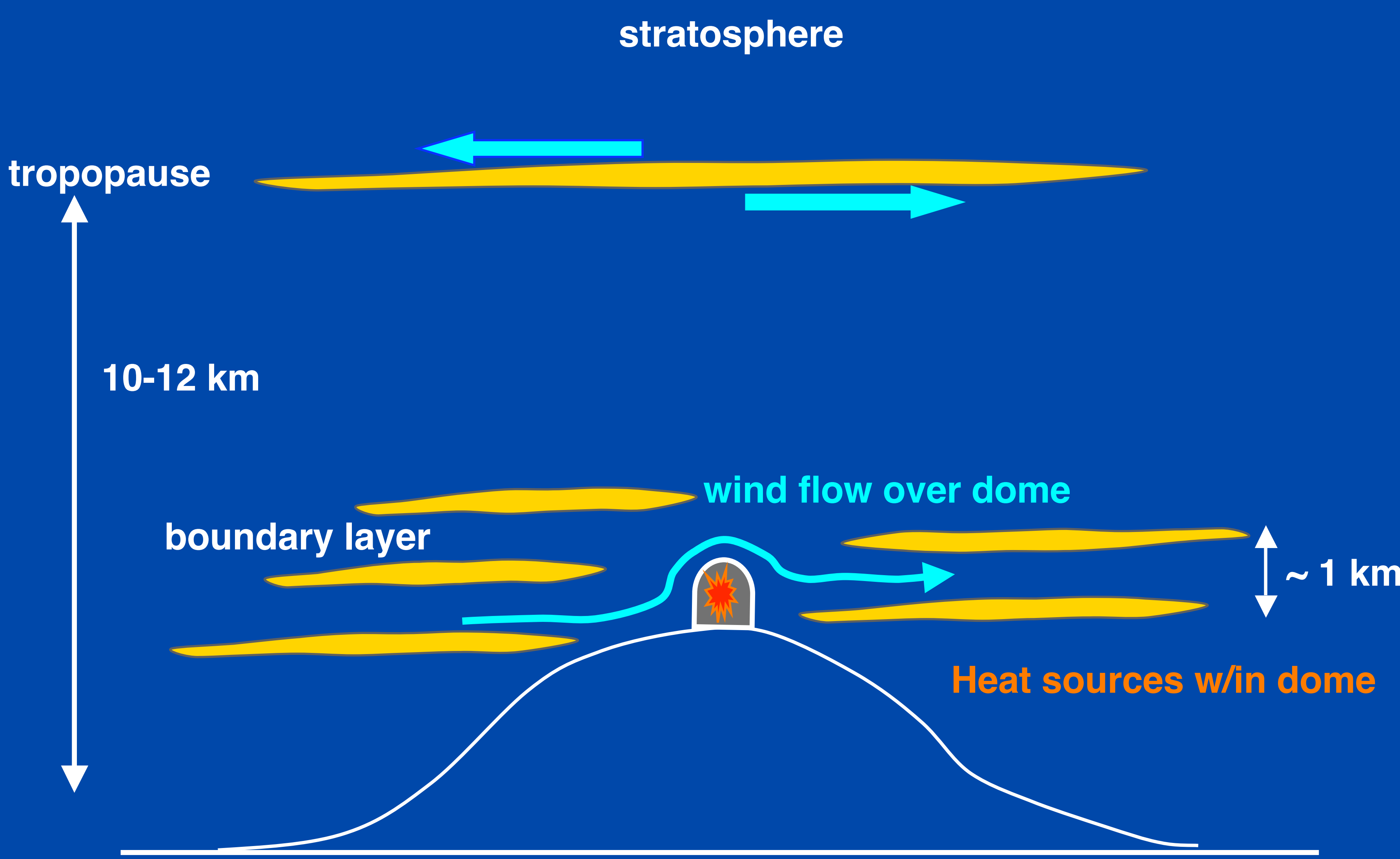
Turbulence has to be similar on path to reference star and to science object

Common path has to be large

Anisoplanatism sets a limit to distance of reference star from the science object



