Tutorial 4: Spectrum of a Jet

This tutorial is based on the lecture on Jets. Most of the needed equations and the equations here are from the lecture notes.

Note that the last part of the lecture notes on the jet are in *German*. An english version is available on: http://www.mpifr-bonn.mpg.de/staff/hfalcke/AGN/jet-engl.ps

Here we assume we have a maximal jet (i.e. $c_s = c/\sqrt{3}$) and the electron's distribution have a powerlaw distribution: $dN/d\gamma = K\gamma^{-p}$. γ_1 and γ_2 are the minimal and maximum Lorentz factor of the electrons. For a maximal jet, minimal Lorentz factor γ_1 is about 300.

- Derive the magnetic field $B(z)$ in the jet at the point $(r,\ z)$, starting from 1) equipartition of particle's energy, and 2) assuming the mass accretion rate is constant. (please read the notes if you don't know how to start)

- The absorption coefficient (dimension: m^{-1}) *describes* the loss of intensity in a beam as it travels a certain distance in a medium. After a certain distance, the photons (of the beam) will be absorbed ("medium's opacity is high"). The quantity to describe a medium's *opacity* is the optical depth τ (dimensionless). When $\tau > 1$, we say the medium is optical *thick*, and for $\tau < 1$ the medium is optically *thin*. If the side of the jet is approached as plane-parallel, we have:

  $$\tau = 2r\alpha/\sin i$$

  where r is the radius, α is the absorption coefficient and i is again the inclination angle. The absorption coefficient for the synchrotron radiation, is given by:

  $$\alpha_{cgs} = 4.5 \times 10^{-12} \frac{1}{\ln(\gamma_2/\gamma_1)} B_G^4\, \nu_{GHz}^{-3} \qquad \mathrm{cm}^{-1} \tag{1}$$

  $$\alpha_{si} = 4.5 \times 10^{6} \frac{1}{\ln(\gamma_2/\gamma_1)} B_T^4\, \nu_{GHz}^{-3} \qquad \mathrm{m}^{-1} \tag{2}$$

  where the magnetic field is given in Gauss or Tesla, and the frequency is in 10^9 Hz (this expression is same as in lecture notes but with k, set to unity)
  According to the definition of τ, the radiation we see comes from the layer where: $\tau \leq 1$. Derive the expression for z where $\tau = 1$

- Synchrotron emission is emission by charged particles moving in a magnetic field. The emissivity is given by:

  $$\epsilon_{cgs} = 5.5 \times 10^{-19} \frac{1}{\ln(\gamma_2/\gamma_1)} B_G^{3.5}\, \nu_{GHz}^{-0.5} \qquad \mathrm{erg/(s\ \ cm^3\ Hz)} \tag{3}$$

  $$\epsilon_{si} = 5.5 \times 10^{-6} \frac{1}{\ln(\gamma_2/\gamma_1)} B_T^{3.5} \nu_{GHz}^{-0.5} \qquad \mathrm{J/(s\, m^3\ Hz)} \tag{4}$$

  Calculate spectrum from one pancake-slice of this jet (take $p = 2$), and then the integrated spectrum of the jet by adding all the slices together.