Assignments, Cosmology 2012/2013, Week 47/48 (S. Larsen)

These are the assignments for the 8th week of the course *Cosmology*.
Hand-in assignment: Problem 8.1

Apparent and absolute astronomical magnitudes

In the class we briefly discussed the magnitude scale that is still in common use among astronomers. There is an important distinction between *apparent* and *absolute* magnitudes. The former quantify how bright stars (or other objects) appear in the sky to an observer, while the latter are a measure of the intrinsic luminosity of an object.

Brighter objects have numerically *smaller* magnitudes. Vega, one of the brightest stars in the sky, has an apparent (visual) magnitude of $m_V = 0$, and even brighter objects (e.g. the Sun, Moon, Venus) have negative apparent magnitudes. The faintest stars visible to the unaided eye have $m_V \approx +6$. In general, the difference between the apparent magnitudes of two stars (m_1 and m_2) is related to the fluxes of the radiation (F_1 and F_2) as

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right) \quad (1)$$

Thus, a difference of 5 magnitudes corresponds to a flux ratio of a factor of 100. A difference of one magnitude corresponds to a flux ratio of $10^{1/2.5} = 2.512$. This "flux ratio" was defined by Norman Pogson in 1856 after quite some debate among astronomers as to what would be the appropriate definition. If the magnitude $m_{\rm Ref}$ and flux $F_{\rm Ref}$ of a reference star are known, we can write the magnitude scale as

$$m = -2.5 \log_{10} F + zp \quad (2)$$

where zp is a zero-point,

$$zp = m_{\rm Ref} + 2.5 \log_{10} F_{\rm Ref} \quad (3)$$

Absolute magnitudes are usually denoted with capital letters to distinguish them from apparent magnitudes. This raises the issue of confusion with masses. Here we will use caligraphic font for absolute magnitudes, $\mathcal{M}$, and standard font for masses, M. The absolute magnitude is defined as the apparent magnitude that would be observed if the object we placed at a reference distance of 10 pc. The Sun has an absolute visual magnitude of $\mathcal{M}_V = +4.8$, the brightest individual stars have $\mathcal{M}_V \approx -9$, and Type Ia supernova reach $\mathcal{M}_V \approx -19$ at peak brightness. A Type Ia supernova is thus about 3×10^9 times more luminous than the Sun at visible wavelengths! The relation between apparent magnitude and flux is replaced with an equivalent relation between absolute magnitude and luminosity L:

$$\mathcal{M} = 2.5 \log_{10} L + \text{const}$$

The difference between absolute and apparent magnitude, the *distance modulus*, is then a measure of the distance: In the absence of any obscuring matter, the flux is $F = L/4\pi D^2$ for distance D, so that

$$\begin{aligned} m - M &= -2.5 \log_{10}\left(\frac{L/4\pi D^2}{L/4\pi(10\text{pc})^2}\right) \\ &= 5 \log_{10} D(\text{pc}) - 5 \end{aligned}$$

Finally, it should be noted that magnitudes (and fluxes, luminosities, etc.) often refer to specific wavelength regions. For example, the Johnson *UBV* system makes use of three filters (U=Ultraviolet, B=Blue and V=Visual), which transmit wavelengths near 360 nm, 440 nm and 550 nm, respectively.

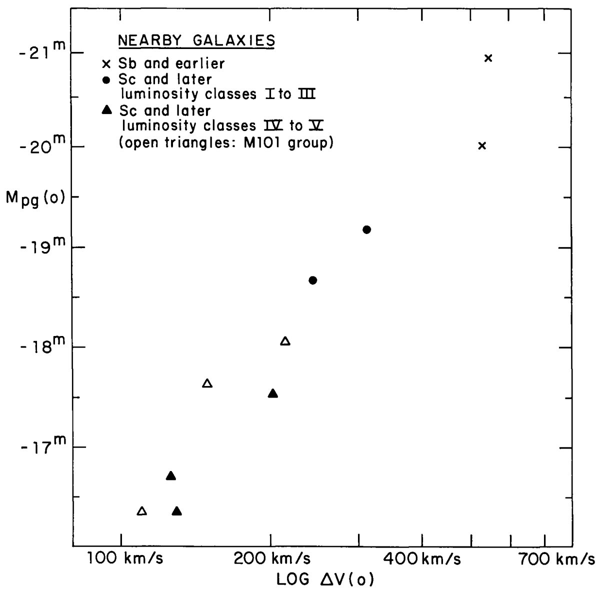
Problem 8.1

The cosmological distance ladder includes a very wide range of distance indicators. Some of these have a relatively solid theoretical foundation while others are of a more empirical nature. Even though the Hubble constant is now determined with very good accuracy, the peculiar motions of galaxies with respect to the Hubble flow can be substantial well beyond distances where the classical calibrators such as Cepheids are observable, and it is therefore still of interest to determine distances by other means, independently of Hubble's law. Supernovae are observable at cosmological distances, but the chance that one appears in any given galaxy at a given point in time is small.

The *Tully-Fisher relation* (see figure below) is a popular distance indicator for spiral galaxies. It relates the luminosity (or absolute magnitude) of a galaxy to its rotation speed, the latter originally coming from radio observations of atomic hydrogen (HI). It may be written as

$$\mathcal{M} = -10 \log_{10} v_{\rm circ} + \text{const}$$

where $\mathcal{M}$ is the absolute magnitude and $v_{\rm circ}$ is the rotation speed. Given the flat rotation curves observed for spiral galaxies, $v_{\rm circ}$ is a fairly well defined quantity observationally. If $v_{\rm circ}$ is observed, then the intrinsic luminosity of the galaxy follows from the Tully-Fisher relation, and the distance can then be determined by comparing the observed flux with the luminosity.



The Tully-Fisher relation (Tully & Fisher 1977)

- Show that the Tully-Fisher relation can be understood if a few simple assumptions are made:
  - The measured rotation speeds represent circular motion
  - Spiral galaxies have a constant *mass-to-light ratio* (Υ)

- They also have a constant *surface brightness* (luminosity per unit area), μ

Hint: start by showing that these assumptions lead to a relation of the form $L \propto v_{\rm circ}^4$, and then rewrite this in terms of magnitudes.

- The figure suggests that a linear fit to the data points would pass through $(v_{\rm circ}, \mathcal{M}) = (100\,{\rm km\,s^{-1}}, -16)$. Now, a galaxy is observed to have $v_{\rm circ} = 200$ km s^{-1} and an apparent magnitude of $m = 11$. What is the distance of the galaxy?

- It turns out that spiral galaxies do not, in fact, have a constant surface brightness. Zwaan et al. (1995) found that *low surface brightness* galaxies, whose surface brightness is 4 times lower than for "normal" galaxies, follow the same Tully-Fisher relation as galaxies with high surface brightness. What can you say about the mass-to-light ratios of these LSB galaxies?

Problem 8.2

Spiral galaxies are observed to have flat rotation curves ($v_{\rm circ} \approx$ const) to large radii.

- Assuming the matter is distributed in a spherically symmetric dark matter halo, show that the flat rotation curves imply a density profile of the form $\rho(r) \propto r^{-2}$.