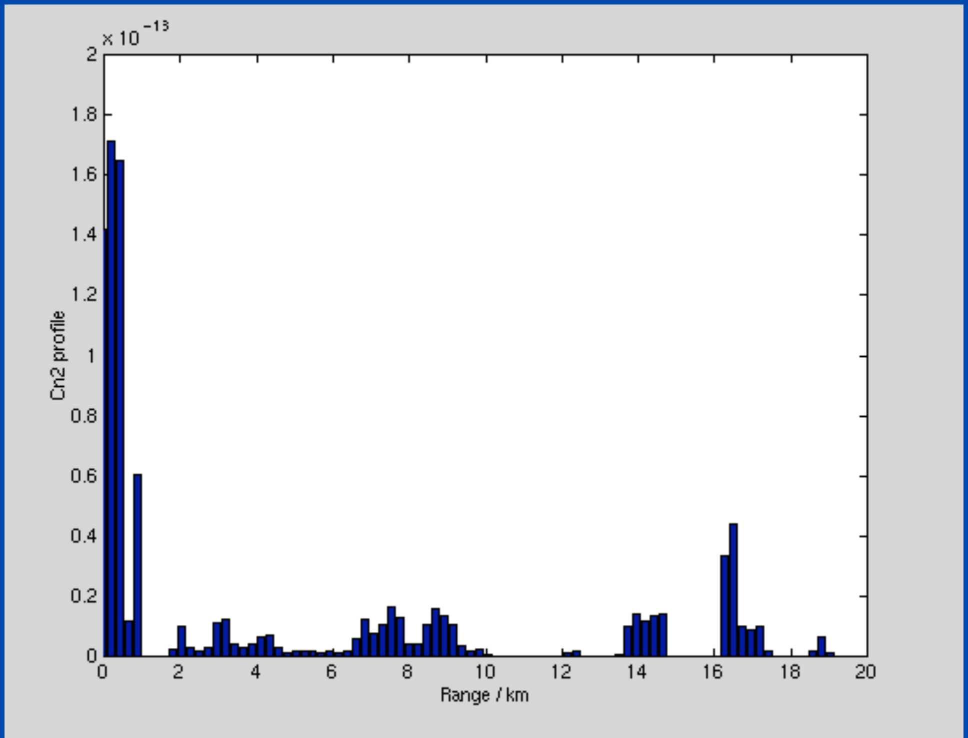
Turbulence arises in many places



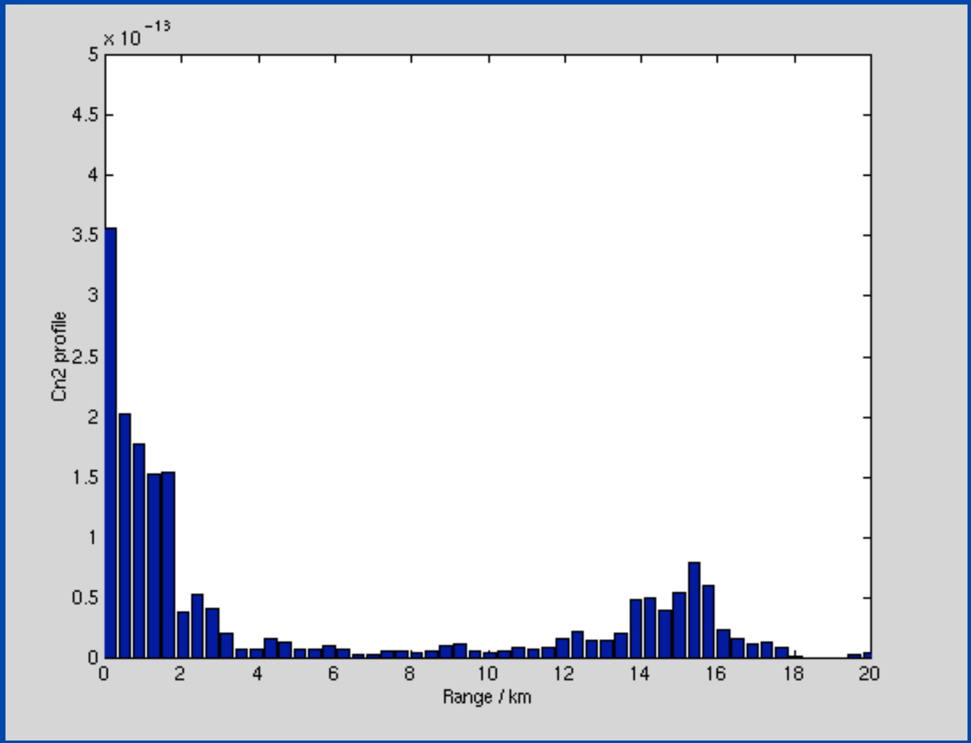
Credit: Claire E. Max, UCSC

Turbulence profiles from SCIDAR

Eight minute time period (C. Dainty, Imperial College)



Siding Spring, Australia



Starfire Optical Range, Albuquerque NM

Credit: Claire E. Max, UCSC

Isoplanatic angle

$$\theta_0 = 0.314 \cos z \left(\frac{r_0}{\overline{h}} \right)$$

h = "mean" altitude of turbulent layers

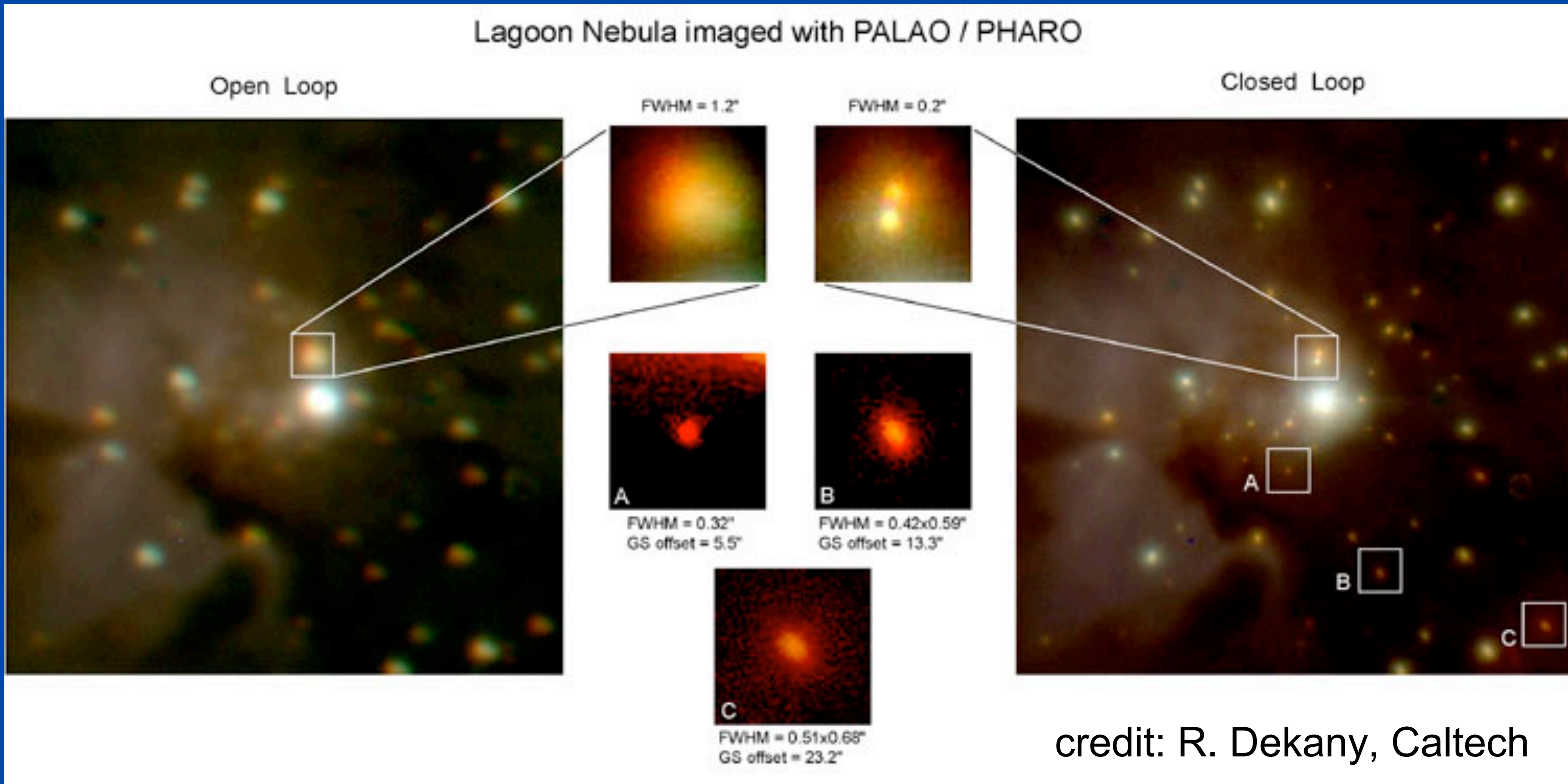
Example:

r_0 = 50 cm, h = 2 km, z=0

θ_0 = 16 arcsec

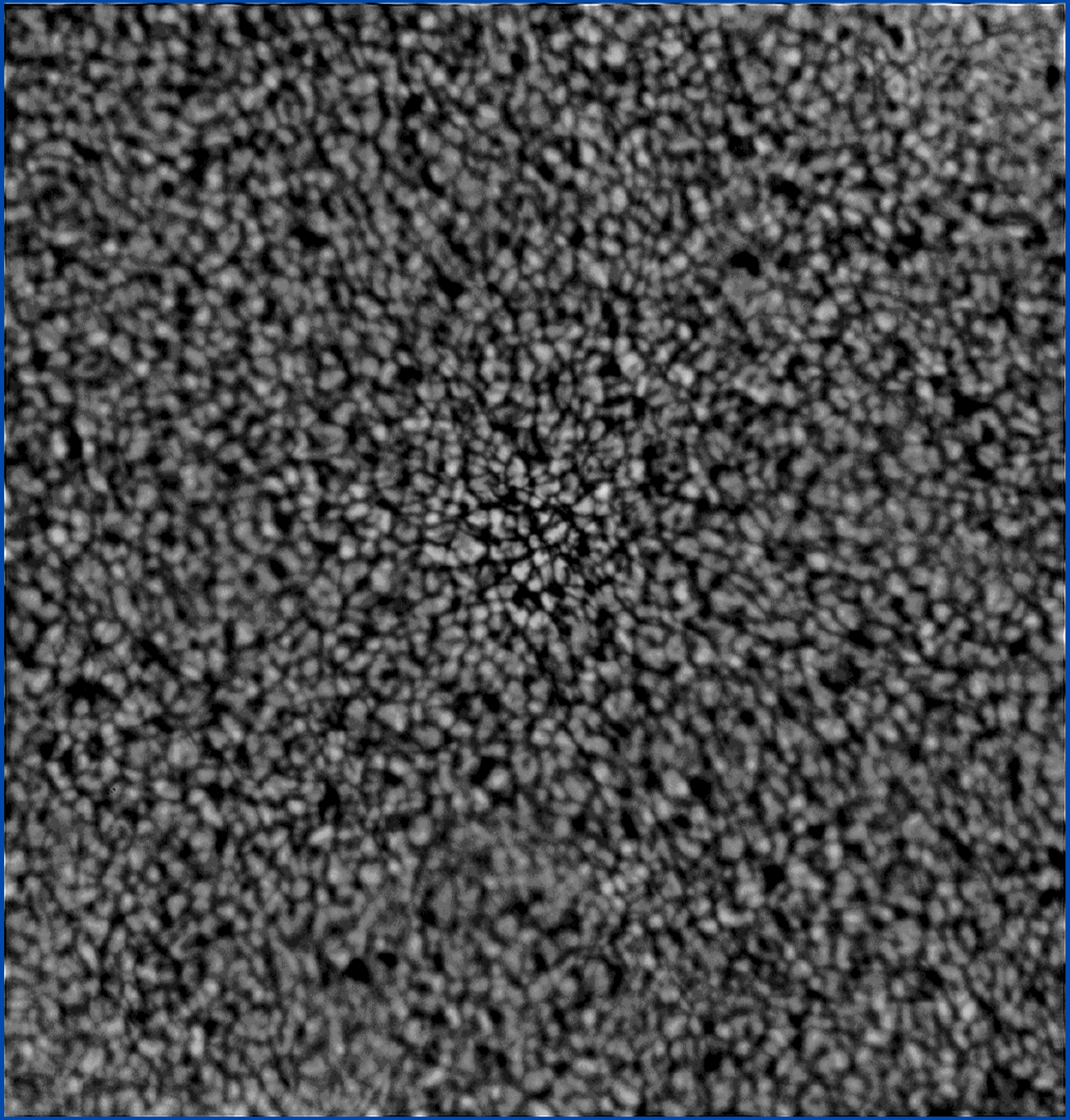
AO systems have a small field-of-view!

Anisoplanatism: how does AO image degrade as you move farther from guide star?



credit: R. Dekany, Caltech

- Composite J, H, K band image, 30 second exposure in each band
- Field of view is 40”x40” (at 0.04 arc sec/pixel)
- On-axis K-band Strehl ~ 40%, falling to 25% at field corner



More about anisoplanatism:

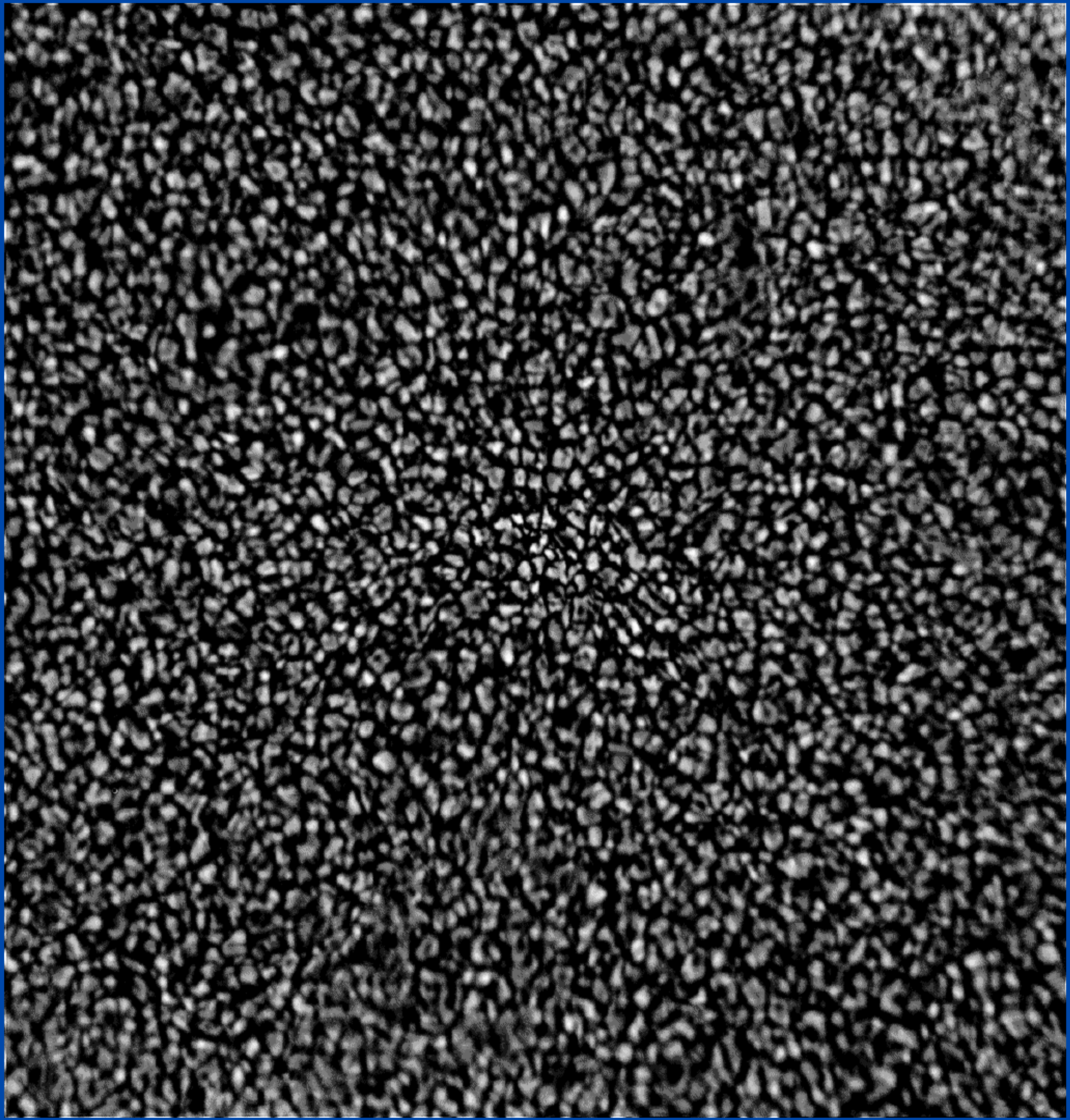
AO image of sun in visible light

11 second exposure

Fair Seeing

Poor high altitude conditions

From T. Rimmele



AO image of sun in visible light:

11 second exposure

Good seeing

Good high altitude conditions

From T. Rimmele

Summary

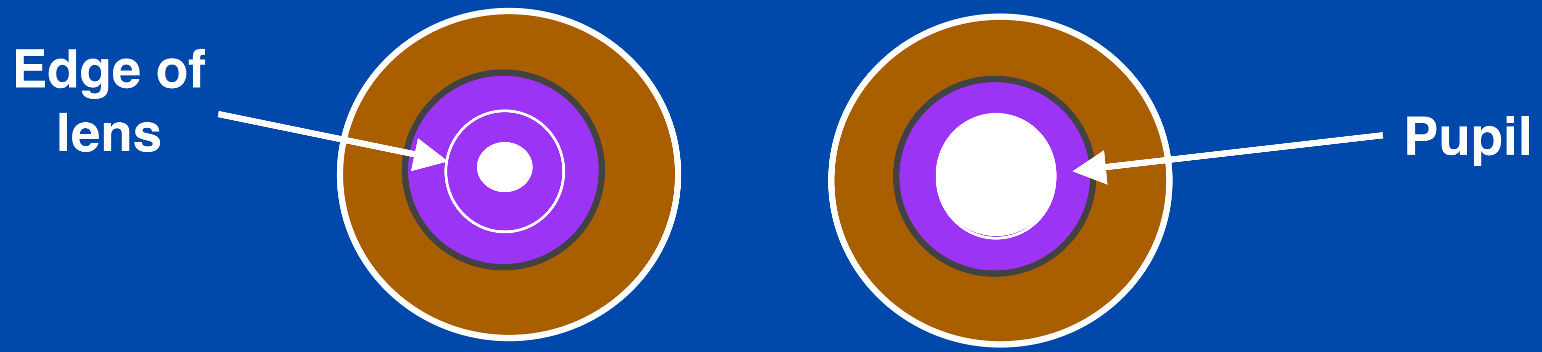
- Adaptive Optics: technique to correct for turbulence in the Earth's atmosphere
- Requires bright *guide star* to sense wavefront errors (can be natural or artificial)
- Much harder at short wavelengths due to:
    - Small r_0 (large number of actuators needed)
    - High correction frequency
    - Small isoplanatic angle

Implications

- Going to shorter wavelengths:
  - r_0 decreases → more actuators
  - τ_0 decreases → higher correction frequency (brighter guide star needed)
  - θ_0 decreases → more difficult to find (natural) guide stars!
- All existing AO systems restricted to the near-IR. Extending AO to the optical is extremely challenging!

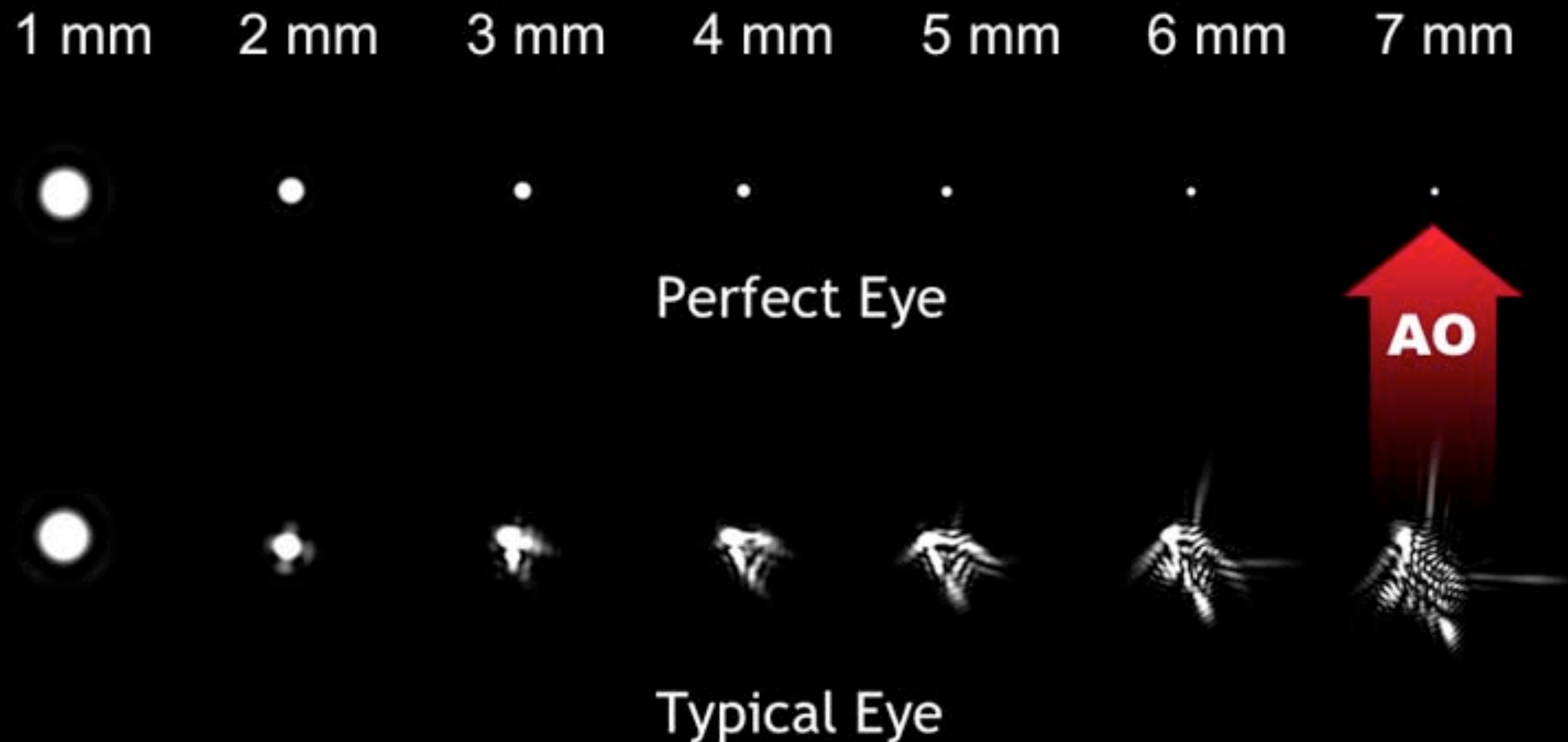
Why is adaptive optics needed for imaging the living human retina?

- Around edges of lens and cornea, imperfections cause distortion
- In bright light, pupil is much smaller than size of lens, so distortions don't matter much
- But when pupil is large, incoming light passes through the distorted regions



- Results: Poorer night vision (flares, halos around streetlights). Can't image the retina very clearly (for medical applications)

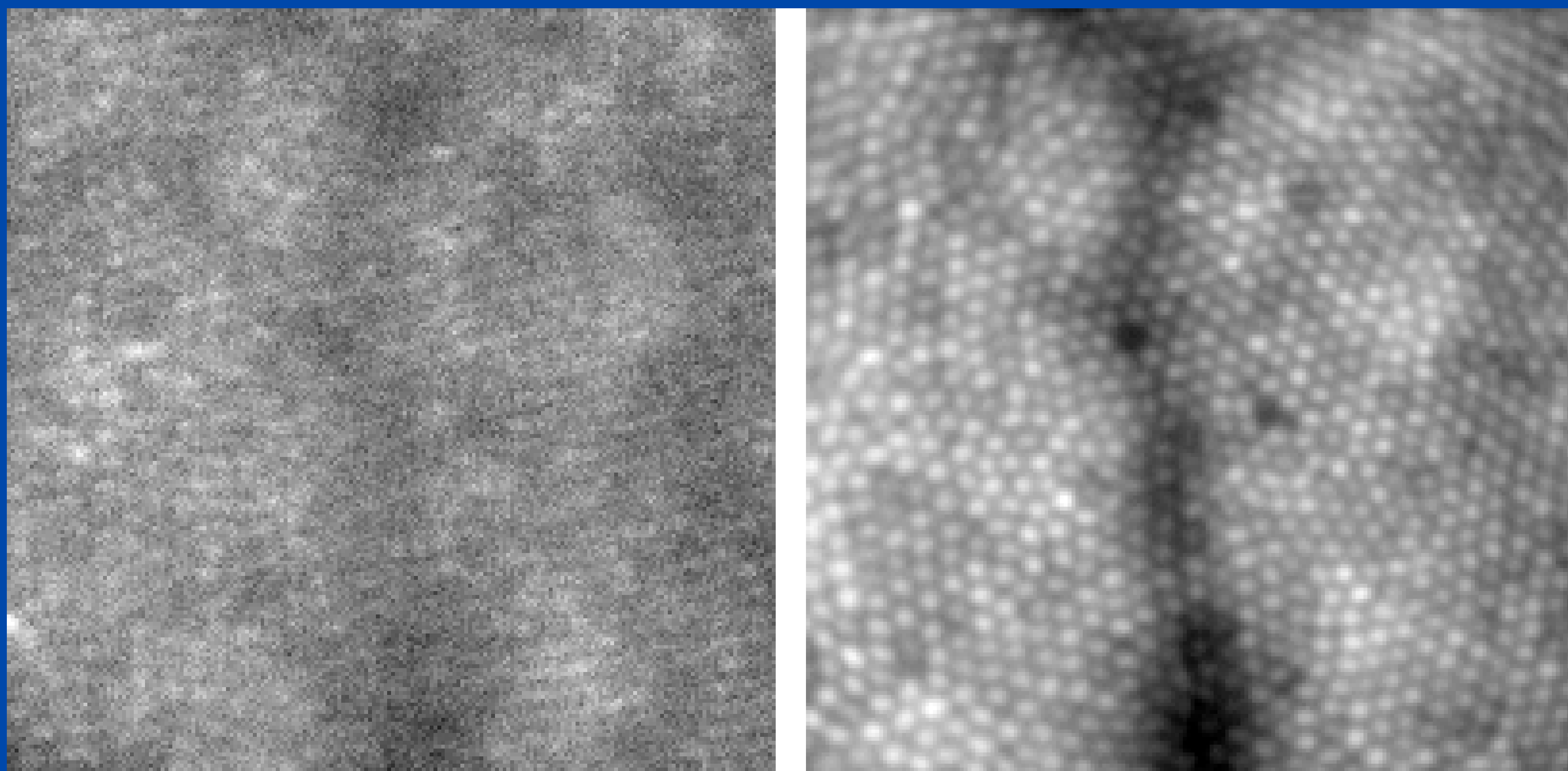
Point Spread Function vs. Pupil Size



C. of Austin Roorda

Adaptive optics provides highest resolution images of living human retina

Austin Roorda, UC Berkeley



Without AO

With AO:
Resolve individual cones
(retina cells that detect color)